ACTIVE ABSORPTION OF FREE AND DIFFUSE SOUND FIELDS

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1. INTRODUCTION

It has long been recognised that the absorption of sound plays a fundamental role in the active control of acoustic fields[1]. The purpose of this paper is to briefly investigate this process in connection with the active absorption of free and diffuse sound fields. The maximum cross sectional area of absorption for these two examples is derived. This is only possible because the acoustic intensity of the sound field bears a simple relationship to the square pressure for these limiting class of sound fields.

2. BASIC EQUATIONS

Consider a secondary source of sound $q_s(\omega,\mathbf{r})$ which is irradiated in some monochromatic primary sound field $p_p(\mathbf{r})$ of hitherto unspecified spatial characteristics. Assume that the secondary source is located at some point $\mathbf{r}_s$ whose volume velocity density is concentrated at a point according to $q_s(\omega,\mathbf{r}_s) = q_s(\omega)\delta(\mathbf{r}_s - \mathbf{r})$ where δ symbolises a Dirac delta function. Following the work of Levine[2], the sound power output W_s from the point secondary source may be constructed from the expression

$$W_s = \frac{1}{2}\mathcal{R}\{\, p(\mathbf{r}_s)q_s^*(\omega)\} \tag{1}$$

For small amplitude oscillations, linear superposition applies so that the total pressure at the secondary source point may be represented by

$$p(\mathbf{r}_s) = p_p(\mathbf{r}_s) + q_s Z_{rad} \tag{2}$$

where Z_{rad} represents the radiation impedance of the secondary source. Substituting the decomposed pressures into equation (1) for the secondary source sound power output yields a quadratic function of the complex secondary source strength q_s of the general form

$$W_s = q_s^* A\, q_s + b\, q_s^* + b^* q_s \tag{3}$$

The coefficients A and b may be identified as $A = 1/2\,\mathcal{R}\{Z_{rad}\}$ and $b = 1/4\, p_p(\mathbf{r}_s)$. The properties of this quadratic function have been extensively investigated with optimisation problems of this type[3]. The optimal secondary source strength $q_{so}(\omega)$ which minimises this function with respect to its real and imaginary parts simultaneously has been derived[3] which is given by equation (4)

$$q_{so} = -A^{-1}b = -\frac{p_p(\mathbf{r}_s)}{2\mathcal{R}\{Z_{rad}\}} \qquad (4)$$

Note that the secondary source strength is exactly in anti-phase of the primary field pressure at the secondary source point. Substituting $q_{so}(\omega)$ into equation (3) yields the minimum secondary source sound power output W_{smin} of the form

$$W_{smin} = -q_s^* A^{-1} q_s = -\frac{|p_p(\mathbf{r}_s)|^2}{8\mathcal{R}\{Z_{rad}\}} \qquad (5)$$

providing $\mathcal{R}\{Z_{rad}\} \geq 0$, which is guaranteed since sound power absorption is not possible in the *absence* of any external sound field. Negative radiation resistance is the condition for sound power absorption which will now be investigated in relation to its effectiveness for absorbing some important examples of idealised primary sound field.

3 THE SOUND POWER ABSORPTION OF FREE FIELD PLANE WAVES

The free field radiation resistance of a point monopole source at a single frequency $\mathcal{R}\{Z_{rad}\}$ is given by[4]

$$\mathcal{R}\{Z_{rad}\} = \frac{\omega^2\rho}{4\pi c} \qquad (6)$$

where ρ and c are the ambient density and sound speed in the medium respectively. For a free field plane wave, the primary pressure field takes the particularly simple form

$$p_p(\mathbf{r}) = Ae^{-j\mathbf{k}.\mathbf{r}} \qquad (7)$$

where A denotes the complex pressure amplitude of the plane wave. Putting the secondary source at the origin of co-ordinates $\mathbf{r}_s = 0$, from equation (4), the optimal secondary source strength is given by

$$q_{so} = -\frac{2A\pi c}{\omega^2\rho} \qquad (8)$$

The minimum sound power output W_{smin}, which is equivalent to finding the maximum sound power absorption, may be determined by substitution into equation (5) to give

$$W_{smin} = -\frac{|A|^2\pi c}{2\omega^2\rho} \qquad (9)$$

Recall that the sound intensity I_P of the incident plane wave in the direction of the propagation is proportional to the square of the pressure according to

$$I_P = \frac{|A|^2}{2\rho c} \qquad (10)$$

One can therefore express the maximum sound power absorption as the product of the plane wave sound intensity I_P and the cross sectional area of absorption S_{absorb} thus

$$W_{Smin} = -I_P \frac{\lambda^2}{4\pi} \tag{11}$$

The cross sectional area of absorption S_{absorb} is therefore equal to $\lambda^2 / 4\pi$ which can be easily remembered by noting that this area corresponds to a circle normal to the incident plane wave front which has a radius r equal to the reciprocal of the wavenumber k^{-1} where $k = \lambda/2\pi$. Thus, one can write

$$S_{absorb} = \frac{\lambda^2}{4\pi} \tag{12}$$

which takes the form of a circle with radius r where

$$r = \frac{\lambda}{2\pi} = k^{-1} \tag{13}$$

This result was originally deduced by Nelson et - al[5] and is further investigated in reference [6]. The absorption process outlined here is necessarily causal since the free space absorption of sound only depends on the primary field pressure at the secondary source point. This may be readily verified by inverse Fourier transforming equation (8). Despite the fact that the point monopole secondary source can only match itself to a single point in the wavefield, the effective cross sectional area of absorption is large. This finding suggests that the influence of the perfectly absorbing source extends much further than its physical dimension which indicates that acoustic energy is somehow diffracted towards the point source[6].

4. DIFFUSE FIELD SOUND POWER ABSORPTION

In a diffuse field environment the radiation impedance of a point source comprises two contributions. The first is the free space radiation impedance Z_{0rad} which quantifies the in-phase part of the acoustic pressure produced *immediately* at the source point per unit volume velocity. The second is the diffuse field impedance Z_{Drad} which quantifies the in-phase part of the acoustic pressure produced at the secondary source point radiated via wall reflections per unit volume velocity, where the subscript 'D' denotes 'Diffuse'. One can therefore write

$$Z_{rad} = Z_{0rad} + Z_{Drad} \tag{14}$$

The minimum sound power absorption can therefore be written more explicitly as

$$W_{Smin} = -\frac{|p_p(r_s)|^2}{8\mathcal{R}\{Z_{0rad} + Z_{Drad}\}} \tag{15}$$

where $\mathcal{R}\{Z_{0rad}\} = (\omega^2\rho/4\pi c)$. Above the Schroeder frequency, the sound field may be regarded as 'diffuse' for which the real and imaginary parts of the diffuse field transfer impedance are normally distributed, zero mean independent random variables[7]. One can no longer speak in terms of the level of sound power absorption with absolute certainty, as in the free field case, but only in terms of the expected value of a large number of similar experiments at varying source positions. One can show that the formal expectation of the secondary source sound power absorption with respect to source position, namely $< W_{Smin} >$ is infinite[8]. This finding arises because of the significant probability that $\mathcal{R}\{Z_{0rad}\}$ and $\mathcal{R}\{Z_{Drad}\}$ are exactly equal and opposite causing to W_{Smin} to become singular. Physically this condition occurs when the energy flowing from the source equals the energy flowing into the source via reflections from the enclosure walls. However, in absolute terms this occurrence is very unlikely, see reference [8]. We now seek to obtain an estimate for the mean value based on those source positions for which $\mathcal{R}\{Z_{0rad}\} > \mathcal{R}\{Z_{Drad}\}$ where equation (15) may be represented as a Taylor series expansion. It is believed that this condition is sufficiently un-restrictive to enable one to derive a mean value which is representative of the average behaviour of the vast majority of possible outcomes. To second order in $\mathcal{R}\{Z_{Drad}/Z_{0rad}\}$, the minimum secondary source sound power absorption may be approximated by

$$W_{smin} \approx -\frac{|p_p(r_s)|^2}{8\mathcal{R}\{Z_{0rad}\}}\left[1 - \frac{\mathcal{R}\{Z_{Drad}\}}{\mathcal{R}\{Z_{0rad}\}} + \frac{\mathcal{R}^2\{Z_{Drad}\}}{\mathcal{R}^2\{Z_{0rad}\}} - \ldots\right] \quad \text{for } \mathcal{R}\{Z_{Drad}\} < \mathcal{R}\{Z_{0rad}\} \tag{16}$$

The expectation of the sum of terms is simply the sum of expectations irrespective of whether the terms are correlated or uncorrelated. Recalling that $\mathcal{R}\{Z_{0rad}\} = \omega^2\rho / 4\pi c$ which is clearly independent of secondary source position, the space average of equation (16) may therefore be approximated by

$$< W_{smin} > = -\frac{< |p_p(r_s)|^2 > \pi c}{2\omega^2\rho}\left[1 + \frac{< \mathcal{R}^2\{Z_{Drad}\} >}{\mathcal{R}^2\{Z_{0rad}\}} - \ldots\right] \quad \text{for } \mathcal{R}\{Z_{Drad}\} < \mathcal{R}\{Z_{0rad}\} \tag{17}$$

where $< \mathcal{R}\{Z_{Drad}\} > = 0$ since all odd moments of the series expansion, i.e., the moments which characterise the asymmetry of the probability density function about the mean (skewness) have been set equal to zero. The second non-vanishing expectation term in the series expansion has a well defined physical significance. Given that $\mathcal{R}\{Z_{Drad}\}$ is a zero mean, Gaussian random variable, then $< \mathcal{R}^2\{Z_{Drad}\} >$ is equal to the variance σ^2_{rad} of the radiation resistance with respect to varying source position and $\mathcal{R}\{Z_{0rad}\}$ is the mean μ_{rad} so that one can write

$$\frac{< \mathcal{R}^2\{Z_{Drad}\} >}{\mathcal{R}^2\{Z_{0rad}\}} = \frac{\sigma^2_{rad}}{\mu^2_{rad}} \tag{18}$$

Equation (18) is known as the 'relative' variance of the secondary source input impedance which is directly related to the noise modal bandwidth overlap factor $M(\omega)$ according to[9]

$$\frac{\sigma^2_{rad}}{\mu^2_{rad}} = \frac{27}{16M(\omega)} \tag{19}$$

$M(\omega)$ is defined as the average number of modes having a natural frequency within the modal bandwidth $\Delta\omega_N$ defined by $\Delta\omega_N = \int_0^\infty |A(\omega)|^2 d\omega \,/\, |A(\omega)|^2_{max}$ which is $M(\omega) = \Delta\omega_N n(\omega)$ where $n(\omega)$ symbolises the asymptotic modal density for which oblique modes are completely dominant and $A(\omega)$ is the second order modal response function[4]. The diffuse field sound intensity I_D is readily derived by averaging the plane wave intensity given by equation (10) over all possible elemental solid angles in the spherical co-ordinate system[4] to produce the result

$$I_D = \frac{<|p_p(r_s)|^2>}{8\rho c} \tag{20}$$

Equations (17), (19) and (20) may be combined to yield the following approximate relationship

$$< W_{smin} > \approx -I_D \frac{\lambda^2}{\pi} \left[1 + \frac{27}{16M(\omega)} \right] \tag{21}$$

We note that at a frequency equal to the Schroeder frequency, the critical frequency above which an enclosed sound field exhibits 'diffuse' behaviour[7], the modal overlap factor $M(\omega)$ is approximately equal to 10 and increases still further as the cube of the excitation frequency[8]. To a reasonable approximation therefore, this term can be disregarded to give

$$< W_{smin} > \approx -I_D \frac{\lambda^2}{\pi} \tag{22}$$

The cross sectional area of absorption for a perfectly absorbing point monopole source in a diffuse field environment is therefore approximately equal to λ^2/π. The isotropy of the diffuse wavefield suggests that the reverberant radiation bombards the secondary source from all angles equally. The area of absorption must therefore take the form of a sphere of surface area $4\pi r^2$ which has the point secondary source located at its centre. Equating $4\pi r^2$ to λ^2/π enables one to solve for the radius r of absorption of the hypothetical sphere on the surface of which, on average, all sound is absorbed. Solving for r gives

$$S_{absorb} \approx \frac{\lambda^2}{\pi} \tag{23}$$

which takes the form of a sphere with radius r where

$$r \approx \frac{\lambda}{2\pi} = k^{-1} \tag{24}$$

It is intriguing to observe that the radius of this notional sphere of absorption is equal to the radius of the circle of absorption for the plane wave example outlined previously. Note that the diffuse field cross sectional area of absorption S_{absorb} is exactly four times the area of absorption for an incident plane wave in the free field $\lambda^2/4\pi$. The free field and diffuse field cross sectional areas of absorption are fully consistent with the idea that the point monopole source extends a sphere of influence of radius $\lambda/2\pi$, on whose surface all sound is absorbed. In the case of an incident plane wave, the normal projection of the sphere onto the plane wave front is precisely the circle of absorption identified in equation (13). A fully diffuse field will, however, see the full benefit of the hypothetical sphere and its cross sectional area of absorption is therefore increased accordingly. Inspection of equation (5) shows that the optimal secondary source cross sectional area of absorption S_{absorb} is also insensitive to the proximity of the enclosure walls. For example, in the case of a point secondary source on an enclosure wall, the square pressure is doubled[10]. At the same position however, the radiation impedance of the secondary source is also effectively doubled by the impedance contribution from a co-located 'image' secondary source. The ratio of terms which determines the maximum sound power absorbed according to equation (5) therefore remains constant and is consequently independent of the source position[8] with respect to the room boundaries.

The optimal absorption of sound is only possible by presenting to the oncoming primary wavefield an apparent optimal impedance. From the point of view of the incident sound field, it is entirely irrelevant whether this impedance is active or passive in origin. One passive device which has been successfully applied to a range of different problems at *low* frequencies is the Helmholtz resonator. This arrangement may be optimally tuned to a given frequency and the absorption of sound maximised at that frequency. It is generally well known that the absorption cross sectional area of this device at the resonant frequency in the diffuse field is also λ^2/π where λ is the wavelength at resonance[11]. The obvious advantages of using an active source is that, in principle, one is able to obtain maximum sound power absorption over a band of frequencies simultaneously whereas the Helmholtz resonator is a high Q system carefully tuned to a single given frequency. The Helmholtz resonator may therefore be regarded as the passive analogue of the optimally absorbing point monopole source.

5. REFERENCES

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