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A NEW CABLE ELEMENT USING THE ABSOLUTE COORDINATE FORMULATION

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In this paper, based on the previous studies on the finite elements of gradient deficient absolute nodal coordinate formulation (ANCF), a new spatial transverse isotropic cable element is proposed to numerically study under the frame of multibody dynamics. Cable element is popular nowadays because the numerically stiff behavior resulting from shear terms in existing ANCF beam elements that employ the continuum mechanics approach to formulate the elastic forces and the resulting locking phenomenon make these elements less attractive for cable structures. Also in practical engineering, cable has a high tension strength and a low bending resistance, which means ignoring the bending strain is acceptable and feasible. And assuming that cable material is transverse isotropic can greatly improve the computational efficiency. Based on above asumption, a new gradient transverse isotropic cable element of ANCF is proposed. The elastic force of the finite elements and their Jacobian J can be further derived Once the formulation of the element strain energy is deduced based on ANCF. Both generalized-alpha method and scaling technique are utilized to develop calculating program aimed to solve dynamics equations precisely and efficiently. Finally, numerical example of capturing mechanism is given to demonstrate the applicability and effectiveness of the proposed element of ANCF for flexible multibody system dynamics.
Keywords: transverse isotropic cable, multibody dynamics

1. Introduction

It has been a hot spot in the field of multibody dynamics research for rope large deformation dynamics problems at home and abroad. Generally modeling methods for ropes can be divided into two types: discrete models and continuous models [1]. Discrete models discrete ropes into rigid bodies or lumped mass point which are connected by revolute hinges, spherical hinges or spring elements. For continuous models, its essence is that the rope is treated as a slender elastic beam with large deformation, and then system equations are derived based on Newton’s law or Hamilton’s law. However continuous models suffer from a lack of generality proved by carful researches expers have investigated. In this case, gradient-deficient elements attract the attention of academic circles.

In this paper, based on the previous studies on the finite elements of gradient deficient absolute nodal coordinate formulation (ANCF), a new spatial transverse isotropic cable element is proposed to numerically study under the frame of multibody dynamics.

2. A new spatial transverse isotropic cable element of ANCF

According to the works by Gerstmayr and Shabana [2], Liu [3] proposed a spatial curved slender-beam element based on Euler-Bernoulli beam assumption as shown in Figure 1..

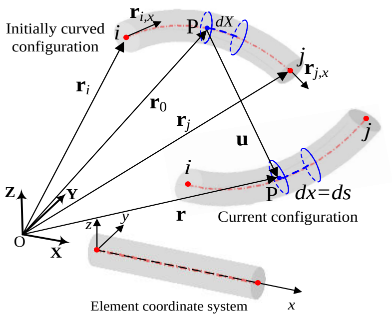


Figure 1: A spatial curved cable element in initial and current configurations

From Figure 1,the absolute nodal coordinates for the beam element can be casted as

$$\mathbf{e}=\begin{bmatrix}\mathbf{r}_i^{\mathrm{T}} & \mathbf{r}_{i,x}^{\mathrm{T}} & \mathbf{r}_j^{\mathrm{T}} & \mathbf{r}_{j,x}^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} \tag{1}$$

where $\mathbf{r}_k\,(k=i,j)$ is the position vector of node i, $\mathbf{r}_{k,x}$ is the partial derivative vector of $\mathbf{r}_k$ of node i to ξ direction, $\mathbf{r}_{k,x}=\partial\mathbf{r}_k/\partial x$.

The numerically stiff behavior resulting from shear terms in existing ANCF beam elements that employ the continuum mechanics approach to formulate the elastic forces and the resulting locking phenomenon make these elements less attractive for cable structures. Also in practical engineering, cable has a high tension strength and a low bending resistance, which means ignoring the bending strain is feasible. And assuming that cable material is transverse isotropic can greatly improve the computational efficiency. Based on above asumptions and works, a new gradient cable element of ANCF which can represent the transverse isotropic characteristic is proposed.

Set 2-3 plane is the isotropic plane, axis 1 is perpendicular to 2-3 plane (1, 2 and 3 refer to the elastic principal directions).For transverse isotropic materials, we have

$$E_2=E_3,\quad \nu_{21}=\nu_{31},\quad \varepsilon_2=\varepsilon_3 \tag{2}$$

where the constants $E_i\,(i=1,2,3)$ is Young's modulus of elasticity in the three elastic principal directions. $\nu_{ij}\,(i,j=1,2,3)$ denotes the Poisson's ratio.

Then the longitudinal strain is calculated as

$$\varepsilon^P=\frac{1}{2}\left(\frac{\mathbf{e}^T\mathbf{S}'^{\mathrm{T}}\mathbf{S}'\mathbf{e}}{l^2}-1\right) \tag{3}$$

where **S** denotes the shape function, and *l* is the length of the element. Thus the longitudinal stress of point P is calculated by

$$\sigma^P=\frac{E_1^2(\nu_{32}-1)\varepsilon^P-2E_1E_2\nu_{21}\varepsilon_2}{2E_2\nu_{21}^2-E_1+E_1\nu_{32}}=E_1\varepsilon^P \tag{4}$$

Therefore, through the volume integral of the initial configuration to the element, once the formulation of the element strain energy is deduced based on ANCF, the longitudinal strain energy and elastic force vector formulation for the cable element can be expressed

$$\begin{aligned} U&=\frac{1}{8}\int_0^1 E_1Al\left(\frac{\mathbf{e}^{\mathrm{T}}\mathbf{S}'^{\mathrm{T}}\mathbf{S}'e}{l^2}-1\right)^2\mathrm{d}\xi \\ \mathbf{F}^e&=\frac{1}{2}\int_0^1 E_1A\frac{\mathbf{e}^{\mathrm{T}}\mathbf{S}'^{\mathrm{T}}\mathbf{S}'}{l}\left(\frac{\mathbf{e}^{\mathrm{T}}\mathbf{S}'^{\mathrm{T}}\mathbf{S}'\mathbf{e}}{l^2}-1\right)\mathrm{d}\xi \end{aligned} \tag{5}$$

where *A* is the cross-section area.

Based on ANCF, the final dynamic equations for a constrained rigid-flexible multibody system can be expressed in a compact form as a set of differential algebraic equations with a constant mass matrix as following [3]

$$\begin{cases}\mathbf{M}\ddot{\mathbf{q}}+\mathbf{\Phi}_{\mathbf{q}}^{\mathrm{T}}\boldsymbol{\lambda}+\mathbf{F}(\mathbf{q})=\mathbf{Q}(\mathbf{q}) \\ \mathbf{\Phi}(\mathbf{q},t)=\mathbf{0}\end{cases} \tag{6}$$

where **M** is the constant mass matrix of the system, **F**(**q**) is the elastic force vector, which is a non-linear function of nodal coordinates, **Φ**(**q**,t) represents the vector that contains the system constraint equations, **Φq** is the derivative matrix of constraint equations with respect to the generalized coordinates **q**, **λ** is the Lagrange multiplier, **Q**(**q**) is the external generalized forces.

3. Numerical simulations

Numerical examples are given to demonstrate the applicability and effectiveness of the proposed element of ANCF for flexible multibody system dynamics. Due to the limited space, only one example, which is capturing dynamics of flexible ropes on space large-scale end effector, is shown in this paper.

As shown in Figure 2, two rigid rings consisting the fixed ring and the rotating ring are located in the socket, which will drag three flexible ropes to lock the position and attitude of shaft. The shaft and the fixed ring are fixed on the satellite. The rotating ring that is parallel to the fixed ring rotates at the speed of some certain function. When the shaft is locked, the position and attitude of satellite are adjusted to a predetermined status.

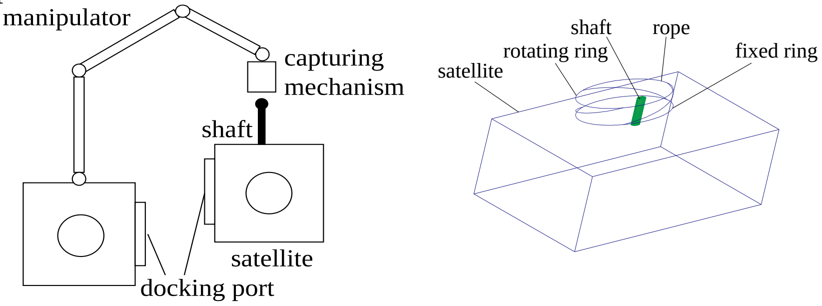
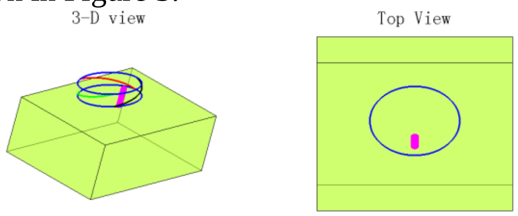
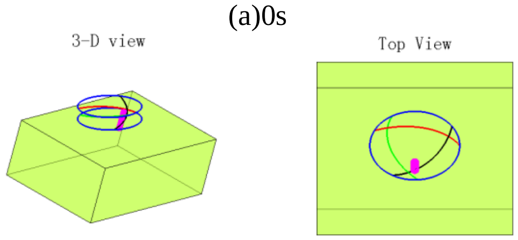


Figure 2: Sketch of capturing process.

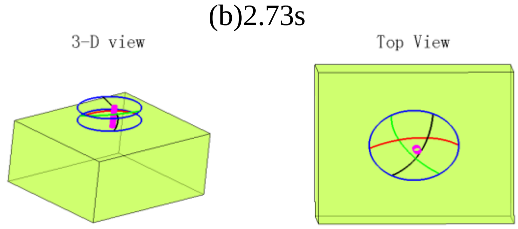
Then the numerical simulation for capturing process of flexible ropes can be performed. And the configuration sketches are shown in Figure 3.



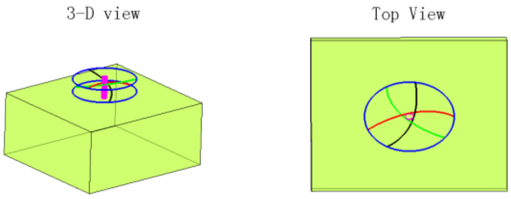
(a)0s



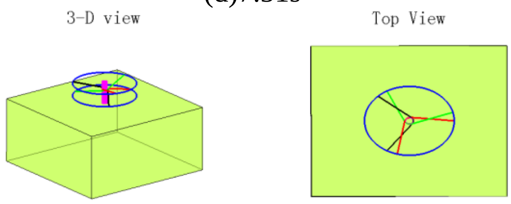
(b)2.73s



(c)6s



(d)7.51s



(e)11.76s

Figure 3: Dynamic deformation of gate and flow field of water for model I (Unit: m/s)

At t=2.73s, the second rope touches the target matching pole. As three ropes tightening, the region enclosed by ropes gets smaller and the target matching pole moves toward the center of rings. At t=7.51s, all three ropes touch the pole and with the influence of all three contact forces, the position and attitude of the pole is continuously rectified. And then at t=11.76s, the target matching pole has been correctly locked and cannot escape away from three ropes. Here the angle between the pole axis and the ring axis should be less than 0.3° . Due to the limited space, other research conclusions will be stated in the presentation.

4. Conclusion

In this investigation, a new spatial curved finite element for nonlinear dynamic analysis of transverse isotropic cable problems is proposed in the frame of the finite absolute nodal coordinate formulation. It is shown that this formulation that can be used in the large deformation and rotation analysis of cables leads to a constant mass matrix and zero centrifugal and Coriolis inertia forces. The strain energy for the spatial curved cable element is derived by using the definition of Green-Lagrange strain tensor in continuum mechanics. The assumption on small strains can be relaxed in the final strain energy formulation. The longitudinal strain energy and elastic force vector formulation for the cable element can be deduced. The generalized-alpha method is used to solve the huge set of system equations. Finally, the validation and performance of the transverse isotropic cable element of ANCF are illustrated by numerical case studies to validate the effectiveness of the proposed element for flexible multibody system dynamics. In future research, the proposed element will be applied to study the practical engineering problem based on ANCF to study the complex dynamic behaviors of the large scale space structures.

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