Proceedings of the Institute of Acoustics

ACOUSTIC SCATTERING BY SHIELDED OBSTABLES

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1. INTRODUCTION

This paper considers numerical methods for solving steady state acoustic fluid-structure interaction problems. The elastic structure may include flooded regions, as in the case of a sonar dome. This is a structure which is designed to be nearly acoustically transparent, isolating a sonar array from external flow.

The behaviour of vibrating structures in vacuo is well predicted using finite element techniques, see ref [1]. Finite element methods can also be used to analyse problems with enclosed regions of fluid coupled to structures, see ref [2]. The case of an external fluid of infinite extent may be dealt with by using a combined boundary element/finite element formulation, see ref [3].

The PAFEC finite element analysis system has been extended to include both acoustic finite elements and acoustic boundary elements. To test the program on a situation involving all three types of region, a simple problem with a closed form solution has been devised. The results are reported in this paper.

2. FLUID EQUATIONS

For small amplitude oscillations in an inviscid, irrotational, compressible fluid with no mean flow the pressure distribution satisfies the wave equation

$$\nabla^2 P - \frac{1}{C^2}\,\frac{\partial^2 P}{\partial t^2} = 0 \qquad (1)$$

where C is the acoustic wavespeed. This reduces to the Helmholtz equation

$$\nabla^2 P + k^2 P = 0 \qquad (2)$$

here k=ω/C is the acoustic wavenumber and ω is the circular frequency. The boundary condition

$$P_{,n} = -i\omega\rho V \qquad (3)$$

where ρ is the density, enforces continuity of normal velocity V on a vibration surface. For external problems the total pressure P can be decomposed as

$$P = P_I + P_s \qquad (4)$$

where P_I is the free field incident pressure and P_S is the scattered pressure. The scattered pressure must satisfy the Sommerfeld radiation condition

$$\lim_{r \to \infty} r\,|P_{s,r} - ikP_s| = 0 \qquad (5)$$

which ensures that it consists only of outgoing waves.

3. BOUNDARY ELEMENT FORMULATIONS

Invoking the divergence theorem it can be shown that

$$\epsilon P(\underline{x}) = \int_\Gamma (P(\underline{y}) g_{,n(\underline{y})}(\underline{x},\underline{y}) - g(\underline{x},\underline{y}) P_{,n(\underline{y})}) d\Gamma(\underline{y}) + P_I(\underline{x}) \qquad (6)$$

where

$$g(\underline{x},\underline{y}) = exp(-ikr)/4\pi r \qquad (7)$$

is the free space Green's function, r=|$\underline{x}$-$\underline{y}$| and $4\pi\epsilon$ is the solid angle in the fluid region. To obtain numerical solutions the surface Γ can be divided into patches over which the pressure P and its normal derivative $P_{,n}$ are interpolated using the same shape functions [N]. Equation (7) becomes

$$\epsilon P(\underline{x}) - \sum_{i=1}^{n} \int_{P_i} \frac{\partial g}{\partial n_y}[N] d\Gamma\{P\} = -\sum_{i=1}^{n} \int_{P_i} g[N] d\Gamma\{P_{,n}\} + P_I(\underline{x}) \qquad (8)$$

where P_i is the ith patch of the boundary element and n is the number of patches. Taking the collocation point $\underline{x}$ to be at each nodal point in turn a set of linear equations ensues, which can be written in matrix form as

$$[H]\{P\} = [G]\{P_{,n}\} + \{P_I\} \qquad (9)$$

The surface Helmholtz formulation described above works well apart from at a characteristic set of frequencies which become dense in the higher frequency range. At any of these critical frequencies the surface Helmholtz formulation does not provide a unique solution. Two boundary element methods, CHIEF and CONDOR are commonly used to overcome this problem.

CHIEF, combined Helmholtz integral equation formulation, was first proposed by Schenck [4]. The equations of the surface Helmholtz formulation are augmented by some additional equations, taking $\underline{x}$ at some interior points in equation (6), when $\epsilon=0$. The resulting overdetermined system is then solved by least squares methods.

CONDOR, composite outward normal derivative overlap relation, was first introduced by Burton and Miller [5]. This method combines the surface Helmholtz formulation and its normal derivative,

$$\epsilon P_{,n}(\underline{x})=\int_{\Gamma}(P(\underline{y})g_{,n(\underline{y})n(\underline{x})}-g_{,n(\underline{x})}P_{,n(\underline{y})})d\Gamma(\underline{y})+P_{I,n}(\underline{x}) \tag{10}$$

It can be shown that if a complex combination of equations (6) and (10) is taken then the resulting formulation does not suffer from any characteristic frequencies. The difficulty in the CONDOR method is evaluating the highly singular integrals. In the PAFEC acoustic system a method suggested by Meyer et al. [6] is employed. This is based on the relation

$$\int_{\Gamma}P(\underline{y})g_{,n(\underline{x})n(\underline{y})}d\Gamma=\int_{\Gamma}(P(\underline{y})-P(\underline{x}))g_{,n(\underline{x})n(\underline{y})}d\Gamma-P(\underline{x})\int_{\Gamma}\underline{n}_x\cdot\underline{n}_y(ik)^2 g\,d\Gamma \tag{11}$$

4. ACOUSTIC BOUNDARY ELEMENTS IN PAFEC

In the PAFEC program acoustic boundary elements for 3D problems can be constructed from generally curved eight-noded quadrilateral and six-noded triangular patches. The interpolation functions can be taken as either quadratic or constant. There is the ability to model planes of symmetry, permitting the use of small models in situations with reflective cyclic symmetry. Axisymmetric problems are treated quite efficiently using an axisymmetric boundary element constructed from three-noded quadratic line patches.

All the above types of boundary element can be used for the surface Helmholtz formulation and CHIEF methods. Only boundary elements constructed from constant pressure patches can use the CONDOR formulation.

5. SCATTERING BY A RIGID OBJECT

The scattering of a plane harmonic wave by a rigid sphere was analysed, as this can be compared directly with a closed form series solution. The sphere was taken to have a radius of 1m. Two frequencies were chosen, resulting in ka=1 and ka=π, where a(=1) is the radius of the sphere. The fluid medium was taken to be water with properties density=1000 kgm^{-3} and acoustic wavespeed=1500 ms^{-1}. The two meshes shown in figs 1 and 2 were used. The loading was taken to be an incident harmonic pressure wave, travelling in the x-direction, with unit amplitude. The results computed were directivities, or normalized far field pressures. First equation (9) was solved to determine the pressure distribution on the sphere's surface, and equation (6) was evaluated taking $\underline{x}$ to be at infinity, but normalized by a factor of r, for each direction. At ka=1, which is well below the first critical frequency, both meshes give good results. Polar directivity plots produced using the constant pressure patch mesh are shown in fig 3. At ka=π, which is the first critical frequency, the surface Helmholtz formulation fails to give correct results for both meshes. Figs 4 and 5 show the results for the axisymmetric mesh and the 3D mesh respectively. It is interesting to note the different manners of failure; the axisymmetric mesh produces directivity values about 100 times too large, whereas the constant pressure patch mesh gives results of the right order of magnitude, but completely the wrong distribution. Fig 6 shows how the CHIEF method, taking one interior collocation point at the centre, corrects the results for the axisymmetric mesh. Using the CONDOR method on the 3D mesh gave similarly good results.

6. COUPLING TO STRUCTURAL F.E.

The equations of motion for the structure can be written in the form

$$([S]-\omega^2[M])\{u\}=\{F\}+\{F_p\} \tag{12}$$

where [S] is the structural stiffness matrix, [M] the structural mass matrix, {U} a vector of displacements, {F} a vector of mechanically applied forces and $\{F_p\}$ a vector of forces representing the pressure distribution in the surrounding fluid. These resulting acoustic forces can be expressed in terms of the vector $\{P_b\}$ of pressures on the boundary element mesh by

$$\{F_p\}=-[T_b]^T\{P_b\} \tag{13}$$

for a suitable coupling matrix $[T_b]$. It is necessary to construct a matrix [E], such that $[E]^T\{U\}$ gives the normal displacements at the degrees of freedom on the boundary element mesh. Equation (9) becomes

$$[H]\{P_b\}=\omega^2\rho[G][E]^T\{u\}+\{P_{bI}\} \tag{14}$$

Details of the construction of the matrices $[T_b]$ and [E] are given in ref [7].

7. INCORPORATING FLUID FINITE ELEMENTS

The PAFEC acoustics system contains acoustic finite elements. These have one degree of freedom at each node, the pressure. They can be coupled to structural finite elements. The equation for the fluid region is

$$([S_a]-\omega^2[M_a])\{P_f\}=-\omega^2[T_f]\{u\} \qquad (15)$$

where $[S_a]$ and $[M_a]$ are square symmetric matrices, $\{P_f\}$ is a vector of pressures on the acoustic finite element mesh, $[T_f]$ is a coupling matrix similar to $[T_b]$ and $\{U\}$ is a vector of structural displacements as before. This equation can be derived using Galerkin weighted residuals methods, see ref [8], or using variational methods. In the PAFEC program there are both 3D and axisymmetric acoustic elements.

Combining equation (15) with equations (14), (13) and (12) and including an extra term to account for the forces from the pressure distribution in the finite fluid region acting on the structural mesh gives

$$\begin{bmatrix} [S]-\omega^2[M] & -[T_f]^T & -[T_b]^T \\ -\omega^2[T_f] & [S_a]-\omega^2[M_a] & [0] \\ \omega^2\rho[G][E]^T & [0] & [H] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{P_f\} \\ \{P_b\} \end{Bmatrix} = \begin{Bmatrix} \{F\} \\ \{0\} \\ \{P_{bI}\} \end{Bmatrix} \qquad (16)$$

solving this set of equations gives the response of the combined system caused by an incident wave and/or some applied forces.

8. NUMERICAL RESULTS

To test the acoustics program a simple problem with a closed form solution was considered; a spherical shell, containing a layer of internal fluid and a concentric spherical rigid baffle is excited by an incident wave in the external fluid, see fig 7. The rigid sphere was taken to have a radius of 2.25 m. The inner and outer radii of the spherical shell were taken to be 2.5 m and 2.55 m respectively. The fluid properties were taken as above. The structural properties were taken to be density=1200 kgm^{-3}, Young's modulus=5.7e9 Nm^{-2} and Poisson's ratio=0.34. The frequency of excitation was 1000 Hz. The ka value for this is 9.42, taking a to be the radius of the baffle. To compare results, the pressure distribution on the baffle was examined. Ref [10] describes how this problem can be solved by a series, using the separability of the wave and elasticity equations in spherical coordinates. Fig 8 gives a comparison of the pressure distribution with and without the surrounding shell.

The finite element/boundary element mesh used to analyse the problem is shown in fig 9. All elements used were axisymmetric. The external fluid was discretized by an acoustic boundary element constructed from 40 three-noded line patches. The shell was modelled using 40 isoparametric eight-noded axisymmetric quadrilaterals, see ref [9]. The internal fluid required 80 eight-noded isoparametric quadrilateral axisymmetric acoustic elements. The acoustic boundary element used the CHIEF method, with 5 internal collocation points. Fig 10 gives the comparison between the closed form and the f.e./b.e. solutions. The level of agreement is good.

9. CONCLUSIONS

It has been shown that a combination of structural finite elements, acoustic finite elements and acoustic boundary elements can be used to model complicated fluid-structure interaction situations. These techniques have been successfully implemented in the PAFEC program.

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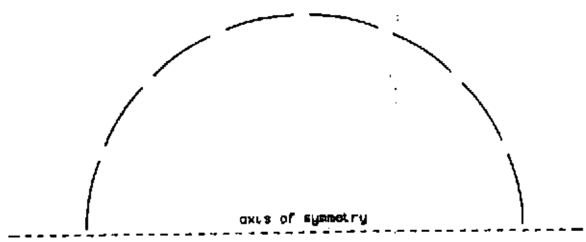
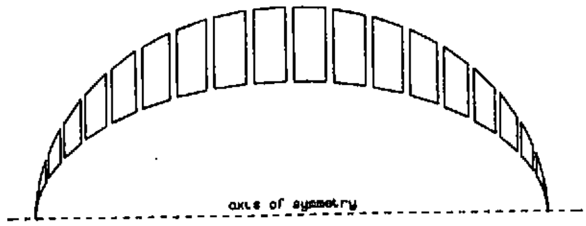
Fig 1



axisymmetric mesh used to analyse
scattering by rigid sphere
eight quadratic line patches

Fig 2



3D mesh used to analyse
scattering by rigid sphere
twenty generally curved
constant pressure patches

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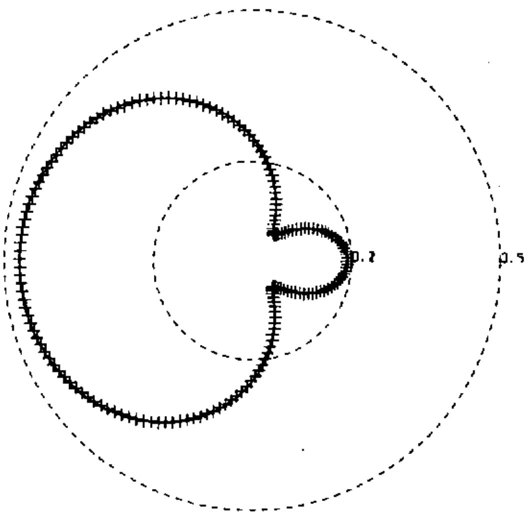
Fig 3

Directivity pattern for plane wave scattered
by rigid sphere (ka=1)

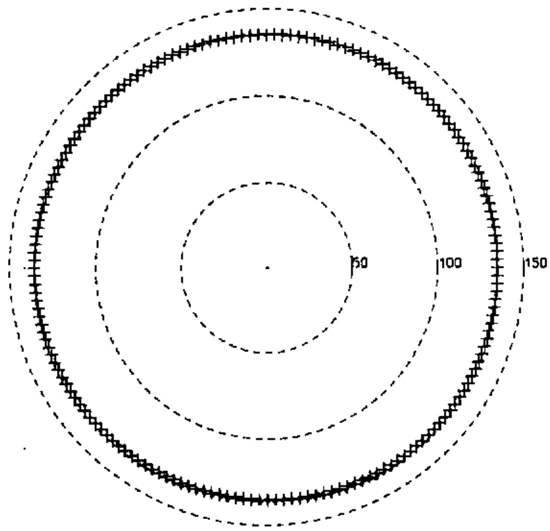


——— closed form solution

+ boundary element solution
using surface Helmholtz formulation
with 20 constant pressure patches

Fig 4

Directivity pattern for plane wave scattered
by rigid sphere (ka=3.14159)

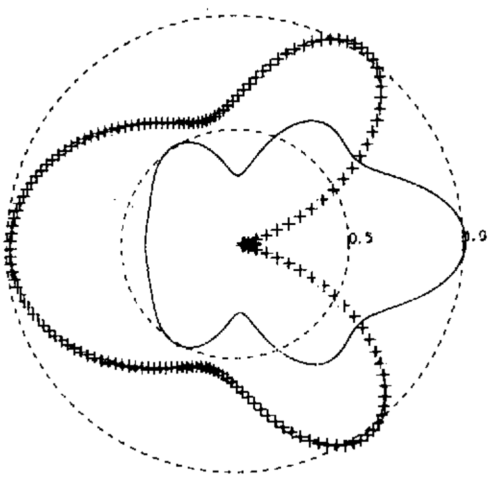


——— closed form solution

+ boundary element solution
using surface Helmholtz formulation
with 8 axisymmetric quadratic line patches

Fig 5

Directivity pattern for plane wave scattered
by rigid sphere (ka=3.14159)

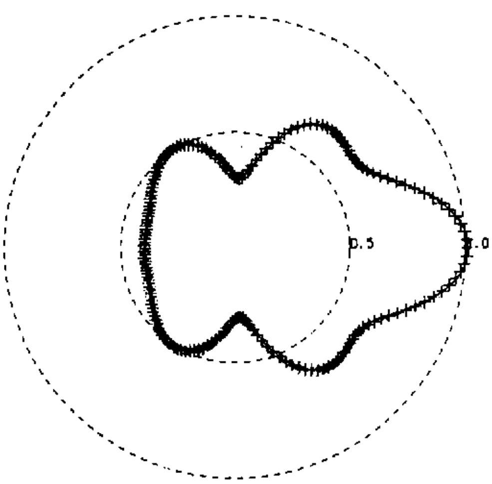


——— closed form solution

+ boundary element solution
using surface Helmholtz formulation
with 20 constant pressure patches

Fig 6

Directivity pattern for plane wave scattered
by rigid sphere (ka=3.14159)



——— closed form solution

+ boundary element solution
using CHIEF method
with 8 axisymmetric quadratic line patches
and 1 interior collocation point at centre

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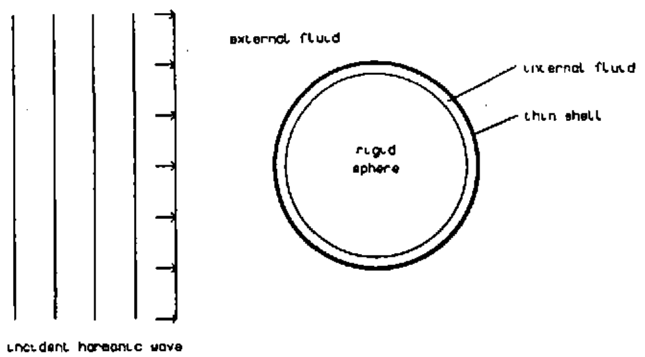
Fig 7



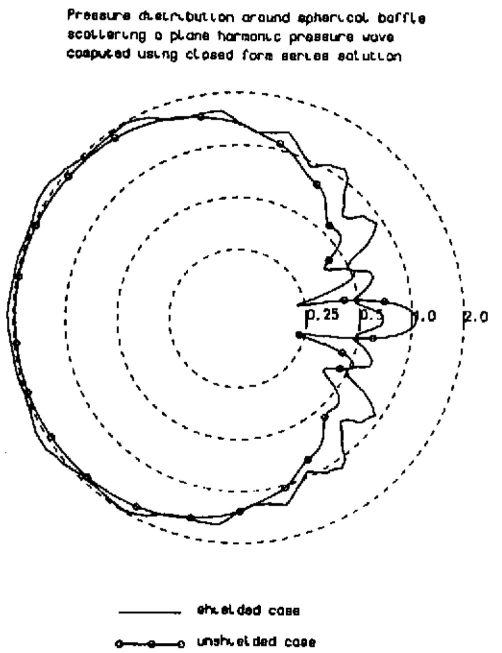


Fig 8

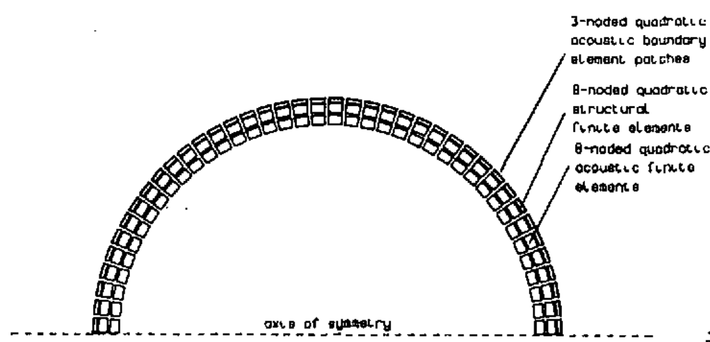


Fig 9

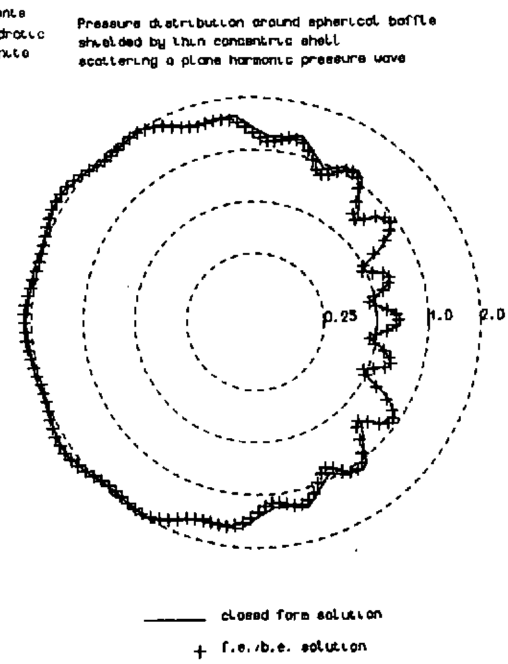


Fig 10