INTRODUCTION

This paper starts with a discussion of the problem of decay function estimation. It is shown that the bias caused by exponential time averaging can be eliminated by using estimation methods based on impulse response measurements. The signal-to-noise ratio can be increased by using a m-sequence excitation. The calculations are carried out by means of FFT technique. Modulation Transfer Function, MTF, and Speech Transmission Index, STI, may also be estimated using the impulse response. An analysis and a development of methods suggested by M. R. Schroeder [1], [2] will here be presented.

BIASED AND UNBIASED DECAY FUNCTION ESTIMATES

There are several problems associated with the "interrupted noise" decay function measurement technique. One is the bias that arises when using time averaging of the squared microphone signal. Let the exciting noise be turned off at t = 0. The microphone signal, denoted y(t), then becomes a nonstationary random signal. The decay function is then given by

\[ r(t) = \frac{E[y^2(t)]}{E[y^2(0)]} = \int h^2(\tau) \, d\tau / \int h^2(\tau) \, d\tau, \tag{1} \]

where \( E[\cdot] \) denotes ensemble average and \( h(\tau) \) the impulse response of the current system. The conventional measurement technique uses in general not only averaging over a number of experiments in order to cancel the variance but also averaging over some time. This latter averaging is in general exponentially (or approximately exponentially) weighted. It has the following ensemble mean value:

\[ r_e(t) = \frac{1}{T_e} \int_0^t r(\tau) \exp\left(-\frac{(t-\tau)}{T_e}\right) \, d\tau, \tag{2} \]
where $T_e$ is the time constant. Let the decay function be ideal i.e.

$$r(t) = \exp(-t(6 \ln 10)/T_R), \quad t \geq 0$$

(3)

where $T_R$ denotes the reverberation time and define a function

$$f(t) = \frac{r_e(t)}{r(t)}$$

(4)

This function is found to be increasing and constrained by

$$1 \leq f(t) < \frac{1}{1 - T_e \cdot (6 \ln 10)/T_R}$$

(5)

This means a biased decay function estimate. The bias is not always negligible. If e.g. $T_R = 0.6$ s and $T_e = 1/32$ s the maximum bias become 5.5 dB and may thereby affect also the reverberation time estimate. The requirement for unbiased estimate is

$$T_e \ll \frac{T_R}{(6 \ln 10)}$$

(6)

An other way to make unbiased estimates is by using the impulse response $h(t)$ and calculate the decay function $r(t)$, (1).

**ESTIMATION OF THE IMPULSE RESPONSE**

There are several methods available for impulse response estimation.

*Pulse excitation gives low signal-to-noise ratio, SNR, but only background noise generated variance.*

Excitation by a "white" noise signal gives high SNR but variance generated not only by the background noise but also by the exciting signal.

*Pseudonoise, i.e. m-sequence, excitation gives high SNR and only background noise generated variance. This means that m-sequence excitation is preferable. It is also feasible for digital signal processing which will be used here. The m-sequence is periodic. The period length is $N = 2^k - 1$, where $k$ is an integer. The estimation procedure means crosscorrelating input and output signals over a number of integer periods. It is natural to choose the sampling rate equal to the clock frequency of the m-sequence. An efficient method to carry out cyclic crosscorrelation is by using Fast Fourier Transform, FFT. However the FFT algorithm needs $2^n$ samples in one period, where $n$ is an integer. This problem can be solved in different ways. Here we suggest the use of FFT with $2^{(k+1)}$ samples where $2^k + 1$ values are equal to zero.*

The FFT algorithm gives the cyclic crosscorrelation sequence, $q(1)$. **
We generate from this sequence a new sequence according to
\[ z(i) = q(i) + q(i + 1 + i) \quad \text{for} \quad i = 0, 1, \ldots, N - 1 \quad (7) \]
which is the wanted crosscorrelation result, i.e. cyclic crosscorrelation over a period of \( N \) samples.

**FROM IMPULSE RESPONSE TO DECAY FUNCTION**

We start with an impulse response estimate given by
\[ h(t) = h(t) + \eta(t) \quad \text{for} \quad 0 \leq t \leq T_0 \quad (8) \]
where \( \eta(t) \) is zero-mean noise with limited variance \( \sigma_\eta^2 \). The decay function estimate is given by
\[ f(t) = \frac{1}{h_0^2} \int_0^{T_0} \left[ h(t) + \eta(t) \right]^2 dt \quad (9) \]
where
\[ h_0^2 = \int_0^\infty h^2(\tau) \, d\tau \quad (10) \]
The mean value of \( f(t) \) is
\[ m_f(t) = E[f(t)] = \frac{1}{h_0^2} \left[ (T_0 - t) \sigma_\eta^2 + \int_0^\infty h^2(\tau) \, d\tau - \int_0^t h^2(\tau) \, d\tau \right] \quad (11) \]
The first term is an error caused by noise power integration. The third term is a truncation error. By studying the ideal decay function and tolerating relative errors equal to \( e^{-1} \) we find
\[ T_0 \leq T_{0M} = \frac{T_R}{6 \ln 10} \ln \left( \frac{h_0^2}{\sigma_\eta^2} - \frac{6 \ln 10}{T_R} \right) \quad (12) \]
and that \( f(t) \) has a tolerable error only for \( 0 \leq t \leq T_0 - T_R/(6 \ln 10) \)
We note that one has to find rough estimates of \( h_0^2, \sigma_\eta^2 \) and \( T_R \) in order to calculate \( T_{0M} \). This is often possible.

**FROM IMPULSE RESPONSE TO MTF AND STI**

The relation between Modulation Transfer Function, MTF, and the impulse response is , [2], [3], given by
\[ m(f) = \left| \int_0^\infty h^2(\tau) e^{-i2\pi ft} \, d\tau \right| \cdot \frac{1}{h_0^2 (1 + \text{SNR}_\text{IN})} \quad (13) \]
where \( \text{SNR}_\text{IN} \) is the input signal to noise ratio. The frequency range of interest is \( f \leq 20 \, \text{Hz} \) (or eventually 100 Hz). This means that the
squared impulse response may be low-pass filtered. In digital signal processing we can then use decimation techniques and thereby reduce essentially the number of samples required.

Speech Transmission Index, STI, is calculated from $m(f)$ for 18 frequencies from 0.4 Hz to 20 Hz spaced in 1/3-octave intervals. Since the spacing is not linear the FFT algorithm is not suitable. We therefore use the Discrete Fourier Transform, DFT. This is not an essential drawback since the number of samples are few.

REFERENCES

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Measurement of Reverberation Decay Function and Acoustical Impulse Response Using Maximum-Length Sequences and FFT.