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Sound generated by cavitating flows in a duct

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1. Introduction.

The impetus for this work stems from the current interest in measuring the sound produced by cavitation in hydraulic systems in order to monitor the rate of cavitation erosion. Most hydraulic machines, valves and metering orifices operate with a certain amount of cavitation since its complete elimination is not always feasible. There is some evidence to suggest that cavitation erosion rate and sound pressure level produced in the liquid are closely related, Deeprose et al. (1974). Therefore as a first step towards quantifying this relationship we have studied the sound radiated by several cavitating flows in order to determine scaling laws in terms of fluid velocity, cavitation number and flow size. Two configurations were chosen which between them exhibited several distinct types of cavitation, which have been classified by Knapp et al. (1970) as travelling, fixed and vortex types. The two configurations were a venturi-type section and a sudden expansion. These were produced in the parallel-sided working section of a cavitation tunnel by inserting a convergent-divergent wedge with a sharp edge at the throat and a rear facing step with a streamlined upstream portion (figure 1).

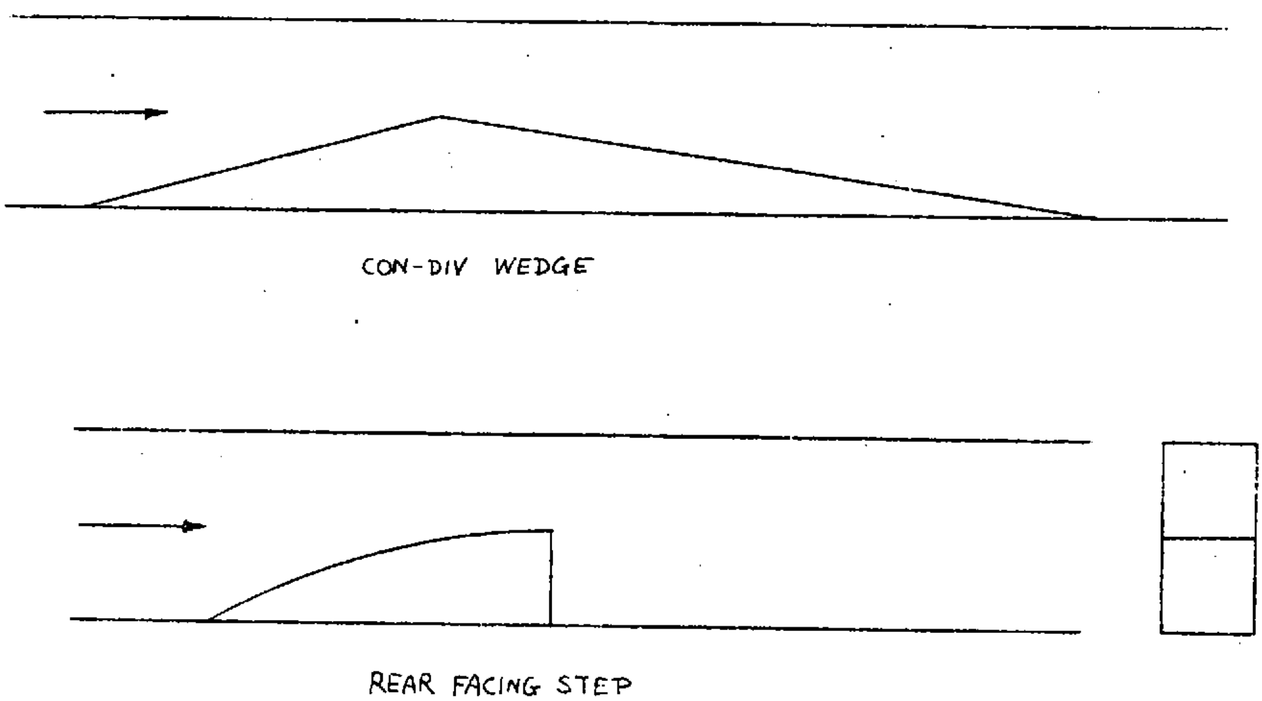

Figure 1

At high cavitation number just below inception, the cavitation produced by the con-div wedge is the travelling bubble type. As the cavitation number is reduced the bubbles coalesce to produce a fixed cavity and at a certain point the cavity pulsates in an approximately periodic manner. For the rear facing step at high cavitation number, the cavitation is of the vortex type, with bubbles forming in vortices produced in the shear layer between the main flow and the separated region. As cavitation number is reduced, the separated region eventually becomes a fixed cavity, which also pulsates at a certain cavitation number. In both cases at low enough cavitation number the cavity becomes indefinitely long and the flow is said to be at breakdown. On the practical side, the con-div wedge is intended to represent the blade of a centrifugal pump impeller and the rear facing step to represent a valve or orifice.

We have also proposed a simple theoretical model for the sound radiation from each type of cavitation, using Rayleigh's analysis of a collapsing spherical cavity as a starting point. From this model we have deduced scaling laws and compared them with the laws actually measured. The measurements were made using a flush mounted quartz piezoelectric pressure transducer which could be positioned at various points in the tunnel working section. We used a Vibrometer pressure transducer type 6QP500 and it was coupled to a Vibrometer charge amplifier type TA-3/C. The signal was analysed using a B & K sound level meter type 2203 as an RMS meter. The cavitation tunnel was designed so that the pressure and flow rate could be controlled independently and therefore the tunnel could be operated either at constant velocity or constant cavitation number.

2. Acoustic power produced by cavitation.

Since cavitation is essentially a volume pulsation, the sound is effectively produced by a distribution of monopoles. The strength of these sources can be estimated by considering the flow field surrounding a collapsing empty spherical cavity. This problem was first investigated by Rayleigh (1917) and he obtained a solution for the motion of the cavity wall as a function of time. He also calculated the pressure perturbation at any point in the liquid and, although he was interested only in the pressure produced near the cavity, his result can be used to obtain the pressure at great distances. Even though the theory assumes incompressible flow, this result is equivalent to the far field sound pressure provided that the sound wavelength is much greater than the size of the cavity. The acoustic energy produced by a single collapsing cavity can then be calculated by integrating the square of the pressure perturbation over the time taken for the cavity to collapse. Unfortunately this result gives infinite energy because the pressure becomes indefinitely large as the cavity approaches zero radius. However in reality the collapse is arrested at a small but finite radius by gas trapped in the cavity. We assume that the minimum cavity radius is always the same fraction of the maximum size, i.e., this ratio is independent of the external pressure. This is equivalent to assuming that the gas content or volume fraction of the liquid is always the same. On making this assumption it is possible to deduce a scaling law for the acoustic energy radiated by a single cavity as it collapses. The acoustic power produced by a distribution of cavitation bubbles is therefore the total amount of acoustic energy produced per unit time, which can be obtained by simply summing the contribution from each cavity assuming it is uncorrelated with the remainder. We find that the scaling law for the acoustic power W is given by,

$$W \sim \frac{\nu\rho}{c} \left(\frac{\Delta p}{\rho}\right)^{3/2} R_m^{\ 3} \tag{1}$$

where ν is the number of cavities collapsing per unit time, Δp is the difference between the ambient pressure and the saturated vapour pressure, and R_m is the maximum radius of the cavity and ρ and c are density and speed of sound of the liquid. This result is effectively given by Ross (1976) and further details of the derivation can be found in Lush and Hutton (1976).

The result (1) applies strictly to the case of radiation in a free field. Since we are concerned with cavitation confined in a duct, we expect that this result requires some modification. The peak frequency of cavitation noise is in the region of 1 kHz (see Lush (1975)) and at this frequency the sound wavelength is 1.5m. Consequently for duct sizes less than about half this amount, only the plane wave mode can propagate. This has a profound effect on the sound pressure variation during the collapse of the cavity and the acoustic power radiated.

In essence for the plane wave case, the fluid motion between the collapsing cavity and the sound waves in the duct is incompressible. Consequently the particle velocity of the plane wave and hence the pressure amplitude are directly proportional to cavity wall velocity. This is in contrast to the free field case where it can be shown that the pressure amplitude is proportional to the square of the cavity wall velocity. Therefore on making the same calculations as in the free field case, we obtain a scaling law for acoustic power for the plane wave mode as follows:

$$W \sim \nu\rho c \sqrt{\frac{\Delta p}{\rho}} \; \frac{R_m^{\ 5}}{A}, \tag{2}$$

where A is the cross sectional area of the duct. We find that this result (2) applies only at high cavitation number when the cavitation bubble volume fraction is very small. It appears that, as soon as the extent of the bubble region increases to a significant fraction of the size of the duct, the sound speed in the duct is reduced. This has the effect of decreasing the sound wavelengths and allowing more duct modes to propagate. The sound speed in a liquid with gas bubbles is very low indeed and since we would expect a large number of modes to propagate, the sound field can be regarded as diffuse. The acoustic energy is the same as in a free field but with the energy channelled down the duct. Therefore equation (1) gives the acoustic power at sufficiently low cavitation number.

3. Scaling laws for sound pressure level.

We now consider the conversion of the results for acoustic power (1) and (2), into scaling laws for sound pressure level, i.e., mean square pressure, since this is the quantity measured in practice. For both the plane wave mode and the diffuse field, the acoustic intensity will be uniform across the duct. In the former case it will be equal to $\overline{p^2}/\rho c$ and in the latter, because the intensity is the same in all directions, it will be one quarter of this. Thus in both cases, the scaling for mean square pressure is obtained by multiplying the power by $\rho c/A$.

Referring to (1) and (2), it can be seen that we require scaling laws for ν, the number of cavities collapsing per unit time, Δp, the difference between ambient pressure and vapour pressure and R_m, the maximum size of the cavity. The scaling laws for these quantities depend upon the type of cavitation involved which in turn depends on the configuration and the cavitation number. The cavitation number, σ, relates pressure p and velocity U in the vena contracta of the flow and it is virtually independent of the configuration. It is defined as,

$$\sigma = \frac{p - p_v}{\frac{1}{2}\rho U^2} \tag{3}$$

where p_v is the saturated vapour pressure at the bulk liquid temperature.

At fairly high cavitation number just below inception, each configuration exhibits a different type of cavitation. The cavitation for the con-div wedge consists of travelling bubbles and that for the rear facing step is the vortex type. As cavitation number is reduced in each case the cavitation becomes a fixed cavity which also pulsates at a certain cavitation number. For the travelling bubble cavitation, we assume that the bubbles are appearing and disappearing at an average rate proportional to the rate of production of eddies, i.e., U/t, where t is the throat width. We assume that the bubbles are collapsing in the region of the stagnation point downstream of the throat where the flow reattaches to the wall. Since the pressure in the throat will be nearly equal to vapour pressure, the pressure difference tending to collapse the bubbles will be proportional to the dynamic pressure $\frac{1}{2}\rho U^2$. The maximum bubble size will be proportional to the length of the bubble region and this may be estimated by assuming that the bubbles effectively form a single vapour cavity, whose free surface lies on the arc of a circle. From this it may be deduced that the cavity length and hence R_m scales as $t\sigma^{-1}$ (see Lush and Hutton (1976) for details).

Similar arguments may be advanced for the vortex type cavitation except in this case the bubble size depends upon the reduction in pressure produced by vortices embedded in the shear layer. This reduction may be estimated from the cavitation number at inception, σ_i, and it may be shown that R_m scales as $t\sqrt{\sigma_i - \sigma}$.

When for the con-div wedge the travelling bubble cavitation gives way to the fixed cavity, the scaling for ν, Δp and R_m remains essentially the same until the cavity begins to pulsate. At this point, the frequency ν becomes inversely proportional to the cavity length and hence depends on σ, i.e., as $U\sigma/t$. For the rear facing step, the scaling for R_m changes when the cavitation becomes the fixed cavity type. By again assuming that the cavity is vapour filled with a free surface lying on the arc of a circle it is possible to show that cavity length and hence R_m scales as $t\sigma^{-\frac{1}{2}}$.

For both the travelling bubble and vortex types, the cavitation number is high enough that only the plane wave mode is propagating. Thus the above results are substituted in equation (2) to obtain the scaling laws for mean square pressure. For the various stages of the fixed cavity type, the cavitation number is low and the sound field is diffuse. In these cases, the results are substituted in equation (1). Because of the complex situation a wide variety of scaling laws are produced and these are summarised in the table.

TABLE

SCALING LAWS FOR MEAN SQUARE PRESSURE		CAVITATION TYPE			
		TRAVELLING BUBBLE	VORTEX	FIXED CAVITY	PULSATING CAVITY
CON-DIV WEDGE	PLANE WAVE	$\rho^2 c^2 U^2 \sigma^{-5}$	—	—	—
	DIFFUSE	$\rho^2 U^4 \sigma^{-3}$	—	$\rho^2 U^4 \sigma^{-3}$	$\rho^2 U^4 \sigma^{-2}$
REAR FACING STEP	PLANE WAVE	—	$\rho^2 c^2 U^2 \times (\sigma_i - \sigma)^{5/2}$	—	—
	DIFFUSE	—	$\rho^2 U^4 (\sigma_i - \sigma)^{3/2}$	$\rho^2 U^4 \sigma^{-3/2}$	$\rho^2 U^4 \sigma^{-1}$

4. Experimental results and comparison with scaling laws.

We have measured the sound pressure level of cavitation produced by the con-div wedge over a range of cavitation number at a fixed velocity (fig.2). These results show essentially the sound pressure for the fixed cavity. At high cavitation number, the variation is σ^{-3} which is the diffuse field result for either travelling bubbles or a fixed cavity which is oscillating at a frequency not connected with the cavity length. At slightly lower cavitation number, the dependence becomes σ^{-2} when the fixed cavity pulsates at a frequency inversely proportional to cavity length. At even lower cavitation number, the dependence becomes σ^{-1} and at this point the cavity is longer than the divergent part of the wedge. The flow becomes similar to that for the rear facing step and therefore the σ dependence alters accordingly to that appropriate for the pulsating cavity behind a step.

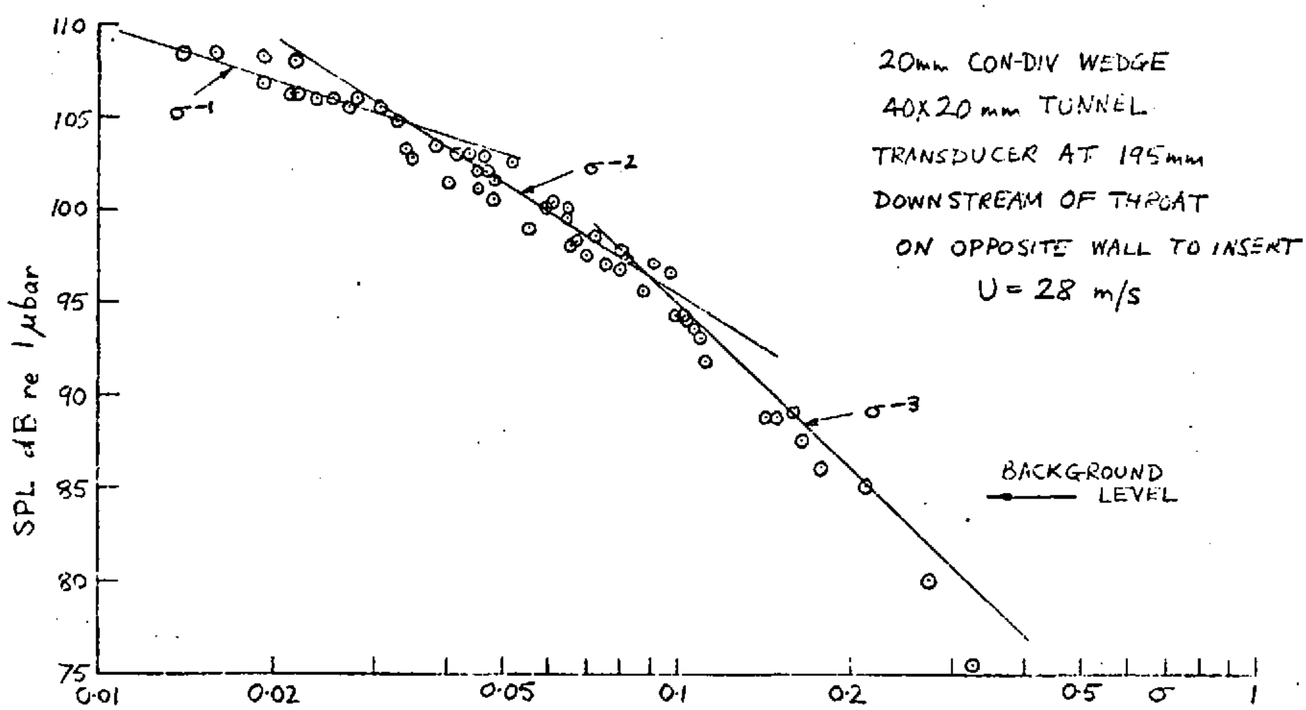

Figure 2

The variation of sound pressure level with σ for the rear facing step (fig. 3) shows clearly the sound produced by vortex type cavitation.

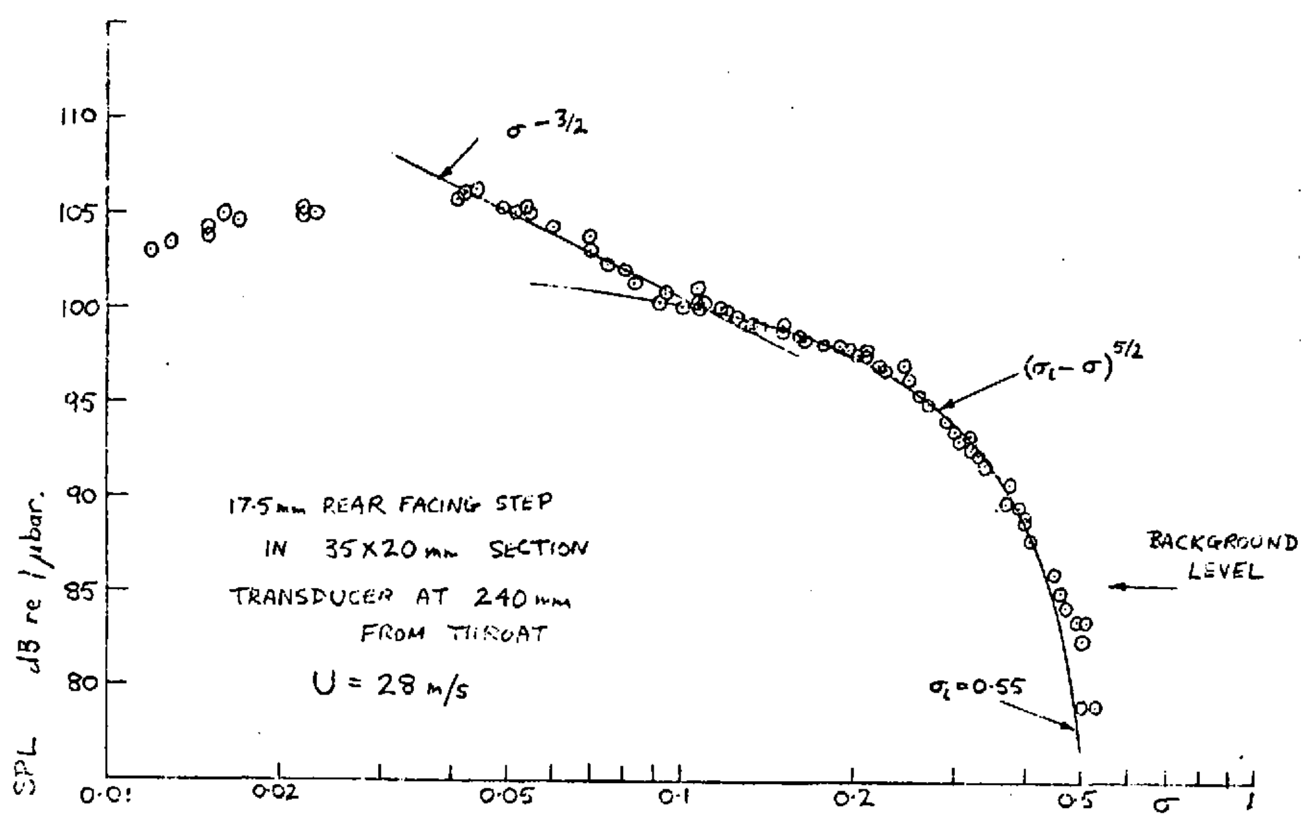

Figure 3

At high cavitation number the variation is that for the plane wave mode, i.e., $(\sigma_i - \sigma)^{5/2}$. This should give way to the diffuse field result at intermediate cavitation numbers but the change will not be detectable because the variation is rather slow anyway. The sound pressure rises when the fixed cavity type begins at about $\sigma = 0.1$ and increases approximately at the rate appropriate to the non-pulsating fixed cavity, i.e., $\sigma^{-3/2}$.

We have also measured the sound pressure as a function of velocity for the con-div wedge at fixed cavitation numbers of $\sigma = 0.1$ and 0.3 (fig. 4). At the lower cavitation number the velocity dependence is very nearly U^4 as predicted for the diffuse field case. Since the diffuse field will revert to the plane wave mode at high enough σ, we might expect the velocity dependence at $\sigma = 0.3$ to be nearer U^2. The result indicates that the variation is about U^3, which is noticeably less than variation at the lower value of σ. Presumably several lower order modes are propagating in addition to the plane wave mode and hence the dependence is between U^2 and U^4. The dependence should approach U^4 as the velocity is increased because the associated frequency increase will allow more modes to propagate.

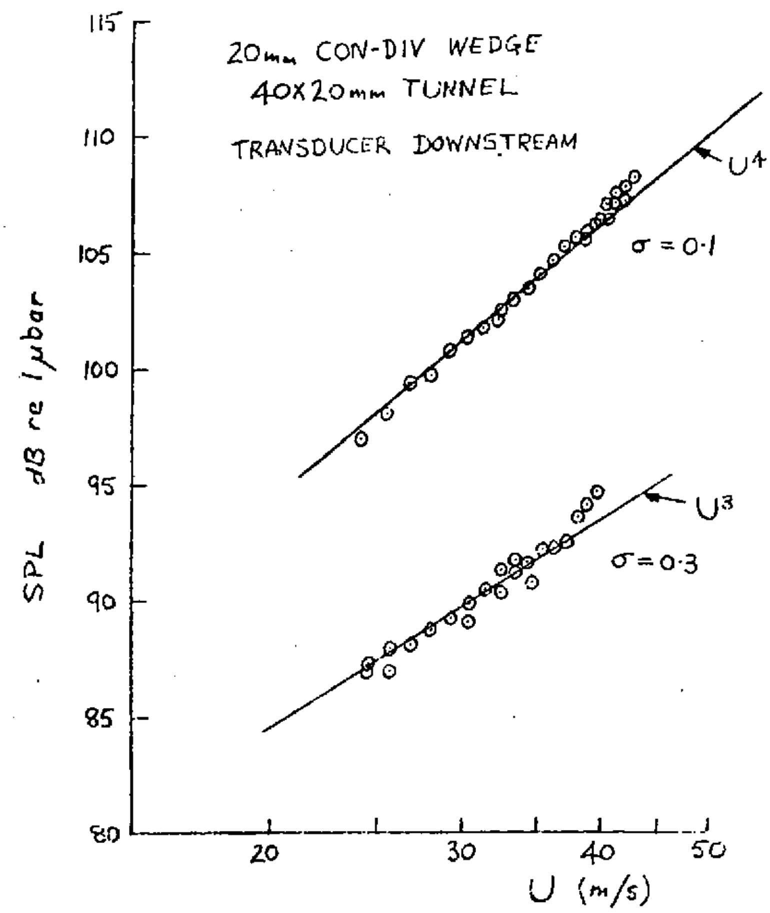

Figure 4

The scaling laws for mean square pressure shown in the table do not involve the length scale of the flow. Measurements of the sound produced by two geometrically similar sudden expansions differing in size by a factor of two indicate that the sound pressure level is very nearly independent of scale (fig. 5). The two tests were made at the same throat velocity and with the transducers in geometrically similar positions. The variations are similar over most of the cavitation number range except near breakdown ($\sigma = 0$), where the cavities are nearing the end of the tunnel working section.

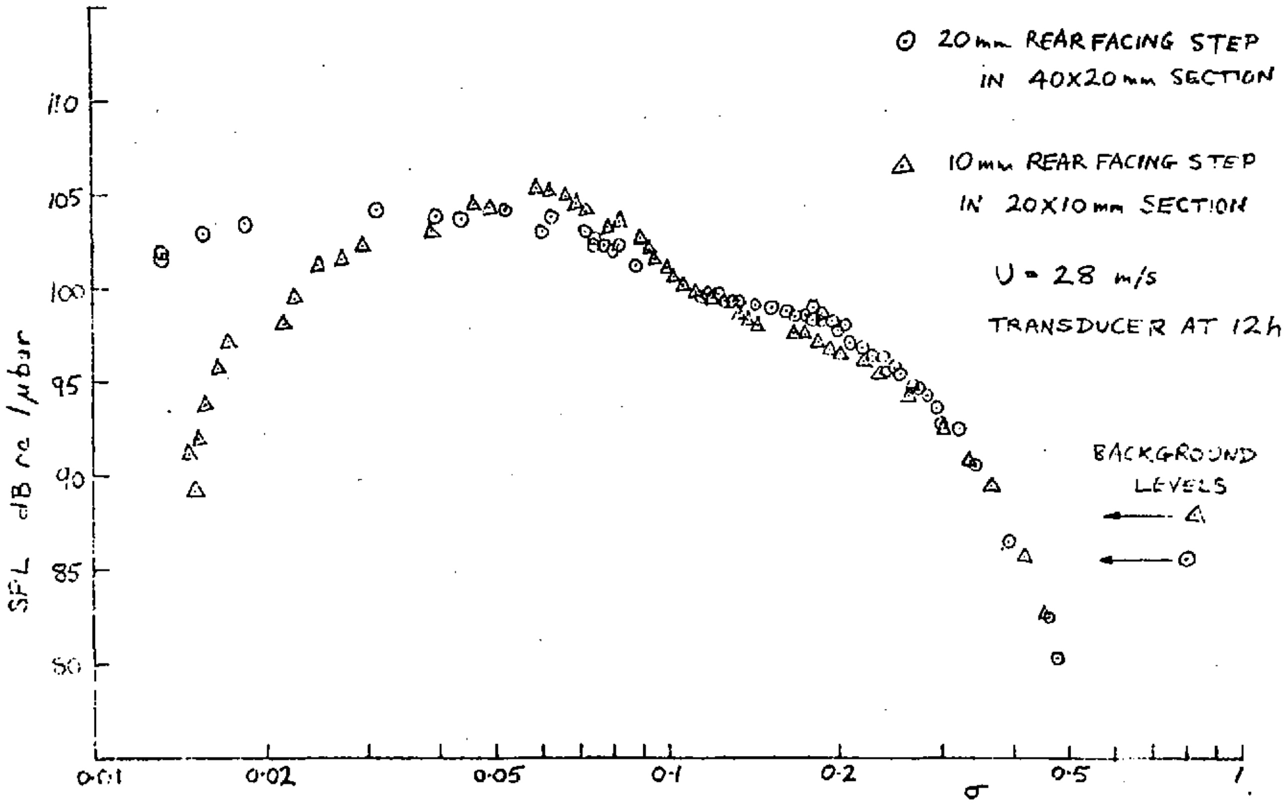

Figure 5

5. Conclusions.

We have found that the cavitation noise produced by venturi-type and sudden expansion configurations can be qualitatively described by scaling laws deduced from Rayleigh's model of a collapsing spherical cavity. In order to apply the general results for acoustic power to these configurations we had to derive simple relations for the number of cavities collapsing (per unit time), the pressure causing collapse and the maximum cavity size. We found that these depended upon the type of cavitation and the configuration.

We found that the effect of the duct on the sound generated could be adequately accounted for by considering the limiting cases of long wavelengths, when only the plane wave is propagating, and that of short wavelengths, when the sound field is diffuse. It appeared that the plane wave result applied at very high cavitation number just below inception when the extent of the cavitation region was very small. At low cavitation numbers, we argued that the increase in the extent of cavitation effectively reduced the sound speed in the duct and allowed more modes to propagate so that the sound field could be treated as diffuse.

There was a wide variety of σ dependences for the sound pressure as summarised in the table. We were able to attribute the differences in σ

dependence to changes in cavitation type or differences in the geometry of the flow. We found four different types of cavitation, namely, travelling bubble, vortex type in a shear layer, fixed cavity and a fixed cavity which pulsated periodically. In general we found that the velocity dependence of the noise at constant cavitation number was U^4, except possibly at high σ where it was nearer U^3. In addition the sound pressure level appeared to be independent of size of configuration.

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