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FUNDAMENTAL NOISE IN MICROPHONES

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SOME FUNDAMENTALS

The study of fluctuations is important for several practical and theoretical reasons:

(i) Fluctuations are a more subtle aspect of physical phenomena than are mean quantities and as such provide a more searching test of our understanding of nature expressed in the fundamental Laws of Physics.
(ii) For the same reason, fluctuations provide a searching test of our reasoning about natural phenomena. This is particularly true of physical reasoning which, in recent years, has tended to be replaced by purely mathematical development; arguably to our detriment since all mathematical physics is at hazard to the accuracy of the abstraction of the physical phenomena to the mathematical formulation.
(iii)The limit of detection or accuracy in the transduction or measurement of a physical quantity is often set by fundamental fluctuations in that quantity itself.

The descriptions mankind makes of physical phenomena may be divided into the mechanistic and the paradoxical. In the first of these, some kind of mechanism, however abstract, is postulated; as for example when we seek solutions to the wave equation of an electron in order to predict its behaviour. In the second kind of description, on the contrary, predictions of physical behaviour are based on avoidance of some paradox, often avoidance of conflict with a general principle which is regarded as firmly established. Although at first sight this second kind of description may appear less satisfying, this is not really so since it enables particular phenomena to be linked to grand distillations of experience, such as the great Conservative Laws of Physics, namely those of Matter, Momentum and Energy, together with the laws of transformation of energy known as Thermodynamics. Thus very powerful general conclusions can be drawn. It is particularly in relation to this kind of approach that physical reasoning comes into its own, and here that we may characteristically recognise the quality we call 'good physics'; a quality which like the proverbial elephant is hard to define but readily recognised.

The avoidance of conflict with the Laws of Thermodynamics requires the Principle of Detailed Balancing. That is to say, in thermodynamic equilibrium there must not only be an equal number of transitions on average into and out of a state, but for any possible transition in one dirction there must be an equal chance of a transition in the opposite direction, since if there were any net circulation around a closed chain of transitions this could be exploited to abstract thermodynamically available energy continuously from a state of equilibrium.

A similar Principle of Detailed Balancing of Fluctuations is required which forbids abstraction of available energy from fluctuations; for example it is not possible to generate DC by rectifying the Johnson noise of a resistor with a semi-conductor diode at the same temperature.

MICROPHONES AND THERMAL FLUCTUATIONS

The limits of sensitivity of many transducers are set by thermal fluctuations, not to be regarded as a deviation from true thermal equilibrium but an essential and integral part of it. The limits of sensitivity of microphones are set in this way. The study and understanding of these thermal limits are of obvious importance to the whole of audio, since almost all reproduced sounds (other than those taken direct from synthesisers) come from signals picked up from microphones.

The scale of sound-level has been chosen so that 0 dBA corresponds roughly to the threshold of hearing. Good commercial microphones have a 'self-noise' commonly in the region 17-20 dBA, and are thus almost ten times less sensitive (expressed as pressure-amplitude ratio) than the ear.

Admittedly the sensitivity of existing microphones suffices for many commercial applications using close microphone spacings and mix-down, but for a more subtle presentation it is at least necessary to match the performance of our own ears; for example, in order to record lute or clavichord with a reasonable sense of acoustic space and perspective. The example of the ear itself establishes that such performance is possible in principle, and the question is how much further the improvement can go. To answer this requires calculation of the actual limit set by thermal fluctuations.

In view of the importance of the question, the state of the literature has been for many years less than satisfactory.

The earliest estimate of the fundamental limit seems to have been that of Sivian and White [1]. They quote from Rayleigh [2] the acoustic radiation resistance of a disc of radius a, small compared with the wavelength and in an infinite baffle, as $R = 2\pi^3 a^4 \rho f^2/c$ in which ρ is the density of the medium, c the velocity of sound in it, and f the frequency. They assume, by analogy with the Johnson [3] and Nyquist [4] formula for the noise generated by an electrical resistor, that the mean-square force fluctuation is 4 kTBR in which B is the bandwidth over which the noise is observed and T the absolute temperature. Thus they find the mean-square pressure fluctuation to be

$$\overline{p^2} = 8\pi kT\rho f^2 B/c \qquad (1)$$

and by integration the rms fluctuation between frequencies f_1 and f_2 is

$$P_{rms} = \left[\int_{f_1}^{f_2} \overline{p^2}\, df\right]^{1/2}$$

$$= \left[\frac{8\pi kT\rho}{3c}\,(f_2^3 - f_1^3)\right]^{1/2} \qquad (2)$$

Inserting numerical values for air at room temperature between f_1 = 1kHz and f_2 = 6 kHz, Sivian and White find

FUNDAMENTAL NOISE IN MICROPHONES

$P_{rms} = 5 \times 10^{-11}$ bar = − 86 dB relative to 1 dyne/cm^2.

(Note that 1 dyne/cm^2 was called a 'bar' in the literature of the time, but is actually a microbar in modern terminology.) They quote the threshold of hearing in the same frequency-range as about −76 dB relative to 1 dyne/cm^2, some 10 dB worse than their estimate of the theoretical limit.

Criticisms of Sivian and White's paper are that the fluctuation formula for an acoustic resistance is assumed without proof, and that microphone diaphragms are not baffled as assumed (the radiation resistance of an unbaffled small piston is much smaller and would give quite a different sensitivity limit).

The problem seems to have been next addressed by Hunt [5] who applied the principle of equipartition to the normal modes of an acoustic enclosure, each such mode having an average energy kT. From a knowledge of the density of such modes in space and frequency, the mean square pressure fluctuation is found to be

$$\overline{p^2} = \frac{4\pi kT\rho f^2 B}{c} \tag{3}$$

This is seen to differ from Sivian and White's result, equation (1) by 3 dB. The main criticism of this deviation is that it yields only the fluctuation in the acoustic field without showing (which indeed is not always so) that the transducer is able to reach a limit of sensitivity set by this fluctuation.

An experimental approach was attempted by Olson [6] who constructed a special ribbon microphone for the purpose of observing fundamental thermal acoustic noise. The accuracy of this brave attempt is questionable but Olson concluded that his observations were consistent with equation (3).

Since however Olson was using a ribbon microphone, he was actually observing the fluctuation in the air-particle velocity, not directly the fluctuation in pressure. This reminds us that neither Sivian and White, nor Hunt, had addressed themselves to the whole problem, but only half of it. Some microphones respond to pressure, others to air-particle velocity, and we need to understand the fluctuations in both. (A common misapprehension is to suppose some kinds of microphone to be responsive to pressure-gradient; the effective driving force in such microphones may indeed be a pressure-difference, but for a given acoustic wave-pressure the air-particle velocity is independent of frequency whereas the gradient is proportional to frequency, so that for a flat frequency-response the signal must be compensated so as to be directly proportional to the velocity itself.)

TRANSDUCTION IMPEDANCE, RADIATION RESISTANCE AND PRESSURE FLUCTUATION

While the principle of equipartition of energy shows that the average thermal energy is kT per mode (where k is the Boltzman constant), for practical applications we need to know the rate of fluctuation about this mean; in other words, the fluctuation per unit bandwidth B. The energy in a mode decays because of dissipative elements in the system, and in order to maintain equipartition, energy must be fed into the mode by these elements at the same average rate. In this sense it is dissipation that is the source of the fluctuations.

FUNDAMENTAL NOISE IN MICROPHONES

A general method of calculating fluctuations is by application of the principle of detailed balancing of fluctuations to transduction impedance (Fellgett and Usher [7]). This method is applicable whenever we can devise or imagine, even in principle, a passive transducer between electrical signals and the physical phenomenon we wish to study. Transduction impedance is the extra impedance which a passive transducer presents, over and above the impedance of its component parts, by virtue of the interaction between these parts which causes it to transduce one physical quantity into another. Perhaps the most familiar example is the dynamic impedance of a moving coil loudspeaker. Detailed balancing requires that the resistive part of any electrical transduction impedance should exhibit the same noise (in accordance with the Johnson-Nyquist formula) as any other resistance. Thus the thermal limit of sensitivity can be calculated for any physical quantity for which a passive electic transducer can be imagined.

By this method it is found, for example, that the mean square fluctuation ΔT in temperature associated with a thermal resistance R is

$$\overline{\Delta T^2} = 4kT^2BR \tag{4}$$

Note that this is not directly analogous to the Johnson-Nyquist formula $V^2 = 4kTBR$ for an electrical resistance R. Sivian and White were lucky, however, as the method of transduction impedance does show that the fluctuating force ΔF associated with an acoustic resistance R is indeed

$$\overline{\Delta F^2} = 4kTBR \tag{5}$$

just as they assumed, though without justification in their treatment.

In order to apply the method to pressure-sensitive microphones it is necessary to postulate an elementary pressure-sensitive transducer. This is assumed small compared to the wavelength, as the behaviour of larger configurations can be calculated by integrating over a distribution of small elements.

The required element, sensitive only to acoustic pressure without directivity, is the pulsating sphere; that is, a spherical shell, small compared with the wavelength, and constrained to respond only in the 'breathing' mode whereby its radius may increase or decrease but always maintaining spherical symmetry. It is of course supposed that the output signal corresponds exactly to the changes of radius.

Because of the high degree of symmetry, the acoustic properties of a pulsating sphere are comparatively easy to calculate. Stokes [8] found the acoustic radiation resistance to be

$$R = \frac{16\pi^3 \rho a^4 f^2}{c} \tag{6}$$

in which a is the radius of the sphere. He also showed that the interaction with the medium increases the apparent mass of the radiator, a phenomenon known as the accession of inertia. Substitution of equation (5) into (6), and using the area $4\pi a^2$ of the sphere, yields

$$\overline{p^2} = 4\pi kT\rho f^2 B/c \tag{7}$$

This is in agreement with the fluctuation in the acoustic field itself, as found by Hunt (equation (3)), confirming his result in a manner which fills the logical gap which he left. It does not agree with the result of Sivian and White. In order to understand this, note first that an unbaffled small disc does not respond at all to the acoustic pressure, and so is a quite unsuitable test-object for estimating pressure fluctuations. Sivian and White's assumption of an infinite baffle has two effects:

(a) It causes pressure doubling at the reflecting surface thus created. This increases the fluctuation by 6 dB.
(b) In the form they adopt, it enables thermal-acoustic wave to impinge on the disc from only one hemisphere. Since the fluctuations from the two hemispheres are uncorrelated, this decreases the fluctuation by 3 dB.

Thus the combined effect of the two errors is to increase the estimate by 3 dB, explaining the comparison between equations (1) and (7).

VELOCITY-FLUCTUATIONS AND THE ACCESSION OF INERTIA

If Sivian and White chose a wrong test-object for studying pressure-fluctuations (and compounded the error by adding an unrealistic baffle), an axially-oscillating disc is a suitable elementary receptor responsive to air-particle velocity. It is in fact easier to solve the problem of an axially-oscillating sphere, small compared with the wavelength. This was first done by Rayleigh [2] who found the radiation resistance and accession of inertia to be respectively (with $\omega = 2\pi f$)

$$R = \frac{\pi\rho\omega^4 a^6}{3c^3} \tag{8}$$

$$M' = \frac{2}{3}\pi\rho a^3 \tag{9}$$

Lamb [9] derived these two results by a particularly simple and elegant method which depends on showing that the velocity-potential field due to an elementary dipole is effectively identical with that generated by an axially-vibrating sphere.

Lamb goes on to derive a general result of great importance. He invokes the physical principle that if momentum is supplied to an acoustic medium in any region small compared with the wavelength, the far-field depends only on the momentum and not on the details of how it is applied. From this he is able to show that for all small vibrating bodies

$$R = \frac{\rho\omega^4}{12\pi c^3}\left(V + \frac{M'}{\rho}\right)^2 \tag{10}$$

in which V is the volume of the body and M' the accession of inertia. Thus, knowing the volume V of a body, knowledge of the accession of inertia is equivalent to knowing the radiation resistance, and <u>vice versa</u>. For an axially oscillating sphere, for example

$$R = \frac{\pi\rho\omega^4 a^6}{3c^3}$$

$$M' = \frac{2}{3}\ \pi\rho a^3 \qquad (11)$$

$$V = \frac{4}{3}\ \pi\, a^3$$

which is seen to be consistant with equation (10). The axially-oscillating disc provides a second example. The history of the solution of this more difficult problem is somewhat obscure, but the result was known to Lamb [2]

$$R = \frac{16\omega^4 a^6}{27\pi c^3}$$

$$M' = \frac{8}{3}\ \rho a^3 \qquad (12)$$

$$V = 0$$

which is also constant with equation (10).

In terms of the physically-intuitive equivalence in which force corresponds to voltage and velocity to current, the force applied to the medium by the oscillating body is opposed by the radiation resistance R in series with a mass (inductance) $M = M' + \rho V$ consisting of the accession of inertia plus the mass of medium enclosed by the body; we here take the body to be itself hollow and massless, a sort of rigid soap-bubble, in order to see how the applied force transfers momentum to the medium. The properties of this series equivalent circuit may equally well be represented by a mass (inductance) in parallel with a resistance r given by (Fellgett [10])

$$r = \omega^2 M^2 / R \qquad (13)$$

Substitution into equations (11) or (12) or the general expression (10) yields in each case

$$r = \frac{12\pi\rho c^3}{\omega^2} = \frac{3\rho c^3}{f^2} \qquad (14)$$

which result is therefore true not only for an axially oscillating sphere or disc, but for all such vibrating bodies whatsoever, provided only that the size is small compared with the wavelength. This general result may appropriately be called 'Lamb's Theorem'; a full statement of it is given in Fellgett [10].

It is a result of crucial importance for the estimation of acoustic thermal velocity-fluctuation, since r may be regarded, according to the Thevenin-alternative form of the Johnson-Nyquist formula (equation 5), as associated with a current (velocity) generator of thermal fluctuation. This fluctuation is

$$\overline{u^2} = 4kTB/r = \frac{4\pi kTf^2 B}{3\rho c^3} \qquad (15)$$

and by its derivation this result is independent of the size or shape of the test body (provided it is small); as of course must be the case.

It is interesting to note that, from equation 15, the average thermal acoustic energy-density is

$$I = \rho \Sigma \overline{u_i^2}$$

in which the summation is over the three resolved components of velocity, yielding

$$I = \frac{4\pi k T f^2 B}{c^3}$$

which is in agreement with the value derived form the principle of equipartition of energy (Hunt [5]).

APPLICATIONS AND CONCLUSIONS

From the foregoing arguments (further details of which are in Fellgett [10]) we know not only the thermal-acoustic fluctuation in both pressure and velocity, but can also be sure that 'ideal' pressure- and velocity-sensitive transducers have in principle a noise limit set entirely by these fluctuations in the acoustic field.

It is then obvious that these limits, since they depend only on the behaviour of the medium, must be the same for all transducers small compared with the wavelength. Our equations of course show this; for example the factors of $\underline{a}$ cancel in going from equation (6) to (7), or from equations (11) and (12) to (15). This is directly analogous to the way in which all radio-antennae, small compared with the wavelength, have a sensitivity which (neglecting obvious losses) depends only on the directivity.

The industry and audio practitioners commonly assume, on the contrary, that large microphones are necessarily able to exhibit better sensitivity than small ones. This is 'obvious', and like so many 'obvious' things is untrue. Of course our discussion has been directed solely to fundamental thermal noise-limits, and it does not necessarily follow that practical microphones of all sizes are able to come equally close to these limits. For a ribbon microphone, for example, the resistance of the ribbon and therefore the Johnson-Nyquist noise it generates, are independent of size assuming constant thickness and aspect ratio. For given flux density, the signal generated is the same per unit length independent of size. Therefore the ratio of total signal to noise deteriorates as the microphone is scaled down. The performance of moving coil microphones deteriorates even faster, since the area of cross-section of the coil is here inversely as the square of the size. For capacitor microphones, by contrast, the limits of responsivity set by distortion depend only (the exact manner depending on the configuration and how the signal is generated) on $\Delta x/x$, in which x is the spacing between moving and fixed electrodes and Δx the change in it due to the incident soundwave. With suitable design it is therefore possible to make x as small, and hence the capacitance as large, as we please. In particular, both the capacitance and the fractional change in it may be held constant as the microphone is scaled-down, and there is then no barrier to maintaining the performance constant. This does however require greater precision in fabrication; it is like the progress from a long-case pendulum clock to a wristwatch.

The general conclusion is that the self-noise of good current commercial microphones can in principle be improved upon by some 20 dB, and given the necessary skill microphones may be scaled-down without sacrifice in noise-level. This latter conclusion is of special importance in relation to modern near-coincident array microphones such as the Soundfield.

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