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BEYOND SABINE: INVESTIGATING THE ACOUSTICAL PHENOMENON OF REVERBERATION USING ROOM MODAL DECAY

Panos Economou, Costas Economou and Panagiotis Charalampous

P.E. Mediterranean Acoustics Research & Development Ltd, Cyprus
email: panos@pemard.com

The definition of Reverberation Time (RT) is based on a phenomenon which is far more complex than the definition is able to explain. Sabine in his effort to solve an engineering problem at first tackled the problem empirically and then tried to justify the findings of his empirical work within a theoretical framework by the use of many assumptions. Ever since then, the acoustical community tried to better understand the parameter described as RT confined within the original concept of Sabine which begins to converge on Sabine's idea of RT above the Schroeder frequency. This paper tries to redefine the way we investigate this acoustical phenomenon without Sabine's assumptions which are: (a) That an isotropic and homogeneous acoustical field exists within a room, (b) developed by sound reflections only, (c) due to plane wave sound propagation, (d) RT can be calculated using the borrowed concept of the Mean Free Path from particle physics and optics, (e) the use of sound absorption coefficients and (f) energy summation. This paper studies room modal decay using (i) spherical wave propagation, (ii) surface impedance (iii) a sound field formed by sound reflections and diffractions and (iv) sound pressure summation preserving wave front phase information.

Keywords: Reverberation, Modes, Decay, Diffraction, Impedance

1. Introduction

Wallace Sabine more than a century ago in his effort to apply remedial acoustical measures at the Fogg Lecture Hall [1], part of the Fogg Art Museum in Boston USA, coined the term Reverberation Time (RT) to described the time for sound to die out in a space. At the time no engineering methods were available for the measurement of the acoustical qualities of rooms nor were there any calculation methods to predict them. Therefore he needed to experiment to find out what controls the acoustics of a room. His research concluded that an important controlling factor is Reverberation and the parameter for its control is Reverberation Time (RT). His work produced an empirical formula which can only be applied under the following assumptions:

a) That an isotropic and homogeneous acoustical field exists within a room,
b) developed by sound reflections only,
c) due to plane wave sound propagation.
d) RT can be calculated using the borrowed concept of the Mean Free Path from particle physics and optics,
e) the use of sound absorption coefficients and
f) energy summation.

At the same time he expressed his findings based on fundamental principles of physics [1] and ever since the acoustical community has been debating RT. Based on his findings he was then, later on, able to design the Boston Symphony Hall which is considered to be one of the top three highly rated halls for their acoustics.

There are many subtle yet consequential variations of the definition of RT leading to a lot of interpretations about what RT stands for. According to Wallace Sabine [1, pages 41-43] RT is the time required for sound to decay to inaudibility (which he arbitrarily chose to be), a one billionth (-60dB) of the initial intensity. The current ISO 3382:P1:2009 [2], gives the following definition: "reverberation time, T, duration in s required for the space-averaged sound energy density in an enclosure to decrease by 60 dB after the source emission has stopped". Until 2009 when ISO 3382:1997 [3] was superseded the following definition applied: "reverberation time, T: Time expressed in seconds, that would be required by the sound pressure level to decrease by 60 dB, at a rate of decay given by the linear least-squares regression of the measured decay curve from a level 5 dB below the initial level to 35 dB below." The latest standard also mentions two methods by which to excite the room, the interrupted and the impulse method, while for the estimation of the decay rate the least squares method was recommended and supplemented with the integrated impulse response method. Additionally ISO 354:2003 [4], defines RT as follows: "reverberation time T time, in seconds, that would be required for the sound pressure level to decrease by 60 dB after the sound source has stopped". The plethora of so many definitions and methodologies create confusion and allow various interpretations of results.

2. Theory

2.1 Sound decay constant and Reverberation Time

Assuming that RT is considered the time for sound to decay by 60 dB (Sabine's initial definition), then it may be shown [5] that $RT=3\ln(10)/\langle\delta\rangle$, where $\langle\delta\rangle$ is the average decay constant.

The choice of a decay constant, average or not, is a matter of the approach used in solving this equation. The statistical approach by Sabine, Eyring and others who followed [6], deal with sound as rays or particles and is related to the Mean Free Path and the average sound absorption coefficient of the room surfaces in the various octave bands. In this approach $\langle\delta\rangle = \langle\alpha\rangle Sc/8V$ (where V=room volume, S=room surface area, $\langle\alpha\rangle$=average sound absorption coefficient, c=speed of sound). RT may also be obtained from the slope of the sound pressure level (energy summation) decay plot.

An alternative approach is the wave based analytical solution [5] where the modal decay constant is given below.

$$\delta_n = -\frac{c}{2l_m}\ln|R(\omega_n)|, \ (1) \quad l_m = \frac{l_0}{q_x + q_y + q_z}, (2) \quad q_x = \frac{l_0}{l_x}\cos\varphi_x \ (3)$$

where l_m is the mean free path [7] of a particular mode m while the mode m is represented by the indices n_x, n_y and n_z, l_0 is total path of mode m, $R(\omega_n)$ is the angle dependent plane or spherical wave reflection factor (the major difference between these factors is that the latter is source receiver distant dependent), ω_n is the angular frequency of the mode, l_0 is the total path length and $q_{x,y,z}$ are the number of reflections in the x, y and z directions. By applying the modal decay constant, one is able to calculate RT without having to resort to graphical methods.

Nowadays measuring RT utilizes technology of the highest precision yet Round Robins [8] repeatedly have shown huge discrepancies among results; why? Because there is no clear understanding of what it is expected from RT measurements. Furthermore, even though human perception of RT changes with its sound decay shape, calculation or measurements results do not provide any indications to such varying sound decay effects. No wonder then one may come to different results for RT, depending on the approach (statistical or otherwise) and the method of calculating decay time (graphical or otherwise).

Even though Sabine was a pivotal milestone in the evolution of acoustics, it is about time the acoustical community moved beyond Sabine's era. It is well documented in hundreds of papers and books that this approach is a special case only valid above the Schroeder frequency. The statistical approach is not a general case in dealing with rooms acting as instruments having infinite numbers of resonances. To deal with such a task there is a need to go back to wave based approaches.

2.2 Reverberation is a discontinuous phenomenon.

RT being a parameter associated with free vibration, is not a continuous parameter. This applies in all three domains, frequency, space and time. Only forced vibrations have continuity in all three domains. Bearing this concept in mind one is able to disentangle some of the mysteries related to RT. Take for example RT versus frequency. It is always presented in 1/1 and 1/3 octave bands giving the impression that there are always RT data in any part of the spectrum. This however is not true as Schultz points out in Fig. 1, of his paper [9] reproduced below courtesy of JASA, since the authors could not put it in any better!

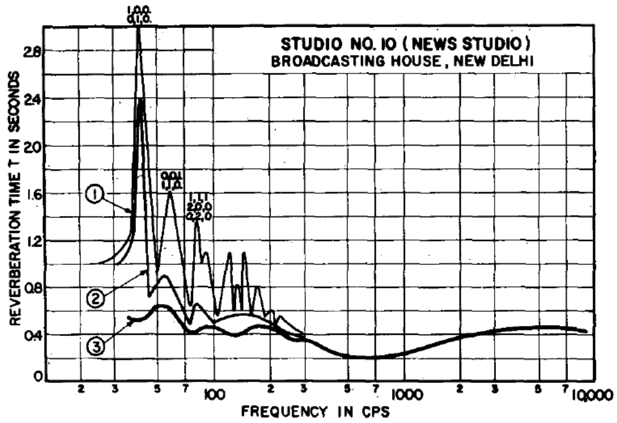

FIG. 1. “Reverberation Time” (?) of small broadcast studio.

Figure 1: Courtesy of JASA, from Theodore Schultz’s paper [9]. The following is an extract from his paper explaining the above curve “The lower curves were obtained by adding progressively more sound absorptive material to the room. The high peaks tend to be levelled off as more absorption is added so that in the lowest curve, which applies to the case with the greatest amount of absorption, the peaks are nearly unnoticeable”

He carries on as follows: “*Now, to begin with, this implies very peculiar behaviour for the absorptive properties of the bare walls of the room: namely, at the frequencies of the lowest resonant modes of the room, there is very little absorption by the walls and hence the reverberation time is high; whereas at frequencies only a few cycles away from the resonance there is a much shorter reverberation time and, therefore, the walls must be much more absorptive. Presumably if the room were a different size, the absorption dips (hence, reverberation peaks) exhibited by the walls would occur at different frequencies corresponding to the new modes of the room. One can hardly believe that the ordinary rigid, bare walls of a room could show such wide (and variable!) differences in absorption within such a narrow frequency range. What, then, is the trouble with these curves? Surprisingly, there appear to be at least three methods of measurement which would give this kind of wrong answer!*”

It is a well-known fact that at specific locations, at the antinodes, there can be no resonance effects. RT being the result of room resonances, can be recorded at only certain locations and frequencies where room natural vibrations (room modes) can be excited. Therefore it is a discontinuous parameter. This is in contrast to forced vibrations where no matter when one finds oneself in a room there would be sound even though resonant frequencies sound much louder. Finally, RT describes the time it takes for a room mode to decay. Therefore in terms of time it is again discontinuous. A waterfall plot in Fig. 2, shows this effect in the frequency-time domain. If you follow time=200ms, for example, one can realize that it is only at certain frequencies that one comes across sound. These regions are characterized by the frequencies related to the Quality Factor (QF) found in any resonant system. The higher RT is, the sharper the bell shape it describes or alternatively, the lower RT is the more nearby frequencies and locations are excited.

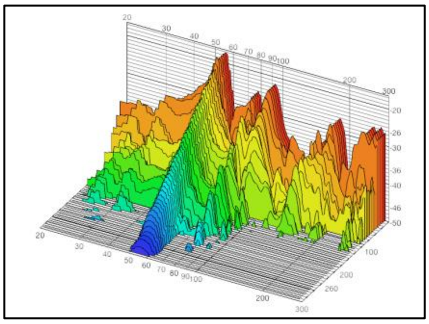

Figure 2: A waterfall plot showing a decaying mode at 50 Hz. It also shows the QF (bell shape) of the decaying mode. It is apparent from the plot that once the source has stopped, after 200ms there is no sound other than the 50 Hz mode, which is called Reverberation and its decay signifies RT (courtesy of AFMG - Ahnert Feistel Media Group).

2.3 Measuring and Predicting RT

Schultz [9] discusses the problems associated with measuring RT, therefore, this paper will only focus on predicting RT. The proper starting point is to deal with RT as a resonant parameter since all other disciplines (mechanical, electronics, optics etc) have very well defined models to deal with such linear time invariant systems. In simple resonators what is of prime importance to be transferred as knowledge in the realm of acoustics are the following:

1. The amplitude at resonance, when the forcing frequency coincides with the natural frequency of the system is governed by the amount of damping present in the system (in terms of room acoustics, sound absorbing material in a room).
2. The higher damping is, the wider the spread of the effect of resonance in terms of frequency. QF embodies all these in the Eq. (4) given below.
3. Wherever there is resonance there is a phase reversal at a slope determined by damping.

Hunt [10] applied these principles to extract the modal decay constant from the frequency response between a source and a receiver in a cavity, from which he was able to extract the sound absorption coefficient in terms of frequency and angle of incidence. From the decay constant he was able to calculate RT.

Elaborating a bit more, in an enclosed cavity the resonance frequency of a room mode will also excite to some extent the surrounding frequencies of the resonant frequency. The extent to which the surrounding frequencies are excited will depend on the Quality Factor of the resonance frequency which is given below in Eq. (4) while the shape of the peak may be modelled by the normal distribution described by Eq. (5):

$$QF = \frac{\omega_n}{\Delta\omega} = \frac{f_n}{\Delta f} = \frac{\pi f_n}{\delta_n}, (4)\ F(f) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(f-f_{mean})^2}{2\sigma^2}}, (5)\ \Delta f = FWHM = \frac{\delta_n}{\pi} = 2\sqrt{2\ln 2}\,\sigma\ (6)$$

Where Δf is equivalent to the Full Width Half Maximum (FWHM) of the resonance peak, $f_n = f_{mean}$ is the mean of the distribution and σ is the standard deviation. Equations (5) and (6) are found in [11]. By substitution the result is,

$$F(f) = \frac{2\sqrt{\pi \ln 2}}{\delta_n} e^{-\frac{(\pi)^2 2\ln 2 (f-f_n)^2}{\delta_n^2}}, (7)\ \ RT(f) = RT_n e^{-\frac{(\pi)^2 2\ln 2 (f-f_n)^2}{\delta_n^2}}\ (8)$$

In Eq. (8) the scaling factor of Eq. (7) is substituted by $RT_n = 3ln(10)/\delta_n$ so that the normal distribution is scaled to the value of RT_n.

3. DISCUSSION AND CALCULATION RESULTS

The results below try to highlight differences which stem from the different approaches in acoustics, i.e, plane vs spherical wave propagation, pressure vs energy summation, impedance vs statistical sound absorption coefficients and sound field built by reflections vs reflections & diffractions. The spherical wave reflection factor is a function of the plane wave reflection factor, corrected for the "fuzziness" due to the curvature of spherical waves which spread the reflection points over the reflecting surface. It is thus additionally distant dependent between the source and receiver. With regard to pressure against energy summation, this again is the result of how nature works, i.e., the microphone receives pressure waves combined with their phases and then circuitry turns them into an energy signal [12]. As far as impedance and absorption coefficient are concerned, according to Morse and Bolt [6], absorption coefficients are not a fundamental material parameter, but rather a combination of material and space. They carry on describing surface impedance as, "*its advantage lies in the fact that its measurement can be specified concisely and uniquely and that its value for a given material has a definite meaning no matter what the distribution of sound inside a room". Finally, the acoustical community acknowledges that sound diffraction effects play an important part in the calculation of room acoustics parameters [6]. Please note that for simplicity, all impedances are calculated based on Delany and Bazley's one parameter porous impedance model [13].

3.1 Sound field based on Pressure and Energy Summation

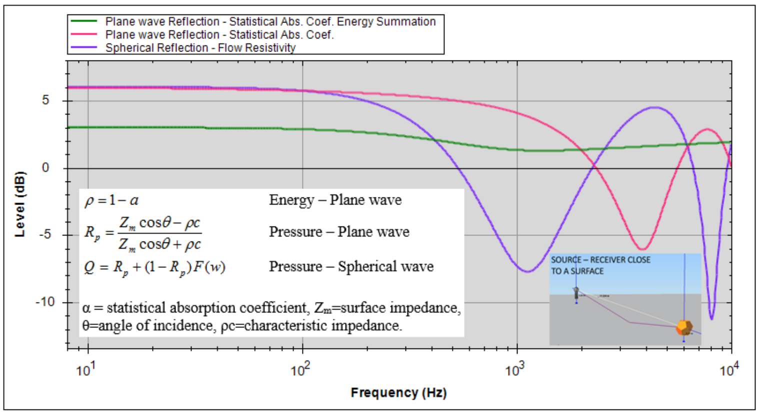

Figure 3: One may see clearly the dramatic differences in sound pressure level brought about by the use of different approaches in acoustics, plane vs spherical wave propagation, pressure vs energy summation and impedance vs statistical sound absorption coefficients.

Figure 3 above shows the calculated results of the sound field at a receiver when a source and receiver are close to a surface of low impedance [14]. It compares energy and pressure summation of sound at the receiver using the corresponding reflection factors (for pressure) and coefficient for energy summation (in this case shown as ρ). For more on the equations see [14]. The flow resistivity used was 200k Pa s m^2.

3.2 Sound absorption coefficient vs angle

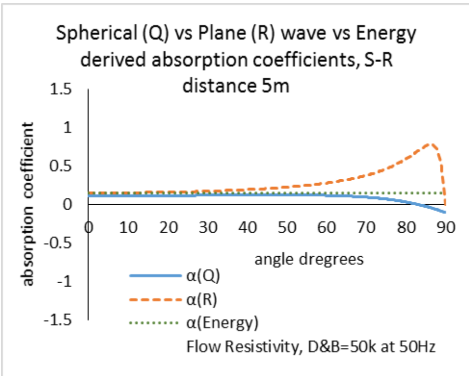

Figure 4: Different methodologies provide different sound absorption coefficients of the same material at different angles.

Figure 4 above shows the effect of the angle of incidence of sound on a material of a flow resistivity of 50k Pa s m^2 at 50 Hz at a source receiver distance of 5m. It compares how the sound absorption coefficient, derived from impedance and expressed in terms of pressure summation (spherical & plane wave propagation) and energy summation (plane wave propagation), varies with angle.

3.3 Sound decay – Plane vs Spherical Reflection Factor

Figure 5 below illustrates sound decay in a small rectangular room with dimensions given in the graph with a uniform flow resistivity of 500k Pa s m^2 at 50 Hz. The spread of the plane wave values is more than the spherical wave. Also and more importantly, the slopes are different.

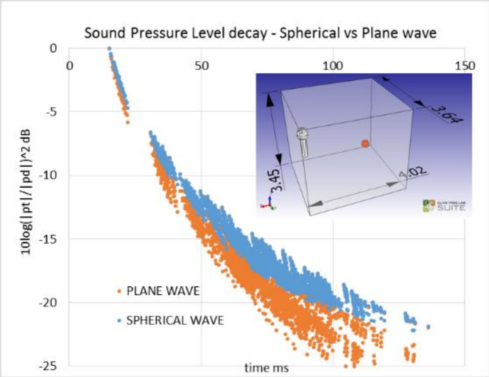

Figure 5: Sound decay in a small rectangular room. Plane vs spherical wave propagation.

3.4 Sound decay using Reflection and Diffraction effects

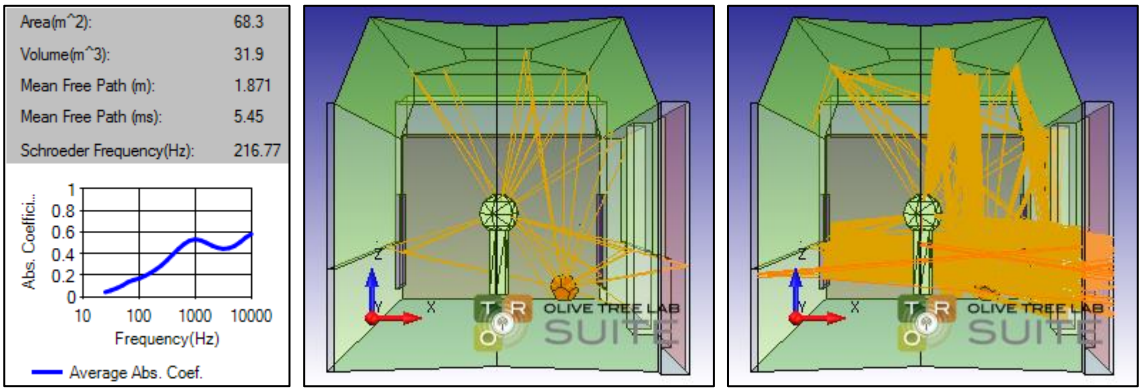

Figure 6: On the left, the acoustical details of a listening room with lots of edges. In the middle sound rays of 1st order diffractions. On the right, rays of 3rd order diffractions. No reflections are shown.

Figure 6 above shows on the left the acoustical details of a heavily sound absorptive small home listening room with lots of edges. The middle picture shows the sound rays of 1st order diffractions while the picture on the right, the rays of 3rd order diffractions, (no reflections are shown). This model was used to calculate the sound decay at 1000Hz at 6th order reflections with and without 3rd order diffractions. Fig. 7 below, compares results showing the effect of calculating sound diffractions in a room. The graphs show sound decay by the integrated method [15] using pressure summation. The level difference between them is most probably due to the few samples taken in time to give meaningful results, however, the graph clearly shows that the scattering effects of sound diffraction from the room edges smooth out the decay (black curve).

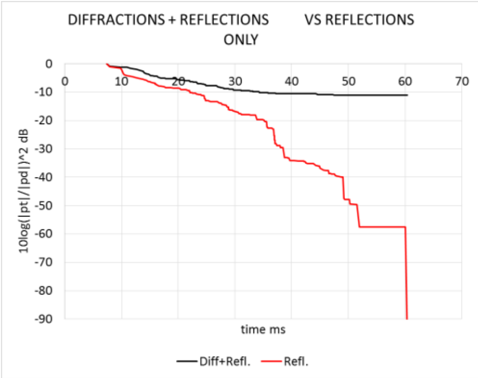

Figure 7: Sound decay with and without sound edge diffractions.

3.5 RT from modal decay constant comparing Q, R, α for High and Low surface impedance

Figure 8 below shows the RT curves for a room of dimensions 7.64x6.16x4.25 meters and surface flow resistivities of 3000k and 200k Pa s m^2. In Fig. 8a it can be clearly seen how prominent the resonance peaks are while Fig. 8b shows that as the surface absorption increases, the resonant peaks begin to overlap with each other and the sound field starts becoming diffused. It can also be seen that the use of energy reflection coefficients significantly underestimates the RT.

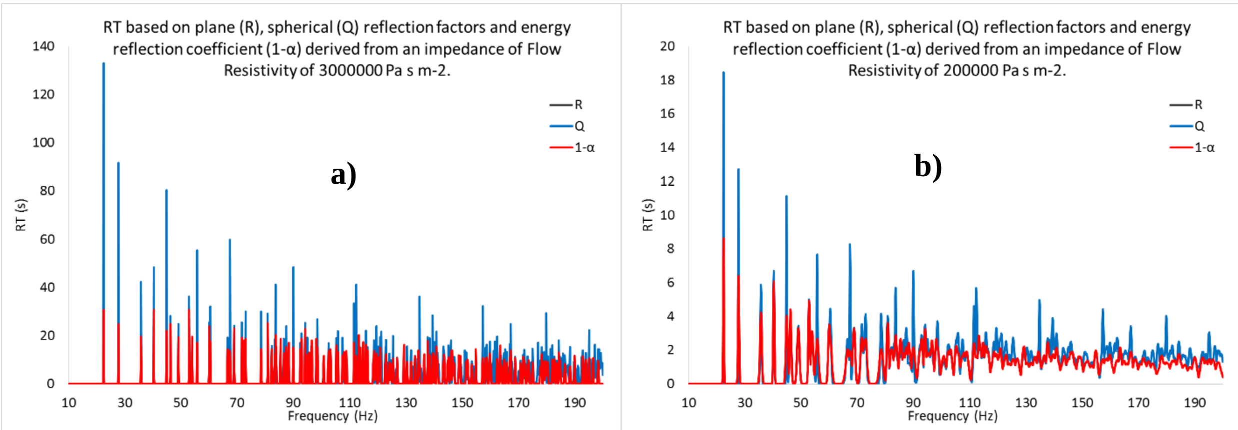

Figure 8: RT graphs for room of dimensions 7.64x6.16x4.25 m and surface flow resistivity of 3000k for (a) and 200k Pa s m^2 for (b). The curves for the R and Q reflection factors are so close they are indistinguishable.

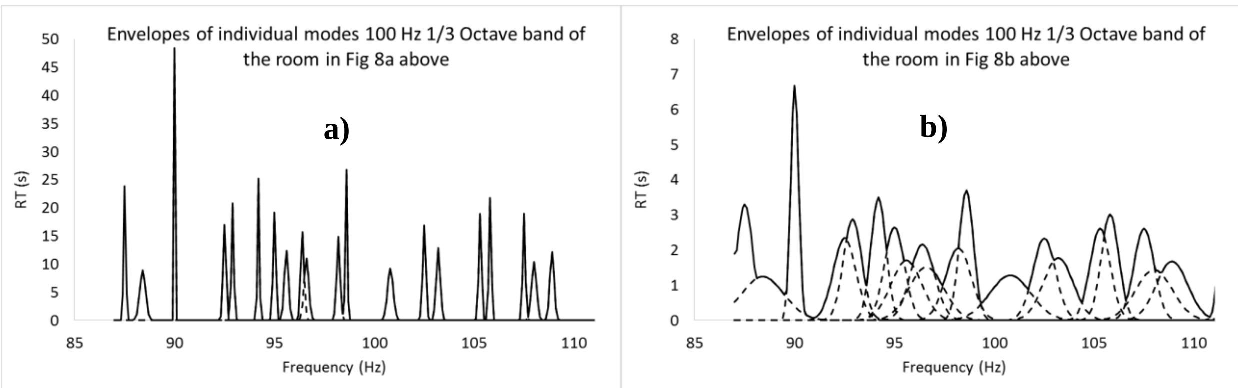

Figure 9: Detailed view of the 100 Hz 1/3 Octave Band frequency range (a) from Fig. 8a above and (b) from Fig. 8b. Spherical reflection factor was used for RT calculations. The dashed curves (-----) represent the RT curves for the individual resonance peaks while the solid curve shows the envelope of the superimposed resonance peaks.

Figure 9 above shows a detailed view of the 100 Hz 1/3 Octave Band frequency range along with the curves of each resonance peak (dashed curves). The difference in the amount of overlap is clearly visible for the cases of a hard surface in Fig. 9a and an absorptive surface in Fig. 9b. Spherical reflection factor was used for RT calculations.

4. CONCLUSIONS AND FUTURE WORK

Morse and Bolt as early as 1944 [6] pointed out that RT depends on the phase of the signal at the time it stops emitting and even if one stops it at a predefined phase, one still has to deal with the source transient response which usually is a loudspeaker diaphragm. This is interpreted by the authors that RT is based on randomness and therefore, it would be hard for anyone to claim to be able to recognise the right RT value of a room, predicted or measured. What this paper proposes is that modal RT calculations are based on fundamental principles and parameters such as the nature of spherical waves, the impedance of surfaces, complex pressure summation and diffraction effects, thus removing the need for as many assumptions as possible from its premise.

Reverberation is not a continuous process in any domain - frequency, spatial or time. It is a resonant phenomenon and as such can only be related to resonant frequencies and locations. Nearby frequencies and locations participate in this process depending on how high or not the damping is. More damping (more sound absorbing materials present in the room) spreads the process to nearby frequencies and locations. RT can be properly calculated from the modal decay constant, based on spherical sound reflection factors, surface impedances, pressure summation and edge diffraction effects. Future work includes calculating the modal decay constant and RT in three dimensional models using wave based geometrical acoustics [16].

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