MODELLING OF SEA SURFACE SCATTERING

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1 : INTRODUCTION

This paper is developed from the author's 1979 model of sea surface backscatter, which analysed the scattering strength as the sum of that from the interface plus that from resonant bubbles created by breaking waves and lying just below the interface. [1]

Because of the lack of in situ bubble data, that model postulated bubble population laws as a function of windspeed, radius and depth entirely from an analysis of surface backscatter in the 1-60 kHz range, assisted by some dimensional arguments.

Recently, independent photographic and acoustic direct data on bubble concentrations have emerged [2,3,4,5,6], covering bubble radii from ~ 30μm to 250 μm (resonant frequencies 12-100 kHz).

One aim of this paper is to incorporate this new information into the bubble scattering model. A second aim is to re-examine interface scattering, in the light of attempts which have been made to model interface backscattering enhancement due to 'long scale' surface waves.

2 : APPROACH AND NOMENCLATURE

We suppose that

$$M_s = e^{-4\beta} M_{SI} + M_{SB} \tag{1}$$

where

M_s = Backscattering Strength, M_{SI} = Interface backscattering strength, M_{SB} = Bubble backscattering strength, and β = one way amplitude absorption loss of sound traversing the layer.

M_S is a function of the following variables:

- θ = incidence angle
- U = 10 metre windspeed
- f = frequency.

The following subsidiary variables are used

- $k = 2\pi f/(\text{sound speed})$
- $\lambda = k \cos \theta$ = vertical component of wavenumber,
- $|\mu| = k \cos \theta$ = horizontal component of wavenumber
- D = Layer e -fold thickness
- $\gamma = \lambda D$
- R = bubble radius
- z = depth
- $N(R,z)$ = Bubble occupation density per radius per volume at depth z (dimensions m^{-4})

$N_o(R) = N(R,0)$

$V(R)$ = Bubble buoyancy ascent speed

$E(\nu,\Theta) \equiv E(\underline{\nu})$ = Surface elevation power spectrum at wavenumber $\underline{\nu}$ normalised to $\iint d\Theta\ \nu\, d\nu\ E(\nu,\Theta) = \sigma^2$.

3 : BUBBLE BACKSCATTERING

3.1 : Resume of Scattering Theory

Reference [1] details a theory for scattering strength of a subsurface bubble layer. In the resonant scattering case, and taking the special case of an exponential distribution of bubbles in depth, the backscattering strength is

$$M_{SB} = M_o \Big\{ 4\, e^{-4\beta} + (1-e^{-8\beta})/4\beta + 2e^{-4\beta}/(1 + 16\,\gamma^2) - 4e^{-2\beta} \sum_{n=0}^{\infty} [(-1)^n + e^{-4\beta}] . [1 + (2\,\overline{n+1})^2\, \gamma^2]^{-1} . (2\beta)^n/n! \Big\} . \quad , \tag{2}$$

where $Mo = \pi\, B.D/(2\,\delta)$, (3)

and the neper attenuation is

$$\beta = \pi^2\, \delta_d\ B.D/(\delta_r . \delta . \cos\Theta), \tag{4}$$

where

$$B = R_f^{\,3}\ N_o(R_f), \tag{5}$$

$$\delta_d = 3.8 . 10^{-4} . f^{1/2}\ (\text{MKS},\ f \lesssim 10^5\ \text{Hz}) \tag{6}$$

is the resonant dissipation loss, $\delta r = 0.013$ = radiation loss,

$\delta \simeq \delta_r + \delta_d$ = total loss, $R_f = 3.2/f$ (MKS) = resonant bubble radius. Eq(2) approximates by integrating over the narrow resonance range of $R \sim R_f$, and corrects a typographical error in reference [1].

For non-resonant bubble scattering, we have to integrate an equation like (2) over all radii, not just close to resonance, and where the terms M_o, β , D, γ depend on the radius. In [1] , several special cases of (2) were analysed: for very thin layers ($\gamma \leq 0.15$), $M_{SB} \propto N(R_f).D^4$, imitating the interface scattering $\cos^4\Theta$ law as $\Theta \rightarrow 90°$, approaching grazing. Conversely, for thick layers, M_{SB} depends only on $N_o(R_f)$.D, the superficial density, eventually approaching a saturation Lambert Law $\propto \cos\Theta$, independent of the bubble density as $N_o(R_f)$ increases. This condition, which occurs practically at 60kHz, arises because the bubble cloud becomes so absorbent that it is not penetrated.

3.2 : Review of Additional Data

In situ photographic bubble counting, reported by Johnson and Cooke [2] , showed that at depths from 0.7 to 4m in ~ 15m/s windspeed, the bubble radius spectrum N(R,z) peaks at ~ 45 μm, the peak position and shape of the spectrum versus R being remarkably independent of depth, the e-folding depth being ~ 1.1m., reducing slightly for the larger bubbles, out to the maximum observed bubble size of 250 μm. For those parts of the radius range containing 3 depth samples, an exponential depth profile appeared consistent.

Similar results had been found for R dependence in a tank experiment reported by Glotov et al, [6] , with a peak at about 60 μm. This method, which uses a bubble 'catcher', and so is a little less direct, was also used by Kolovayev at sea [3] , in a similar or slightly lower sea state to that of Johnson and Cooke. His results show a peak at 80 μm, but again with similar depth gradient, and similar but slightly lower total number of bubbles. Insufficient data on different sea states exists, but a U^3 to U^5 law appears likely [7] .

Thorpe has studied bubble layers by the use of a 248 kHz upward looking beam 4 . His work gives information on concentration against depth for the bubbles near the peak of the distribution, and dependence of concentration and e -fold depth upon windspeed. His analysis is based on extraction of the local volume scattering strength, $M_v(z)$, as a function of depth. M_v should be proportional to bubble density. M_v is found to fit well to an exponential law, confirming the result of Johnson and Cooke. Fig. [1] shows $M_v(o)$.

extrapolated, and D, deduced from Thorpe's data, plotted against windspeed.

The windspeed dependence is $\sim U^3$ for bubble density, and $\sim U^1$ for D, in perfect agreement with the forms taken in [1] . The U^3 law is also sufficiently consistent with the very limited U dependence available from the photographic data to allow its retention.

A comparison between the model in [1] and these independent data may be stated by observing that the predicted windspeed dependence is confirmed, but for 50 μm to 250 μm bubbles, the e - fold depth is much less than predicted, although the acoustically significant superficial density N (R,z).D is in quite close agreement. A modification in the model retaining superficial density but reducing D is thus required.

3.3 : Bubble Dynamics

Thorpe has examined the transport equation for bubble density in detail [4,5,6] . He finds that provided that the transport can be modelled by a turbulent diffusion mechanism, the appropriate equation is in the following form, under stationary conditions, omitting insignificant terms,

$$K \frac{\partial^2}{\partial z^2} N + \left[\frac{\partial K}{\partial z} + V(R) \right] \frac{\partial N}{\partial z} - \phi(R,z,s,K) \cdot \frac{\partial N}{\partial R} = 0, \quad (7)$$

where K(z) is the vertical diffusion coefficient, and ϕ is a complicated function describing the bubble's contraction or expansion by gas exchange and to a lesser extent its change in depth. ϕ can be positive or negative , and depends strongly on the gas saturation, s. Thorpe's study included the case of K constant, and $K \propto z$. The latter is awkward for our purposes since $N \to \infty$ as $z \to 0$. K constant was taken in deriving the Model in [1] , and for large bubbles, where ϕ can be ignored, it gives an exponential depth law. With plausible ϕ terms Thorpe shows that K constant cannot give combined R, z dependence consistent with experiment for 40-250 μm bubbles, however [6] .

Briefly, we now consider the exponential law :

$$K(z) = K_o \exp(\alpha . z), \quad (8)$$

The resulting depth law, if we ignore ϕ for the time being, is then

$$N(R,z) = N_o(R) . [\exp(q e^{-\alpha z}) - 1] / [e^q - 1] . \quad (9)$$

where $q = V(R)/ \alpha K_o$.

This is asymptotically like $\exp(-\alpha z)$ for $q \ll 1$ (small bubbles), but like exp (- V(R).z/K) for q >> 1 (large bubbles). It thus agrees with Ref. [1] in the latter case, but for smaller bubbles gives a depth law independent of bubble size, as is observed experimentally.

Clearly, the exponential dependence of K cannot continue with z indefinitely, but we are in practice only interested in the top metre or so, so this does not matter. Neglect of the Ø term cannot be justified for all conditions, but at least close to the peak, where $\partial N/\partial R = 0$, it would appear reasonable, and it is also negligible for large bubbles, say R $\gtrsim$ 300 µm. (9) can be approximated for intermediate values of q, by retaining an exponential depth dependence, with D faired between the two limiting cases, as quantified in (12) below, with in addition a small creation depth

3.4 : Revised Bubble Distribution Law

The following modified bubble population is proposed, all quantities being in MKS units, for R $\geq$ 25 10^{-6} m

$$N\ (R,z) = S(R).\ \exp(-\ z/D)/V \qquad (10)$$

where the source function is :

$$S(R) = 3.10^{-13}\ U^3\ .\ R^{-4}\ \exp\left[\ -(4.7\ 10^{-5}/R)^5\ \right], \qquad (11)$$

$$D = \alpha^{-1}\left[1 + q\ (q + 1.4)/(q + 5.6)\right]^{-1} + 50.R \qquad (12)$$

$$V = \mathrm{Min}\left\{\left[0.15\ R^{-1/2} + 4.6\ 10^{-7}\ R^{-2}\ \right]^{-1},\ 0.25\right\}, \qquad (13)$$

$$\alpha = 13.5\ /\ U, \qquad (14)$$

$$q = V/K\alpha = 22.4V, \qquad (15)$$

$$K_0 = 3.3\ 10^{-3}\ U. \qquad (16)$$

This approximates the earlier model at low kHz frequencies, but has the additional property of being consistent with the data of Johnson and Cooke and of Thorpe, for smaller bubbles. (13) is a slight modification to the velocity law used in [1], to agree more closely with other recent formulae. The model gives acoustic back scatter consistent with experiment at frequencies from ~ 2 kHz to 60 kHz. In particular, the comparisons with experiment given in [1] are not appreciably altered.

4 : COMPOSITE INTERFACE BACKSCATTER

In [1], the backscattering strength was taken as that derived from the lowest order Rayleigh-Marsh method, which for a saturated sea state is given by

$$M_{SI} \simeq M_{SIO} = 4\ \lambda^2\ E\ (2\mu) \simeq\ 2.10^{-4}\ \cot^4\theta, \qquad (17)$$

Proposals have been made that this should be increased, because the low amplitude short waves at the Bragg wavenumber 2μ ride on the back of the longer waves, with 'local' grazing angles differing from the global angle $\pi/2 - \theta$. The most recent such treatment is that of McDaniel and Gorman [8].

The fundamental problem is to choose a bifurcation wave number, ν_L , splitting the wave spectrum into short and long domains,

$E(\nu) = E_S\ (\nu) + E_L\ (\nu)$ respectively,

for the long waves are then simply treated by computing their slope tensor

$$\sigma_{ij}^2 = \int_0^{2\pi} d\theta \int_0^{\nu_L} \nu\, d\nu \quad \nu_i \nu_j \quad E(\nu, \theta) \tag{18}$$

σ_{ij}^2, in practice just the component in the plane of incidence, $\sigma_{\parallel}^2$, is then used to compute the augmented backscattering strength. Clearly σ_{ij} depends on the value of ν_L chosen, and because of the $\cot^4\theta$ law, at near grazing incidence, the modified M_{SI} is strongly sensitive to $\sigma_{\parallel}^2$. McDaniel and Gorman set $\nu_L = 2\mu$ – i.e. all waves longer than the first order active wave are taken as contributing. This is questionable, since waves only just longer than the Bragg wavelength cannot be pictured as representing a long scale slope.

The question can be investigated in the limit by considering higher order Rayleigh theory. As a function of mean square slope $\sigma_{\parallel}^2$, composite scattering theory predicts.

$$M_{SI}(\theta) = M_{SI} + \tfrac{1}{2} M_{SI}'' \; \sigma_{\parallel}^2 + O(\sigma_{\parallel}^3).$$

For a given long scale wave spectrum, M_{SI}'' is calculable from third order Rayleigh-Marsh terms, and agrees with that given by composite theory in the limit of long scale wavelength going to infinity, but for a finite long scale wave-number, ν, we find that the ratio of contribution to M_{SI} to that due to an infinitely long wave of the same slope is:

$$\eta(\nu) = \frac{\lambda^2}{\nu^2} \Big\langle \mathrm{Re} \Big\{ |\lambda_{\mu+\nu}|^2 E_s(2\mu+\nu) + |\lambda_{\mu-\nu}|^2 E_s(2\mu-\nu) + \big[\tfrac{1}{2}(\lambda_{\mu+\nu} - \lambda_{\mu-\nu})^2 - \lambda.(\lambda_{\mu+\nu} + \lambda_{\mu-\nu})\big] E_s(2\mu) \Big\} \Big\rangle_\theta \div$$

$$\div \big\langle [(3\mu^2 - \lambda^2) E_s(2\mu) - 4\mu\lambda^2 E_s'(2\mu) + \lambda^4 E_s''(2\mu)] \big\rangle_\theta \tag{19}$$

In the above, $\lambda_{\mu\pm\nu} = \sqrt{k^2 - |\mu \pm \nu|^2}$, E_s' and E_s'' are derivatives of power spectrum in the μ direction and $\langle \cdots \rangle_\theta$ implies an average with respect to wave direction of ν.

$\eta(\nu)$ is the effectiveness of wavenumber ν in increasing backscatter. Fig.(2) shows a plot of $\eta(\nu)$ against the long scale/Bragg wavelength ratio, $2\mu/\nu$, for the sea surface short scale spectrum

$E_S(k) \propto k^{-4}$.

It can be seen that as expected $\eta(\nu) \to 1$ as $\nu \to 0$, but for near grazing incidence, moderate wavelengths become progressively inefficient at augmenting backscatter, and that therefore the acoustically effective mean square slope, which may be re-estimated as

$$\sigma_{\parallel}^2 \simeq \int_0^{2\pi} d\theta \int_0^{2\mu} \eta(\nu).\nu^2 \cos^2\theta \; E(\nu)\, d\nu \tag{20}$$

is much less than that found by simply taking all wavelengths at equal value. This has been done for the Pierson-Moskowitz spectrum for a fully aroused sea without regard to azimuth:

$$E(\nu) = \frac{8.1\ 10^{-3}}{4\pi}\ \nu^{-4}\ \exp\left[-0.74\ g^2 / \nu^2 U^4\right] \qquad (21)$$

The resulting $\sigma_{\shortparallel}^2$ was inserted into the composite scatter model of Reference [8], and the results are shown, for various frequency.windspeed products, in Fig. (3). The enhancement is now much less than that found by McDaniel and Gorman using an unmodified $\sigma_{\shortparallel}^2$ estimate. Experiemental backscattering enhancement over lowest order Rayleigh theory thus exceeds that predicted by interface alone, at all frequencies above $\sim$ 1 kHz.

5 : SUMMARY

A revised model for bubble distributions against radius, depth and windspeed has been developed, being simultaneously consistent with average experimental backscattering strengths between about 1 and 60 kHz, and with recent photographic and non-resonant acoustic scatter data. Laws for windspeed dependence of bubble concentration and depth agree well between data sources, and with the author's earlier model. Interface backscattering has been re-examined, and it is found that whilst the interface tends to dominate at low windspeed, any increase in interface backscatter with windspeed is masked by the larger effect of the growing bubble layer. Figs. 4 and 5 illustrate comparisons between the model and experiment.

6: REFERENCES

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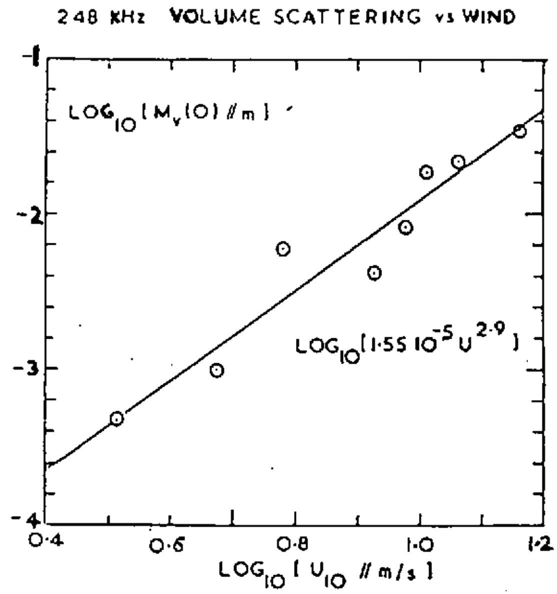

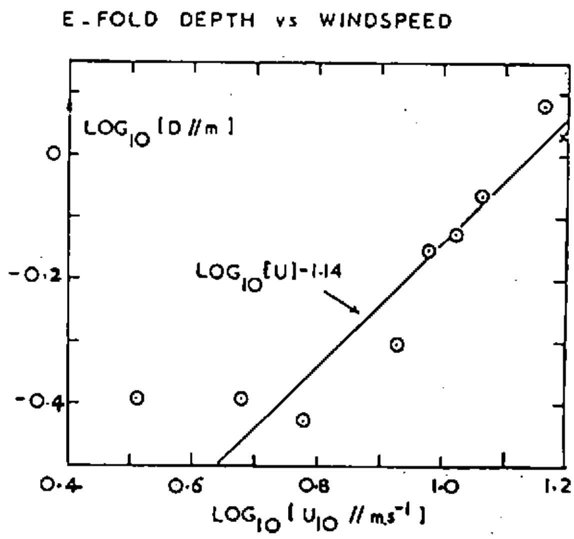

Fig.1 : Volume scattering at 248 kHz, showing the windspeed dependence of M_v (0) and the e-fold depth, D, deduced from Thorpe [4]. M_v (0) is interpreted as non resonant scatter from bubbles of $\gtrsim$ 50μm radius. ⊙ denotes experimental points.

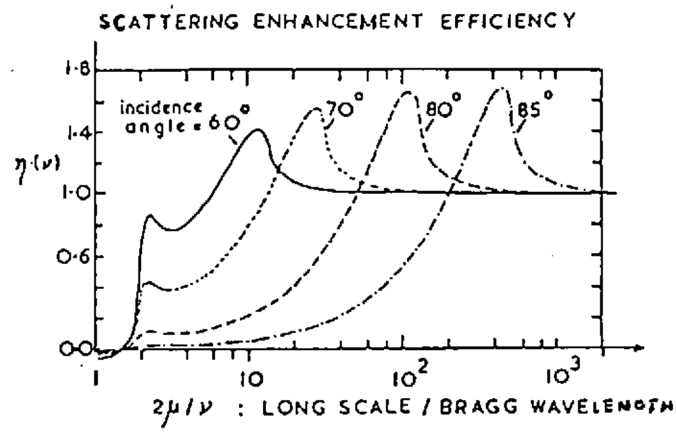

Fig.2 : Illustration of the relative efficiency of longer waves in enhancing backscatter from a k^{-4} short scale surface, as a function of wavelength ratio.

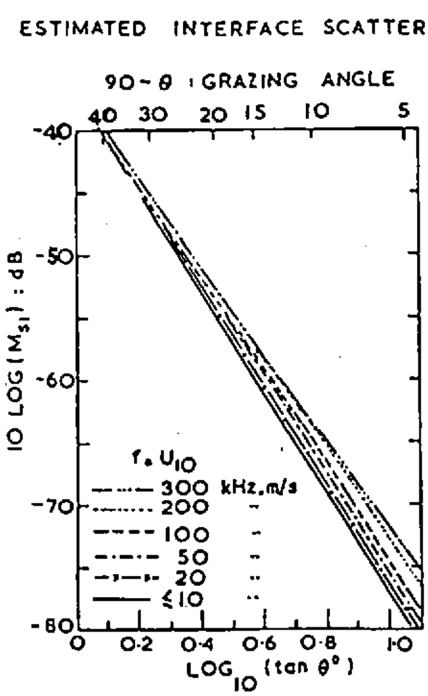

Fig.3 :

Estimated increase in M_s resulting from long scale slopes modulating the local backscatter, and allowing for relative modulation efficiency, for Pierson Moskovitz spectrum, with frequency windspeed product as parameter.

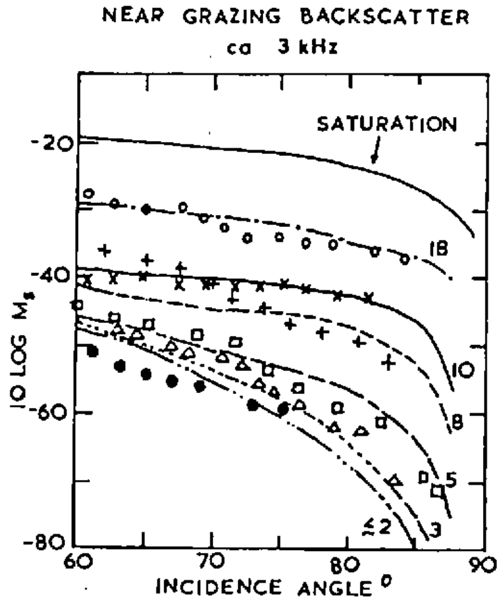

Fig.4 : Backscattering at ca 3kHz.[?]. Lines are from the theory, at m/s windspeeds indicated. Points are experimental, at U_{10} windspeeds :

0 18 m/s, x 10 m/s, + 8 m/s, □ 6 m/s, △ 4 m/s, ● 1 m/s.

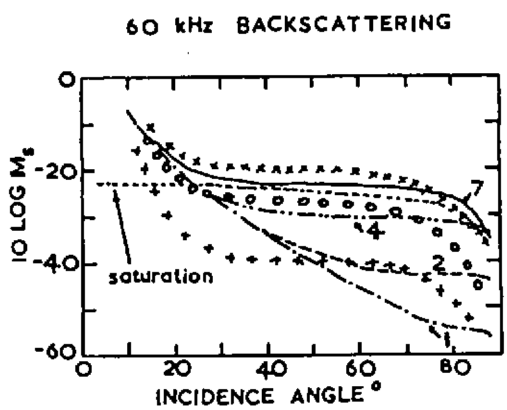

Fig.5 : Backscattering at 60kHz from [9]. Lines are from theory, with m/s windspeeds as indicated. Points are experimental, at windspeeds :

x 7 m/s, 0 4 m/s, + 2 m/s.