REPRESENTATION OF DIRECTIONAL LOUDSPEAKERS IN A FINITE ELEMENT ROOM ACOUSTIC SIMULATION

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1 INTRODUCTION

The finite element method (FEM)[1] is often used for low frequency room acoustic simulation. It is possible to model a loudspeaker source explicitly, but this adds complexity to the model creation and reduces the flexibility of repositioning the source within the room. For an omnidirectional source it is straightforward to use an ideal point source, as in ref[2]. The current work considers some ways of extending this to directional loudspeakers. Ideally a set of point sources should be determined, such that, in combination, they radiate the same pressure field into 3D space beyond some radial distance, as the loudspeaker. One approach would be to start with a set of sources, and adjust the position, amplitude and phase in an optimization procedure to match the desired radiation pattern. This paper considers an alternative, directly constructive approach, based spherical harmonics.

2 REPRESENTATION BY SPHERICAL HARMONICS

Spherical harmonics are a functions of position on a sphere defined by a 'latitudinal angle' θ varying from 0 at the north pole to π at the south pole and 'longitudinal angle' φ varying from 0 to 2π around the equator. They can be used, together with the associated spherical Hankel functions, to characterize a 3D source. The precise definition used for spherical harmonics is not universally agreed, differing between disciplines. For the current work the definitions used by Gumerov and Duraiswami[3] are used together with some of their notation.

$$Y_n^m(\theta,\varphi) = (-1)^m \sqrt{\frac{2n+1}{4\pi} \frac{(n-|m|)!}{(n+|m|)!}} P_n^{|m|}(\cos\theta) e^{im\varphi} \qquad (1)$$

However ref[3] assumes an $e^{-i\omega t}$ variation, whereas this paper assumes an $e^{i\omega t}$ time variation.

With the definition (1), the spherical harmonics are orthonormal when integrated over the unit sphere S_1, i.e.

$$\int_{S_1} \overline{Y_{n_1}^{m_1}(\theta,\varphi)} Y_{n_2}^{m_2}(\theta,\varphi)\, dS = \delta_{n_1 n_2} \delta_{m_1 m_2} \qquad (2)$$

Consider a setup, as in figure 1, where a finite set of sources and/or vibrating surfaces and diffracting edges are radiating into infinite 4π steradian 3D space. The travelling waves within the exterior region are outwardly travelling and satisfy the Somerfeld radiation condition.

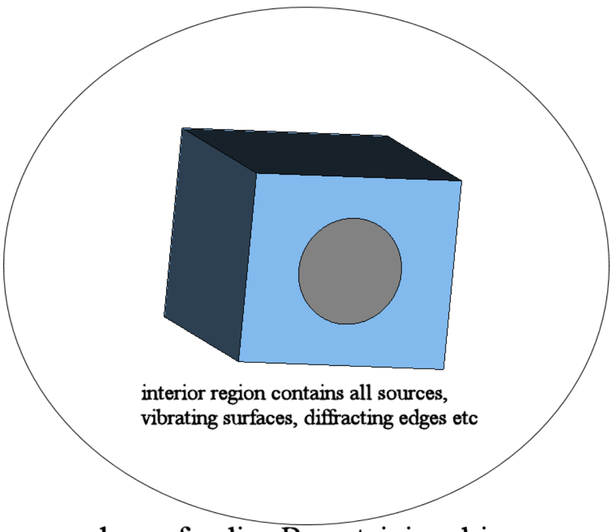

Figure 1, radiation from source contained within spherical surface

For steady state acoustic vibration with wavenumber k, it can be shown using the separabiloity of the Helmholtz equation in spherical coordinates, that the pressure field in the region r>R exterior to the sphere S_R can be represented as

$$p(r,\theta,\varphi)=\sum_{n=0}^{\infty}\sum_{m=-n}^{n}a_{nm}h_n^{(2)}(kr)Y_n^m(\theta,\varphi) \quad (3)$$

where the radial term $h_n^{(2)}(kr)$ is a spherical Hankel function. The function $S_n^m(r,\theta,\varphi)$ defined by

$$S_n^m(r,\theta,\varphi)=h_n^{(2)}(kr)Y_n^m(\theta,\varphi) \quad (4)$$

satisfies the Helmholtz equation in the exterior region, and will be subsequently referred to as a solid spherical harmonic as in ref[3]. Using the orthogonality of the spherical harmonics in (2), the coefficients a_{nm} can be determined by

$$a_{nm}=\frac{\int_{S_R}\overline{Y_n^m(\theta,\varphi)}p(R,\theta,\varphi)\,dS}{R^2h_n^{(2)}(kR)} \quad (5)$$

for any sphere S_R of radius R containing the sources. Thus if the pressure field, both amplitude and phase, is known on a spherical surface surrounding the driver, the solid spherical harmonic coefficient components a_{nm} can be determined from (5), and if each solid spherical harmonic can be approximated by a distribution of point sources then equation (3) can be used to construct a representation of the source using a distribution of monopole sources.

3 REPRESENTATION BY MONOPOLES

3.1 Multipole approach

The term 'multipole' is not consistently defined in the literature. In the context of this paper, as in ref[3], multipoles are taken to be Cartesian coordinate derivatives of monopoles. Thus

$$M_{uvw}\left(\underline{r}\right)=\frac{\partial^{u+v+w}\left(\dfrac{e^{-ikr}}{r}\right)}{\partial x^{u}\partial y^{v}\partial z^{w}} \tag{6}$$

Note that the derivatives are respect to the source position, rather than the receiver position.

It is asserted that each solid spherical harmonic can be expressed as a sum of multipoles, e.g.

$$S_n^m=\sum_{p=1}^{A_{nm}}c_{nmp}M_{u(n,m,p)v(n,m,p)w(n,m,p)} \tag{7}$$

where A_{nm} is the number of terms in the series, c_{nmp} are the complex coefficients and $u(n,m,p)$, $v(n,m,p)$ and $w(n,m,p)$ are the integer orders of differentiation. The assertion is true for the case n=0 because

$$S_0^0\left(r\right)=\frac{i}{\sqrt{4\pi}}\frac{e^{-ikr}}{kr} \tag{8}$$

Suppose that the assertion is true for $n\le N$, for some integer N. It will be shown that the assertion is true for $n\le N+1$. This will complete the proof of the assertion by the principle of induction. From the book by Gumerov and Duraiswami[3], formula (2.2.7)

$$S_{n+1}^m=\frac{a_{n-1}^m}{a_n^m}S_{n-1}^m-\frac{1}{ka_n^m}\frac{\partial S_n^m}{\partial z} \tag{9}$$

where z is the axial, $\theta=0$, direction and

$$a_n^m=a_n^{-m}=\begin{cases}\sqrt{\dfrac{(n+1+m)(n+1-m)}{(2n+1)(2n+3)}} & \text{for } n\ge|m|\\ 0 & \text{for } n<|m|\end{cases} \tag{10}$$

Thus if $S_N^{-N},\ldots,S_N^N$ are expressible as a linear combination of multipoles, then $S_{N+1}^{-N},\ldots,S_{N+1}^N$ are also expressible in this way. The end terms S_{N+1}^{-N-1} and S_{N+1}^{N+1} can be shown to be of the form of (7) using the formulae

$$S_{n+1}^{-n-1}=-\sqrt{\frac{2n+3}{2n+2}}\frac{1}{k}\left(\frac{\partial}{\partial x}-i\frac{\partial}{\partial y}\right)S_n^{-n} \tag{11}$$

$$S_{n+1}^{n+1} = -\sqrt{\frac{2n+3}{2n+2}}\frac{1}{k}\left(\frac{\partial}{\partial x}+i\frac{\partial}{\partial y}\right)S_n^n \tag{12}$$

which can be derived from ref[3] formulae (2.2.9) and (2.2.11). Thus every solid spherical harmonic can be represented as a linear combination of multipoles. Each multipole can be represented as a linear combination of monopoles, using numerical differentiation.

It should be noted that expressions as in (7) are not unique, because each multipole satisfies the Helmholtz equation, and thus can be expressed as a linear combination of 3 higher order multipoles.

3.2 Spherical surface representation

An alternative approach is to have a distribution of monopole sources over a spherical surface. If the source density is given a spherical harmonic distribution on a sphere of radius a

$$\sigma = SY_n^m(\theta,\varphi) \tag{13}$$

then the interior pressure field is a multiple of $j_n(kr)Y_n^m(\theta,\varphi)$ and the exterior pressure field is a multiple of $S_n^m = h_n^{(2)}(kr)Y_n^m(\theta,\varphi)$. It can be shown that to achieve an exterior pressure of S_n^m the required source density is given by

$$S = \frac{i}{k^2a^2 j_n(ka)} \tag{14}$$

A method based on this approach, combined with equation (3), should work apart from when $j_n(ka)=0$. This seems reminiscent of the problem with irregular frequencies for the exterior boundary element method[4,5]. It is suspected that using a source density of SY_n^m on a sphere of radius a, in conjunction with a source density iSY_n^m on a sphere of radius $a-\frac{\lambda}{4}$ would eliminate this issue. This needs further investigation.

4 DRIVER EXAMPLE

The above theory has been investigated on a simple example comprising of a piston of radius 0.0225m in the centre of the front face of a cuboid enclosure (front face :0.09m width x 0.08m height, depth 0.1m). The piston is prescribed unit amplitude velocity. The surrounding air has speed of sound = 340 m/s. The boundary element method was used to compute the radiated sound field on a surrounding spherical surface of radius 0.3m, centred on the middle of the piston, and used in conjunction with equation (5) to compute the solid spherical harmonic components a_{nm} . In fact a sphere of any radius, containing the source, could be used.

Figure 2 is a graph of the pressure at points 4m from the piston centre. Figures 3 & 4 show the pressure and spherical harmonic summations for 4m in front and 4m behind. N is the maximum n in the summation of equation (3). Results clearly improve with increasing N. N=3 is accurate up to about 1500 Hz. N=12 is accurate up to about 5500 Hz.

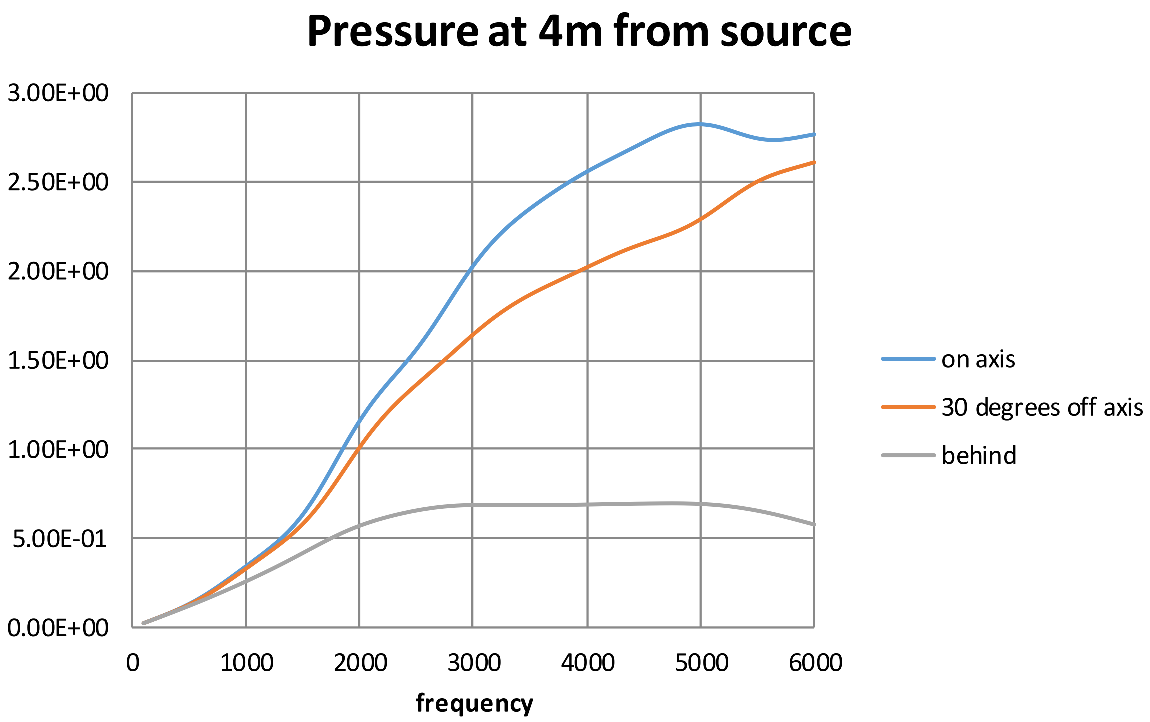

Figure 2, pressure v frequency at points in horizontal plane

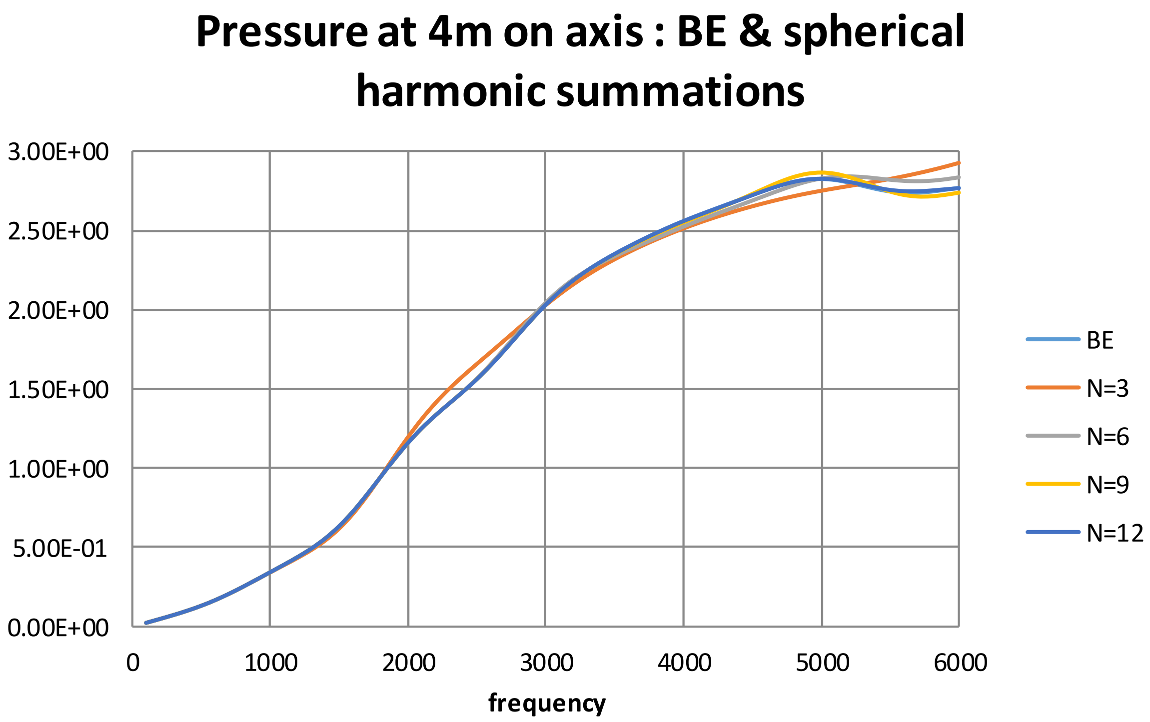

Figure 3, pressure on axis and spherical harmonic summations

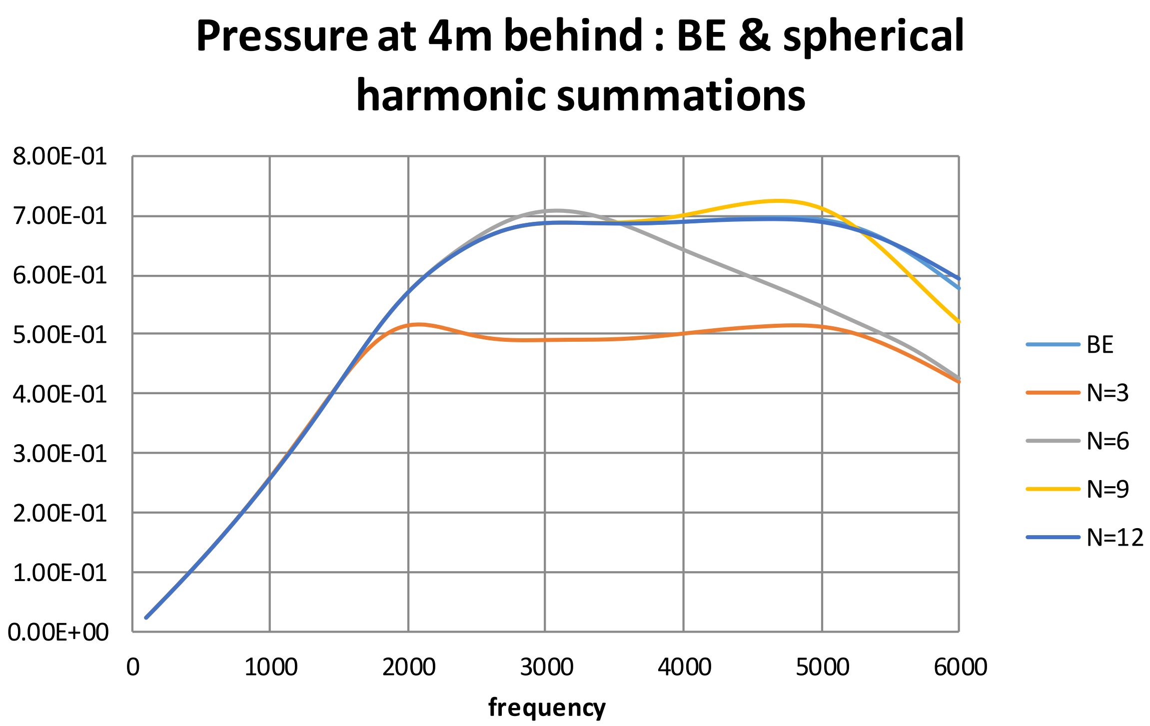

Figure 4, pressure behind and spherical harmonic summations

5 SOURCE REPRESENTATION

The work to date has been testing the multipole approach. The results from the example driver above were processed up to 3000 Hz, i.e. with λ=0.1133m. The solid spherical harmonics up to n=3 were converted to multipoles and subsequently a lattice of monopoles. A spatial difference d=0.02m was used for numerical differentiation. Ideally this should be smaller, particularly at the upper end of the frequency range. Reducing d would increase the accuracy in exact arithmetic, but in finite precision arithmetic the rounding error would increase, particularly for higher order derivatives. The sources were summed at the points 4m in front and 4m behind the piston. The results were also computed with the sources in a finite element mesh modelling a 0.08m x 0.08m x 0.08m region of space with a 25 x 25 x 25 mesh of quadratic bricks, and surrounded by a boundary element extending out to infinity. The finite elements assume a locally quadratic variation of pressure and so a fine mesh is needed in the neighbourhood of the monopole sources.

Figure 5 shows the pressure at 4m on axis in front and figure 6 shows the pressure 4m behind. The pressure in front is more accurate. The pressure behind only has reasonable accuracy up to 1500 Hz.

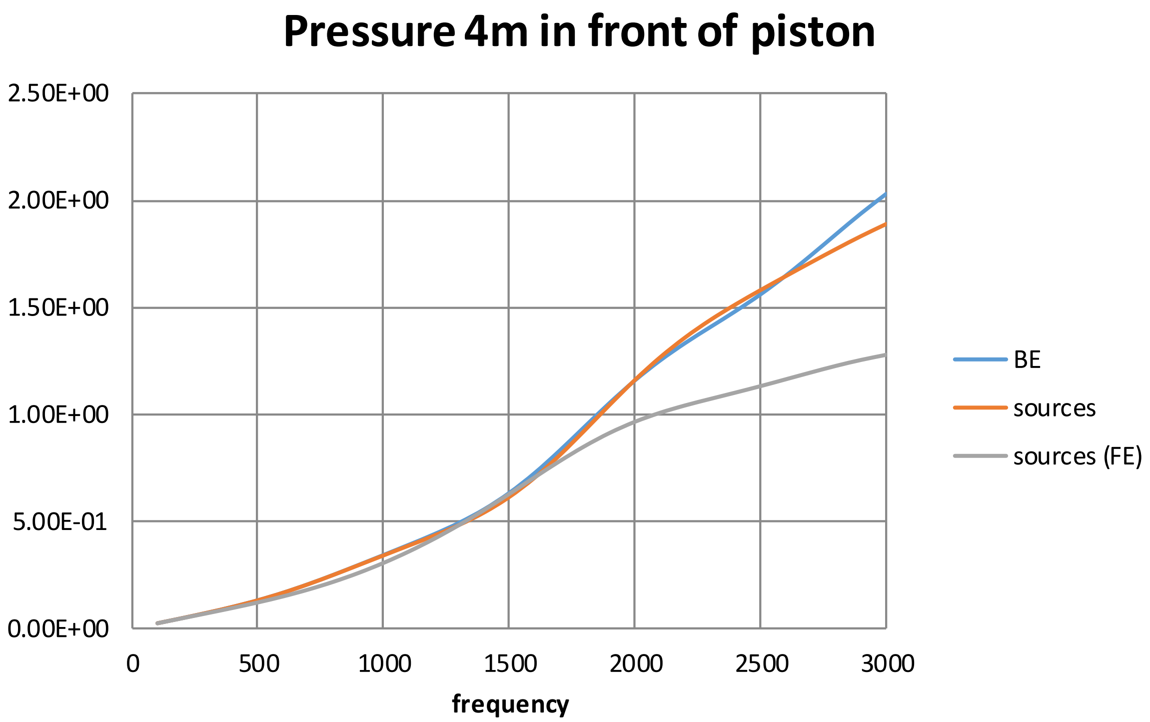

Figure 5, pressure 4m on axis evaluated by sources

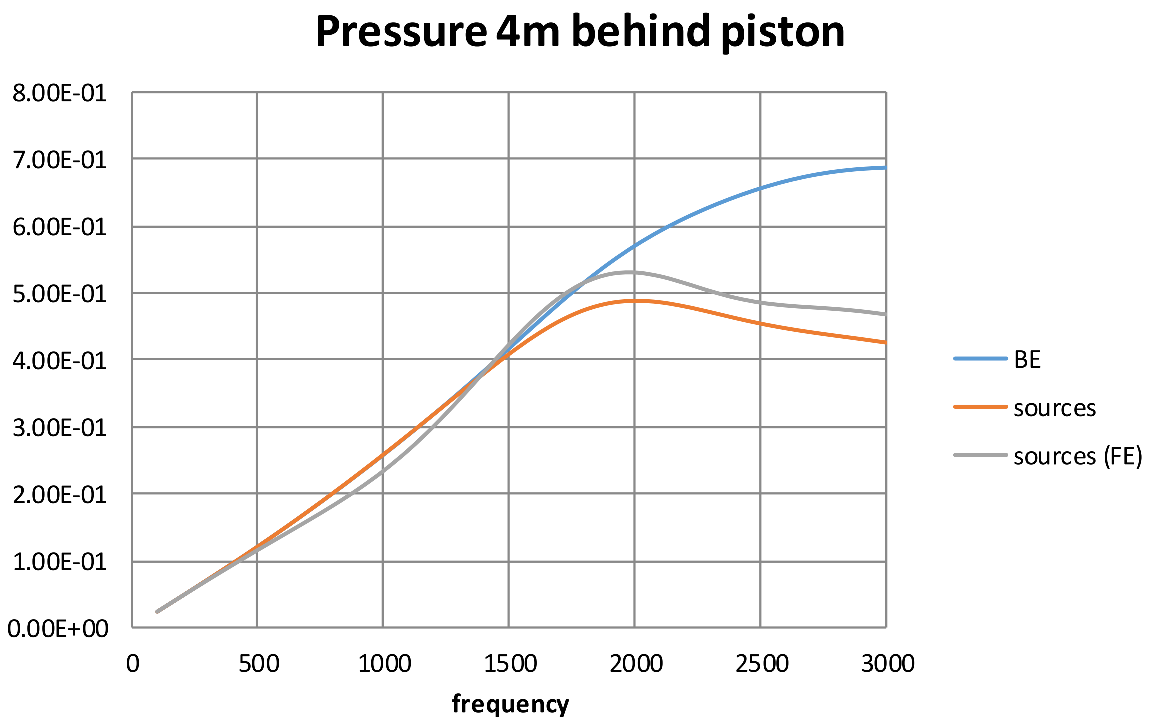

Figure 6, pressure 4m b4ehind piston evaluated by sources

6 CONCLUSIONS

Solid spherical harmonics can be used to characterize a directional driver. Higher orders are needed at higher frequencies. It should be easier to include directional sources in a boundary element model, as the solid spherical harmonics can be directly included as right hand side terms in the equations. Using the multipole-based approach within a finite element model as tested so far has been limited to fairly low frequencies. It is expected that the alternative approach, using a spherical surface with a varying monopole density should work better.

7 REFERENCES

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