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PERFORMANCE LIMITS FOR THE ACTIVE CONTROL OF RANDOM SOUND FIELDS FROM MULTIPLE PRIMARY SOURCES

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1. INTRODUCTION

The application of feed-forward active techniques to the control of sound has to date mostly been limited to cases where there is a single well-defined primary source. The classical problem of plane waves of random sound propagating down a duct is one such case; the field to be controlled can be characterised by the use of a single detection sensor in the duct on whose output signal a feed-forward controller can operate [1]. Similarly, problems of deterministic sound fields such as those associated with engine "boom" in automotive interiors [2] or propeller induced cabin noise in aircraft [3] have well-defined primary sources. Here we address the problem of sound fields generated by sources whose origin is less well-defined. Such instances arise, for example, in the case of tyre or wind noise in road vehicles or boundary layer noise in aircraft. In these cases it is far less easy to define a single detected signal which will characterise the offending signal and on which a feed-forward controller can operate. One is faced with characterising the offending sound field with multiple detected signals, obviously placing detection sensors in locations where they are most likely to sense the outputs of the multiplicity of primary sources contributing to the sound fields.

In this paper we address the multi-channel feed-forward control problem and determine the solution for the optimal matrix of filters which operates on a multiplicity (K) of detected signals to produce the outputs of a number (M) secondary sources and thereby minimise the sound field at (L) error sensors. The problem is firstly dealt with in the frequency domain where we impose no constraint of causality on the control filters. The results of this analysis will give approximate solutions for the maximum performance of the controller in cases where the time taken for sound to propagate from the detection sensors to the error sensors is sufficient to ensure that the impulse response of the optimal control filters is substantially causal. In any event, the results derived will establish an absolute upper bound on the controller performance for a problem with given detected signals and error signals associated with the primary field. The results therefore have considerable practical utility. We also deal with the problem in the time domain and restrict the control filters to have impulse responses which are both causal and finite. It is demonstrated that the problem can be formulated in classical Wiener-Hopf terms and the solution for the optimal controller is expressed in terms of the coefficients of a matrix of digital FIR filters. The appropriate generalisation of the LMS-based stochastic gradient algorithm formulated by Elliott *et al* [4] also follows from this analysis. Some slight modifications to the analysis are incorporated to enable the efficient numerical solution to the equations and thus again provide a useful starting point for the analysis of the potential for active control in any given practical problem.

2. THE UNCONSTRAINED OPTIMAL SOLUTION

Firstly we will deal with the problem in the frequency domain. This enables the derivation of the optimal reductions in level that can be achieved when using filters which are not constrained to be causal. It is useful both in this case and in the causally constrained case (dealt with below) to work with a modified form of the basic block diagram. The general problem is depicted in Figure 1. We assume that there are a discrete number P of uncorrelated primary sources whose outputs are represented by the vector $\mathbf{z}$. The signals due to the primary sources are transmitted via the transfer function matrix $\mathbf{A}$ and detected by L error sensors whose outputs are represented by the vector $\mathbf{d}$. Note that in addition to the sound due to the primary sources, the sensors may be prone to measurement noise represented by the vector $\mathbf{n}_2$. The sound due to the primary sources is also transmitted via the transfer function matrix $\mathbf{B}$ and detected by K detection sensors whose output signals are represented by the vector $\mathbf{x}$. Note that these signals may also contain measurement noise represented by the vector $\mathbf{n}_1$. These signals are passed through a matrix of filters $\mathbf{G}$ to produce the vector $\mathbf{y}$ of signals input to M secondary sources. These signals may also be fed-back via the transfer function matrix $\mathbf{F}$ and result in the corruption of the signals from the detection sensors. We will assume that $\mathbf{F}$ can be determined from measurements on the system and set out to determine the optimal form of the filter matrix $\mathbf{G}$ by firstly determining the optimal form of the filter matrix $\mathbf{H}$. The latter is defined as the filter matrix whose input is the signal vector $\mathbf{x}$ and whose output is the signal vector $\mathbf{y}$ (see Figure 1). Once $\mathbf{H}$ is determined then, in principle, if $\mathbf{F}$ is known, $\mathbf{G}$ can be determined and has the structure illustrated in Figure 2. This is readily demonstrated using manipulations in the frequency domain. We may write

$$\mathbf{y} = \mathbf{Gx} + \mathbf{GFy} \tag{1}$$

where $\mathbf{x}$ and $\mathbf{y}$ are vectors of Fourier transforms of the input and output signals and $\mathbf{F}$ and $\mathbf{G}$ are frequency response function matrices. It follows that

$$\mathbf{y} = [\mathbf{I} - \mathbf{GF}]^{-1}\mathbf{Gx} \tag{2}$$

where $\mathbf{I}$ is the identity matrix and therefore that

$$\mathbf{H} = [\mathbf{I} - \mathbf{GF}]^{-1}\mathbf{G} \tag{3}$$

This expression can be used to deduce $\mathbf{G}$ and it follows, after some rearrangement, that the expression for $\mathbf{G}$ may be written in the alternative forms

$$\mathbf{G} = \mathbf{H}[\mathbf{I} + \mathbf{FH}]^{-1} = [\mathbf{I} + \mathbf{HF}]^{-1}\mathbf{H} \tag{4}$$

which suggests the structure illustrated in Figure 2. (Proof of the second equality in equation (4) follows readily from successive pre-multiplication by $(\mathbf{I} + \mathbf{HF})$ and post-multiplication by $(\mathbf{I} + \mathbf{FH})$). This amounts to a filter implementation which ensures the cancellation of any feedback signals. Note that if $\mathbf{H}$ is constrained to be causal (as in the case dealt with later) then since in any physical system $\mathbf{F}$ must be causal, it is evident from Figure 2 that $\mathbf{G}$ will then also be causal. (The stability of $\mathbf{G}$ is not however guaranteed.) Henceforth we will proceed to determine the optimal filter $\mathbf{H}$ and assume that in practice the problem of feedback can be dealt with using this approach. There is of course a large class of problem in active noise control in which **F** is zero and the determination of the optimal value of **H** is all that is required.

The problem can thus be represented in the form illustrated in Figure 3. The matrix **C** is the matrix of electroacoustic transfer functions relating the M secondary source input signals **y** to the L signals $\hat{\mathbf{d}}$. This vector represents the signals produced by the secondary sources at the L error sensors. The error signals are thus defined by $\mathbf{e} = \mathbf{d} + \hat{\mathbf{d}}$. Before proceeding further we undertake another rearrangement of the block diagram that is illustrated in Figure 3. Again, working in the frequency domain we firstly write

$$\mathbf{y} = \mathbf{H}\mathbf{x} \qquad \hat{\mathbf{d}} = \mathbf{C}\mathbf{y} \tag{5}$$

and expand these expressions in terms of the columns $\mathbf{h}_k$ of **H** and the rows $\mathbf{c}_l^T$ of **C**. This gives

$$\mathbf{y} = [\mathbf{h}_1\ \mathbf{h}_2\ ....\ \mathbf{h}_K]\mathbf{x} \qquad \hat{\mathbf{d}} = \begin{bmatrix} \mathbf{c}_1^T \\ \mathbf{c}_2^T \\ \cdot \\ \cdot \\ \mathbf{c}_L^T \end{bmatrix} \mathbf{y} \tag{6}$$

The signal at the l'th error sensor can be written as

$$\hat{d}_l = \mathbf{c}_l^T\mathbf{y} = \mathbf{c}_l^T[\mathbf{h}_1 x_1 + \mathbf{h}_2 x_2\ .....\ \mathbf{h}_K x_K] \tag{7}$$

which is a scalar quantity which can be transposed to give

$$\hat{d}_l = \mathbf{h}_1^T\mathbf{c}_l x_1 + \mathbf{h}_2^T\mathbf{c}_l x_2 + ...\ \mathbf{h}_K^T\mathbf{c}_l x_K \tag{8}$$

This shows that the block diagram of Figure 3 may be rewritten in the form depicted in Figure 4. The latter shows that the l'th contribution to the vector **d** may be written in terms of the "filtered reference signals" r_{lmk}. These are the signals generated by passing the k'th detected signal x_k through the transfer function c_{lm} which comprises the l,m'th element of the matrix **C**. We will find it convenient to work with these filtered reference signals and further define the composite vectors

$$\mathbf{h} = \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \cdot \\ \mathbf{h}_K \end{bmatrix} \qquad \mathbf{r}_l = \begin{bmatrix} \mathbf{c}_l x_1 \\ \mathbf{c}_l x_2 \\ \cdot \\ \cdot \\ \mathbf{c}_l x_K \end{bmatrix} \tag{9}$$

where **h** is now a vector which contains all the elements of the matrix **H**.

We now seek to determine the optimal vector **h** of filters which ensures the minimum value of the sum of the power spectral densities of L error signals. Thus we define a cost function given by

$$J_\omega = \sum_{l=1}^{L} S_{e_l e_l} = \sum_{l=1}^{L} \lim_{T\to\infty} E\left[\frac{1}{T} e_l^* e_l\right] \tag{10}$$

where e_l is the Fourier transform of the error signal evaluated over some finite duration T. Here we assume that all the signals dealt with are stationary random processes and the expectation operator refers to an ensemble average. Thus the true power spectral density of the signals are derived only from averaging over the ensemble of records of infinite duration. This should be borne in mind when using the results derived below in practice, since practical estimates of the true power spectrum are necessarily based on time averaging over a number of records of finite duration. Since we assume a linear superposition of the primary and secondary sound fields we may put $e_l = d_l + \hat{d}_l$ and since we may use the modified block diagram to write

$$\hat{d}_l = \mathbf{r}_l^T \mathbf{h} = \mathbf{h}^T \mathbf{r}_l \tag{11}$$

the expression for the cost function reduces to

$$J_\omega = \sum_{l=1}^{L} \lim_{T\to\infty} E\left[\frac{1}{T}(d_l + \mathbf{h}^T\mathbf{r}_l)^*(d_l + \mathbf{r}_l^T\mathbf{h})\right] \tag{12}$$

Since the vector **h** comprises the frequency response functions of the (linear, time invariant) filters to be determined, use of the expectation operator shows that the expression for J_ω reduces to the Hermitian quadratic form given by

$$J_\omega = \mathbf{h}^H\mathbf{A}\mathbf{h} + \mathbf{h}^H\mathbf{b} + \mathbf{b}^H\mathbf{h} + c \tag{13}$$

where H denotes the Hermitian transpose and the matrix **A**, vector **b** and scalar c are defined by

$$\mathbf{A} = \sum_{l=1}^{L} \lim_{T\to\infty} E\left[\frac{1}{T}\mathbf{r}_l^*\mathbf{r}_l^T\right] \qquad \mathbf{b} = \sum_{l=1}^{L} \lim_{T\to\infty} E\left[\frac{1}{T}\mathbf{r}_l^* d_l\right] \tag{14}$$

$$c = \sum_{l=1}^{L} \lim_{T\to\infty} E\left[\frac{1}{T} d_l^* d_l\right] = \sum_{l=1}^{L} S_{d_l d_l} \tag{15}$$

The latter can be recognised as the sum of the power spectra of the signals produced at the error sensors by the primary field alone. The optimal value of the filter vector **h** and the corresponding minimum value of the sum of the error sensor power spectra are thus given by

$$\mathbf{h}_0 = -\mathbf{A}^{-1}\mathbf{b} \qquad J_{\omega_0} = c - \mathbf{b}^H\mathbf{A}^{-1}\mathbf{b} \tag{16}$$

The condition for the existence of this minimum is that **A** be positive definite. It is clear that in the absence of the primary field (*i.e.,* all d_l are zero) then the sum of the power spectra due to the secondary sources alone is given by $\mathbf{h}^H\mathbf{A}\mathbf{h}$. This will clearly be greater than zero for all non-zero values of **h** provided all the elements of the vector **x** of detected signals are non-zero. This therefore defines a sufficient condition for the positive definiteness of **A** and the existence of a unique minimum of J. Equations (14), (15) and (16) thus define the optimal values of the filter vector **h** (and thus the optimal values of the elements of the filter matrix **H**) and also the maximum reduction of the chosen cost function that can be achieved. These equations constitute one of the principal results of this paper. To evaluate these optimal results, access is required to both the filtered reference signals r_{lmk} and the primary field signals d_l. In general, the former require a knowledge of both the detected signals x_k and the transfer functions c_{lm}, although in several special cases we shall show below that a knowledge of the transfer functions c_{lm} is not required.

3. RESULTS FOR A SINGLE DETECTION SENSOR AND SINGLE ERROR SENSOR

In this case we have a filter H, a single transfer function C and a single filtered reference signal r. The matrix A reduces to the scalar given by

$$A = \lim_{T\to\infty} E\left[\frac{1}{T}r^*r\right] = S_{rr} \tag{19}$$

and is therefore the power spectral density of the filtered reference signal. The vector **b** also reduces to the scalar given by

$$b = \lim_{T\to\infty} E\left[\frac{1}{T}r^*d\right] = S_{rd} \tag{18}$$

and is the cross power spectral density between the filtered reference signal and the error signal. Note that we may also write

$$S_{rr} = \lim_{T\to\infty} E\left[\frac{1}{T}(xC)^*(xC)\right] = S_{xx}|C|^2 \tag{19}$$

$$S_{rd} = \lim_{T\to\infty} E\left[\frac{1}{T}(xC)^*d\right] = S_{xd}C^* \tag{20}$$

Thus the optimal filter may be written as

$$H_0 = -\frac{S_{rd}}{S_{rr}} = -\frac{S_{xd}}{S_{xx}C} \tag{21}$$

and the corresponding minimum value of the power spectral density of the error signal reduces to

$$J_{\omega 0} = S_{dd} - \frac{S^*_{rd} S_{rd}}{S_{rr}} = S_{dd} - \frac{S^*_{xd} S_{xd}}{S_{xx}} \quad (22)$$

If we write $J_d = S_{dd}$ as the value of the cost function due to the primary field only, then we have

$$\frac{J_{\omega 0}}{J_d} = 1 - \frac{|S_{xd}|^2}{S_{xx} S_{dd}} = 1 - \gamma^2_{xd} \quad (23)$$

where γ^2_{xd} is the *coherence* between the signal from the detection sensor and the signal from the error sensor due to the primary field and measurement noise. This result gives an extremely useful means of easily assessing the potential for active control in any given situation. The coherence (or at least an estimate of the coherence) is readily computed using modern spectrum analysers. Thus, given a signal from a detection sensor (typically placed close to a primary source) and a signal from an error sensor (placed at the position where reductions in level are sought) the computation of $10 \log_{10}(1 - \gamma^2_{xd})^{-1}$ gives the best reduction in level (in decibels) of the error signal that could possibly be produced by the action of active control. This result has the attraction that it can be derived without implementing active control, but it should be emphasised that it places an *upper bound* on the achievable performance of a controller; it assumes the filter is constrained to have neither a causal nor a finite impulse response.

4. RESULTS FOR MULTIPLE DETECTION SENSORS AND A SINGLE ERROR SENSOR

In the case of a random sound field generated by, for example, a number of independent primary sources, we may seek to minimise the power spectral density at one location by operating on K detection signals. One would clearly seek to ensure that the detected signals between them were able to characterise the output of the primary sources. In this case the matrix A can be written as

$$\mathbf{A} = \lim_{T\to\infty} E\left[\frac{1}{T}(Cx)^*(Cx)^T\right] = |C|^2 S_{xx} \quad (24)$$

where S_{xx} is the matrix of cross-spectra of the signals from the detection sensors. The vector **b** reduces to

$$\mathbf{b} = \lim_{T\to\infty} E\left[\frac{1}{T}(Cx)^* d\right] = C^* s_{xd} \quad (25)$$

where the vector s_{xd} is the vector of cross-spectra between the detected signals and the signal at the error sensors due to the primary field and measurement noise. The solution for the optimal filter vector thus reduces to

$$h_0 = -\frac{1}{C} S_{xx}^{-1} s_{xd} \quad (26)$$

and the minimum value of the error sensor spectral density is given by

$$J_{\omega 0} = S_{dd} - \mathbf{s}_{xd}^{H}\mathbf{S}_{xx}^{-1}\mathbf{s}_{xd} \qquad (27)$$

Again putting $J_d = S_{dd}$, this expression can be written in non-dimensional form as

$$\frac{J_{\omega 0}}{J_d} = 1 - \frac{\mathbf{s}_{xd}^{H}\mathbf{S}_{xx}^{-1}\mathbf{s}_{xd}}{S_{dd}} = 1 - \eta^2_{xd} \qquad (28)$$

where η_{xd} is the *multiple coherence function* between the detection sensor inputs and the output from the single error sensor. Again it can be shown that this value varies between zero and unity and its estimation in a given practical situation immediately evaluates the potential for active control. A concise description of the multiple coherence function and its use in the analysis of random signals is given by, for example, Newland [5].

5. THE CAUSALLY CONSTRAINED OPTIMAL SOLUTION

The minimisation of the sum of the time averaged squared error signals using a matrix of filters **H** whose elements are constrained to have causal impulse response functions has been dealt with in a previous paper [6]. In that work it was shown that the vector of causal impulse response functions associated with the composite vector **h** defined above must satisfy a matrix Wiener-Hopf integral equation. This equation is difficult to solve analytically except in certain cases (see, for example, the problems addressed in reference [7]). Here we constrain the optimal filter to have a finite as well as a causal impulse response and analyse the problem in discrete time. This is therefore the case that is of most relevance in practice, where the controller is implemented as a matrix of digital FIR filters.

Thus, working in discrete time, the n'th sample of the lth signal $\hat{d}_l$ can be written as the convolution

$$\hat{d}_l(n) = \mathbf{h}^T(0)\mathbf{r}_l(n) + \mathbf{h}^T(1)\mathbf{r}_l(n-1) + \dots \mathbf{h}^T(I-1)\mathbf{r}_l(n-I+1) \qquad (29)$$

where we have defined a composite tap weight vector and a sampled reference signal vector respectively by

$$\mathbf{h}^T(i) = [h_{11}(i)h_{21}(i)\dots h_{M1}(i)|h_{12}(i)h_{22}(i)\dots h_{M2}(i)| \dots |h_{1K}(i)h_{2K}(i)\dots h_{MK}(i)|] \qquad (30)$$

$$\mathbf{r}_l^T(n) = [r_{l11}(n)r_{l21}(n)\dots r_{lM1}(n)|r_{l12}(n)r_{l22}(n)\dots r_{lM2}(n)|\dots|r_{l1K}(n)r_{l2K}(n)\dots r_{lMK}(n)|] \qquad (31)$$

Note that each filter in the matrix **H** has thus been assumed to have an impulse response of I samples in duration. We now define a further composite tap weight vector **w** which consists of all the I tap weights of all the $L \times M$ filters such that

$$\mathbf{w}^T = [\mathbf{h}^T(0)\ \mathbf{h}^T(1)\ \dots\ \mathbf{h}^T(I-1)] \tag{32}$$

This enables us to write the L'th order vector of sampled error signals as

$$\mathbf{e}(n) = \mathbf{d}(n) + \mathbf{R}(n)\mathbf{w} \tag{33}$$

where the matrix R(n) is defined by

$$\mathbf{R}(n) = \begin{bmatrix} \mathbf{r}_1^T(n) & \mathbf{r}_1^T(n-1) & \dots & \mathbf{r}_1^T(n-I+1) \\ \mathbf{r}_2^T(n) & \mathbf{r}_2^T(n-1) & \dots & \mathbf{r}_2^T(n-I+1) \\ \cdot & & & \\ \cdot & & & \\ \mathbf{r}_L^T(n) & \mathbf{r}_L^T(n-1) & \dots & \mathbf{r}_L^T(n-I+1) \end{bmatrix} \tag{34}$$

We now define a cost function given by the sum of the L time averaged error signals such that

$$J_t = E[\mathbf{e}^T(n)\mathbf{e}(n)] \tag{35}$$

where E denotes the expectation operator then substitution of equation (33) and subsequent expansion shows that

$$J_t = \mathbf{w}^T E[\mathbf{R}^T(n)\mathbf{R}(n)]\mathbf{w} + 2\mathbf{w}^T E[\mathbf{R}^T(n)\mathbf{d}(n)] + E[\mathbf{d}^T(n)\mathbf{d}(n)] \tag{36}$$

This is a quadratic function of the tap weight vector **w** which is minimised by the optimal vector

$$\mathbf{w}_0 = -\{E[\mathbf{R}^T(n)\mathbf{R}(n)]\}^{-1}\{E[\mathbf{R}^T(n)\mathbf{d}(n)]\} \tag{37}$$

and has the corresponding minimum value given by

$$J_{t0} = E[\mathbf{d}^T(n)\mathbf{d}(n)] - \{E[\mathbf{R}^T(n)\mathbf{d}(n)]\}^T\{E[\mathbf{R}^T(n)\mathbf{R}(n)]\}^{-1}\{E[\mathbf{R}^T(n)\mathbf{d}(n)]\} \tag{38}$$

The matrix $E[\mathbf{R}^T(n)\mathbf{R}(n)]$ which has to be inverted is clearly of high order, but by using the definition of the composite tap weight vector given by equation (32) this matrix can be shown to have a block Toeplitz structure and use can be made of recursive algorithms for its efficient inversion [8]. Note that the definition of the composite tap weight vector given here is *not* the same as that adopted in reference [4].

A standard technique for finding the minimum of a quadratic function of the type defined by equation (36) is to use the method of steepest descent where one finds the minimum of the function iteratively by updating the value of the tap weight vector by an amount proportional to the negative of the gradient of the function. Thus in this case, the tap weight vector is updated in accordance with

$$\mathbf{w}(k+1) = \mathbf{w}(k) - \alpha E[\mathbf{R}(n)\mathbf{e}(n)] \tag{39}$$

where $E[\mathbf{R}(n)\mathbf{e}(n)]$ is the gradient of the quadratic function and k denotes each iteration step. In the derivation of the LMS algorithm [9] and in its generalisation given by Elliott *et al* [4] it is assumed that we can approximate the gradient of the function by its *instantaneous* value and thus update the coefficients on a sample by sample basis in accordance with

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \alpha\mathbf{R}(n)\mathbf{e}(n). \tag{40}$$

This therefore generalises the LMS algorithm for use with not only multiple secondary sources and error sensors but also with multiple detected signals. Such an algorithm has clear practical applicability in the active control of sound fields generated by multiple primary sources.

6. CONCLUSIONS

The multi-channel feedforward active control of sound has been dealt with from a general theoretical viewpoint. An analysis of the problem in the frequency domain yields a general solution for the optimal matrix of control filters when there is no constraint of causality on the impulse responses of the filters. Some very useful results are given in particular special cases. In the single channel case, with a single detection sensor and a single error sensor, the maximum reduction in the power spectral density of the error signal that can be achieved is shown to be solely determined by the ordinary coherence function relating the detection and error signals. With multiple detection and a single error sensor, the maximum reduction is equivalently determined by the multiple coherence function. The problem has also been analysed in the time domain and the optimal controller has been determined in terms of the coefficients of a matrix of digital FIR filters. The problem has been formulated to allow its efficient numerical solution. In addition, a generalisation has been presented of the stochastic gradient algorithm enabling the rapid adaptation of the control filters to their optimal solution.

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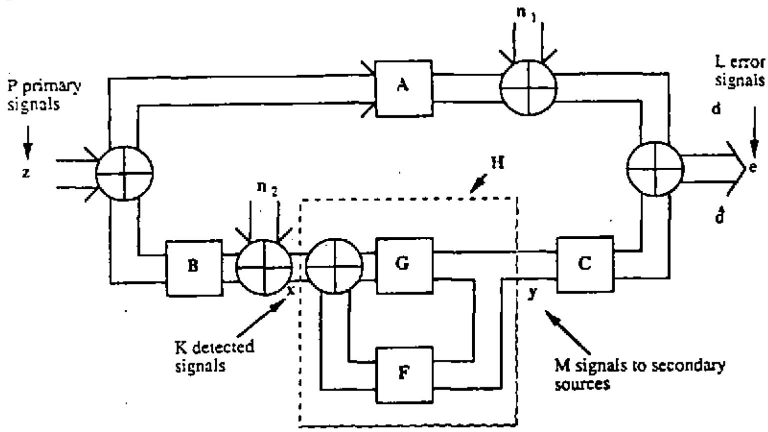

FIGURE 1. The block diagram of the multi-channel feed-forward control problem.

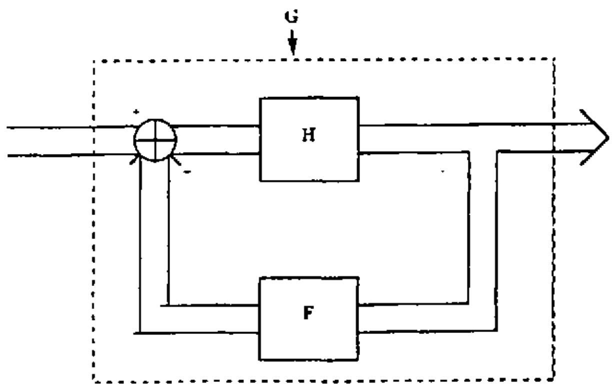

FIGURE 2. The structure of the filter matrix G once the matrix H has been determined. This amounts to a filter structure which ensures the cancellation of feedback from the secondary sources to the detection sensors. Note that if H is causal, G will be causal. The stability of G is not in general guaranteed.

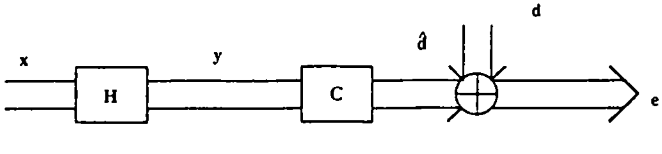

FIGURE 3. The equivalent multi-channel optimal filtering problem.

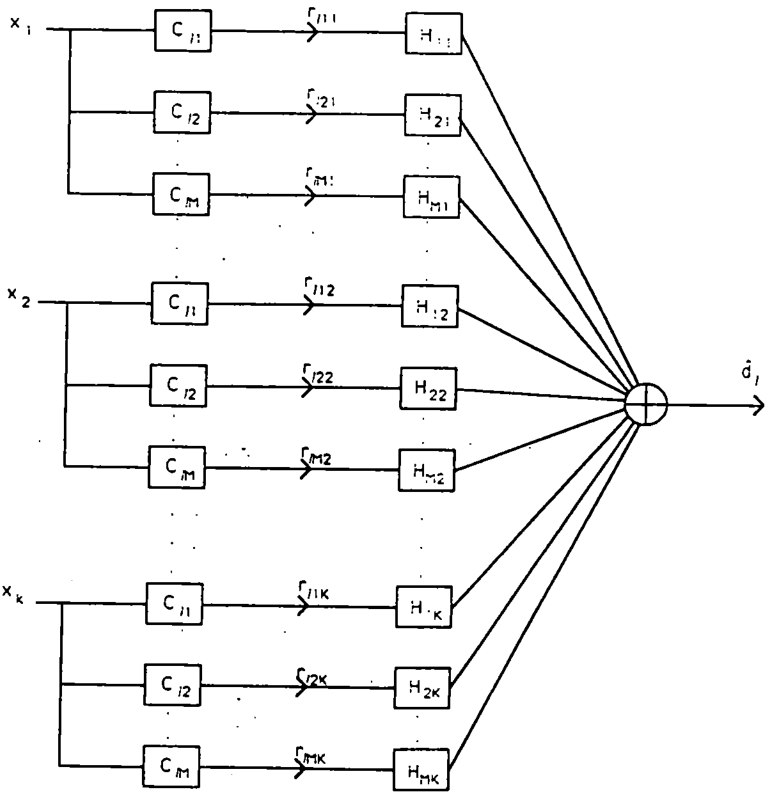

FIGURE 4. The equivalent block diagram showing the generation of the l th secondary signal with the operation of the elements of H and C reversed.