

# inter-noise 83

## DIFFRACTION BY AN ABSORBING BARRIER OR BY AN IMPEDANCE TRANSITION

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### 1. SUMMARY

The presented theory for diffraction by an absorbing barrier combines the known solutions for propagation of sound from a point source over A) a finite impedance ground [1], B) an acoustically hard, wedge-shaped barrier, [2]. The impedances of both sides of the wedge may or may not be similar. An impedance transition is dealt with by taking one side of a wedge as hard and the other as absorbing and setting the top angle to  $180^\circ$ . Results of 1:100 scale model measurements (absorbing wedge) and outdoor measurements [3,4] (impedance transition) are found to agree very well with our calculations.

### 2. EXISTING THEORY

The sound field  $\Psi$  of a point source over a locally reacting ground is described by Attenborough, Hayek and Lawther [1] as:

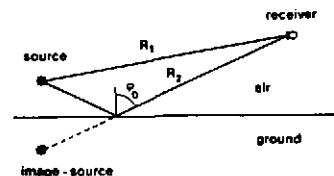


Fig. 1: Definition of symbols.

$$\Psi = \exp(ikR_1)/R_1 + Q \exp(ikR_2)/R_2 \quad (1)$$

$$Q = R_p + (1-R_p)F(z) \quad (2)$$

$$R_p = (\cos \phi_0 - \beta) / (\cos \phi_0 + \beta) \quad (3)$$

$$F(z) = 1 + i\pi^{1/2} z w(z) \quad (4)$$

$$w(z) = \exp(-z^2) \operatorname{erfc}(-iz) \quad (5)$$

$$z = (ikR_2/2)^{1/2} (\cos \phi_0 + \beta) \quad (6)$$

The symbols  $R_1$ ,  $R_2$  and  $\phi_0$  are defined in fig. 1,  $k$  is the wavenumber and  $\beta$  is the normalised, specific acoustic admittance of the ground. The value of  $\beta$  is calculated according to the empirical theory of Delany and Bazley [5], which relates  $\beta$  to a specific flow resistance. Note that  $Q$  is a function of  $R_2$ ,  $k$ ,  $\beta$  and  $\phi_0$ .

Hadden and Pierce [2] derived an expression for the sound field  $\psi_d$ , that has been diffracted by an acoustically hard wedge. Under far field conditions ( $kr_1 r_3/L \gg 1$ ) their solution can be written as:

$$\psi_d = \frac{-1}{\pi} \sum_{n=1}^4 \text{sign}(\phi_n - \pi) A F_v \exp(ikL)/L \quad (7)$$

$$F_v = \frac{\pi}{2} \frac{\sin|A|}{|A|} \left[ 1 + \left( \frac{2r_1 r_3}{L^2} + \frac{1}{2} \right) \cos^2|A|/v^2 \right]^{-1/2} \underline{w}(b) \quad (8)$$

$$\underline{w}(b) = \exp(-b^2) \text{erfc}(-ib) \quad (5)$$

$$b = \left( \frac{ikr_1 r_3}{2L} \right)^{1/2} \left| 2 \cos|A| [v^2 + \left( \frac{2r_1 r_3}{L^2} + \frac{1}{2} \right) \cos^2|A|]^{-1/2} \right|; \quad \arg(b) = \pi/4 \quad (9)$$

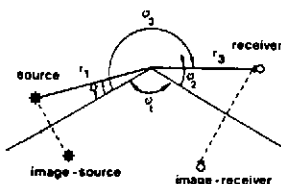


Fig. 2: Definition of used symbols.

$$L = [(r_1 + r_3)^2 + (z_1 - z_3)^2]^{1/2} \quad (10)$$

$$A = A(\phi_n) = \left( \frac{\pi}{2} \right) (-\phi_{ex} - \pi + \phi_n) + \pi H(\pi - \phi_n) \quad (11)$$

$$H(x) = 1 \quad (x \geq 0), \quad = 0 \quad (x < 0) \quad (12)$$

$$v = \pi/\phi_{ex}, \quad \phi_{ex} = 2\pi - \phi_t \quad (13)$$

$$\phi_1 = |\phi_3 - \phi_1|; \quad \phi_2 = \phi_3 + \phi_1; \quad \phi_3 = 2\phi_{ex} - (\phi_3 + \phi_1); \quad \phi_4 = 2\phi_{ex} - |\phi_3 - \phi_1| \quad (14)$$

The geometrical symbols are shown in fig. 2. When the receiver in fig. 2 is moved to one of the lines of sight ( $\phi = \pi$ ) the contribution of the additional ray, that can be constructed in the sense of geometrical acoustics, is added to the total field. Because the sign-function alters at a line of sight, the total field is continuous. For  $\phi_t \rightarrow 180^\circ$  the diffracted field  $\psi_d$  vanishes, as it should do.

### 3. EXTENSION OF THE THEORY.

The four terms appearing in equation (7) can physically be interpreted as the contributions of the sound fields travelling from source to receiver ( $n=1$ ), from image source to receiver ( $n=2$ ), from source to image receiver ( $n=3$ ) and from image source to image receiver ( $n=4$ ). Equation (7) can thus be rewritten as:

$$\psi_d = (D_{s-r} + D_{is-r} + D_{s-ir} + D_{is-ir}) \exp(ikL)/L \quad (15)$$

For both the ground and the wedge an image point is used to describe the reflection of a sound field against a surface. We can account for the absorbing property of such a surface by multiplying the contribution of an image point with a reflection coefficient  $Q$ . For extension of the theory we use this consideration. Just for brevity, all ray paths are assumed to be perpendicular to the barrier edge. Hence, we propose the diffracted sound field of an absorbing barrier to be given by:

$$\psi_d = (D_{s-r} + Q_s D_{is-r} + Q_r D_{s-ir} + Q_s Q_r D_{is-ir}) \exp(ikL)/L \quad (16)$$

$$Q_s = Q_s(\rho_s, k, \beta_s, \pi/2 - \varphi_1); \quad Q_r = Q_r(\rho_r, k, \beta_r, \pi/2 - \varphi_2) \quad (17)$$

This approximation is in accordance with Kirchhoff diffraction theory. The diffracted field of an impedance transition is calculated by taking it as a wedge with a top angle of  $180^\circ$  and different impedances on each side. A minor disadvantage of this approximation is, that  $\psi_d$  does not totally vanish for  $\varphi = 180^\circ$  and  $\beta = \beta \neq 0$ . However, for such a situation of a homogeneous absorbing ground, the theory of ref. 1 is taken and this problem is not met at all. The distance parameters  $\rho_s$  and  $\rho_r$  of  $Q_s$  and  $Q_r$  give rise to another problem. Continuity on the lines of sight requires  $\rho = \rho_s + r_1$ , and calculations for the impedance transition are performed accordingly. However, for a receiver position far into the shadow zone of a wedge, the top of the wedge is considered as a "receiver" for the calculation of  $Q_s$  ( $\rho = r_1$ ) and then radiates as a secondary source to the actual receiver ( $\rho = r_2$  with  $\rho = r_3$ ). Further investigations on this point will be carried out in the near future. The significant advantage of the model is that it meets the reciprocity condition. The model is also analogous to that of Jonasson for a screen on absorbing ground [6], the finite impedances of each side of the wedge can be accounted for separately and it is applicable for top angles of the wedge upto  $180^\circ$  (if not  $\beta = \beta \neq 0$ ). Furthermore it can be incorporated easily into complete sound propagation models (e.g. the advanced model described by De Jong, Moerkerken and Van der Toorn [7], which itself has nonreciprocal expressions for sound diffraction by absorbing barriers and impedance transitions).

#### 4. COMPARISON OF CALCULATIONS WITH MEASUREMENTS

Calculations were carried out for 1/3 octave band center frequencies. Our 1:100 scale model measurements with a spark source gave 1/3 octave band results. A flow resistance of  $300 \cdot 10^{-4} \text{ Nsm}^{-2}$  resulted from curve fitting of the effect of a homogeneous surface. The diffraction effects of four wedge-types were investigated: acoustically hard surfaces at the source side and the receiver side (HH-wedge), absorbing surface at the source side and the other side acoustically hard (AH-wedge) and the two other combinations (HA-wedge and AA-wedge). The calculations and measurements shown in figs. 3a and 3b agree very well with each other, even when the source is very close to a wedge surface. Note that in fig. 3a the curves for a HA-wedge and a AH-wedge overlap, as the reciprocity principle requires. Outdoor measurements at transitions between road surfaces and grass land were performed by Van der Toorn [3] and Rasmussen [4]. They reported values for the specific flow resistance of  $100 \cdot 10^{-4}$  and  $225 \cdot 10^{-4} \text{ Nsm}^{-2}$  respectively. To emphasize the clear diffraction effect of the transitions, theoretical results for homogeneous grass surface and hard surface are included for comparison in figs. 4a and 4b. Except for a small shift of the interference dip in fig. 4a (probably caused by the use of a loudspeaker as an approximate point source), calculations and measurements agree very well again.

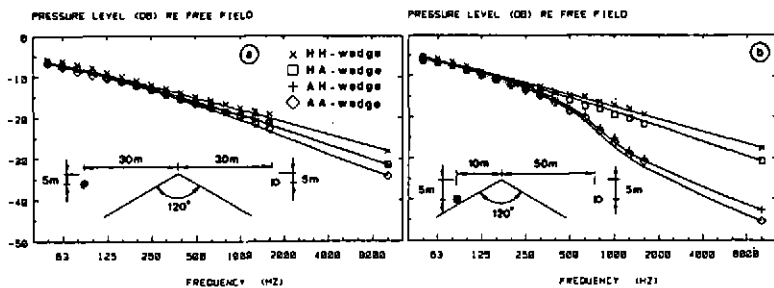


Fig. 3: Calculations (—) and 1:100 scale model measurements (\*\*\* ) for diffraction by a wedge. Parameters are given for scale 1:1.

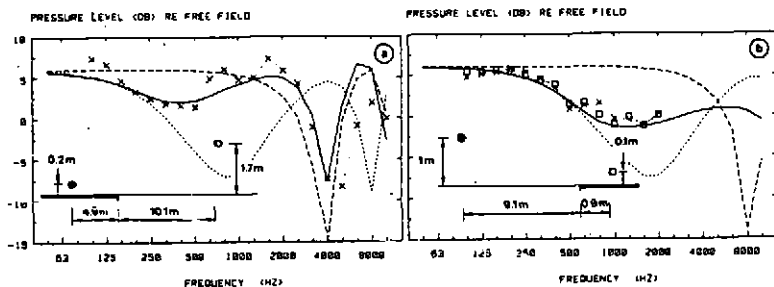


Fig. 4: Calculations (—) and outdoor measurements by Van der Toorn [3] (\*\*\*, fig. 4a) and by Rasmussen [4] (\*\*\* and ooo, fig. 4b) of the effect of an impedance transition. Theoretical curves for a homogeneous absorbing (.....) and hard (----) surface are included for comparison.

## 5. REFERENCES

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