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MEASUREMENTS OF SCATTERING FROM A SUSPENSION OF SPHERICAL SCATTERERS

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INTRODUCTION

The scattering of sound from a sphere is a classical problem in acoustics. Since Rayleigh's[1] pioneering work a number of theoretical and experimental studies[2-10] have been conducted to examine the interaction of sound with a body underwater. Particularly the early analysis of Faran[2],Hickling[3] and Neubauer etal[4] established a framework for the interpretation of the scattering from a sphere.In more recent years the theoretical and experimental developments [11-20] have investigated scattering for spheroids under increasing complex geometries and detailed the numerous resonances stimulated. These studies have concerned themselves with single particle scattering and it is only in the last few years that this analysis has been applied[21-25] to the problem of scattering by a suspension of scatterers.

The work reported here examines the interaction of sound with a homogeneous suspension of nominally spherical particles with a narrow size distribution. Initially measurements were obtained on single particles to establish the form function[4] for the material. This was required for the interpretation of the suspension measurements. Observations of the ensemble root-mean-square backscattered pressure level and attenuation were recorded for a variety of conditions. A number of parameters were varied and data were collected for a selection of pulse lengths between 10-200us, over a range of suspended concentration, 0.06-1.1Kgm^{-3}, covering distances between 0.02-1.50m from the transceiver.

Comparison of the theoretical predictions with the experimental data is conducted using the form function description of single sphere scattering.An integral is conducted over the insonified volume and the rms scattered signal evaluated. The particles in suspension are treated as uniform in size and the concentration sufficiently dilute that multiple scattering effects are not significant.The assumption is also made that the scatterers are sufficiently dispersed that the phase is random and uniformly distributed over 2π, and the scattered signal is therefore incoherent.

Although the data set obtained to date is still relatively limited, comparison of the theoretical predictions with the experimental measurements show good agreement for the parameter changes examined.

INSTRUMENTATION

The evaluation of the applicability of the backscatter theory was conducted using the sediment tower shown in Fig1a. This consisted of a hopper and an impeller mounted on a perspex tower with a collection tray at the base. To obtain an absolute estimate of the suspended sediment concentration pump samples were taken. This was carried out by pumping typically 0.015m^3 of suspension out of the tower, passing it through a filter and measuring the dry weight of material collected.

The acoustic transceiver was mounted near the base of the tower. Sediment was prevented from settling on the transceiver by passing a horizontal jet of water just over its surface. The transducer used in the tower operated at 3MHz, was 10mm in diameter and had a -3dB half beamwidth of 1.4°. The arrangement for collecting the data is shown in Fig1b. The envelope of the transceiver signal, backscattered from the sediment in suspension, was digitized and fed directly into a microcomputer. The system sampled the backscattered signal at 256 contiguous range bins to yield an acoustic profile of the returned echo.Typically 200 profiles were collected and the rms ensemble average backscattered pressure profile computed.These mean profiles were employed for comparison with the theoretical developments. Also obtained using a hydrophone placed at the top of the tower were measurements of the attenuation due the presence of the suspended sediments. Usually 100 estimates of the attenuation were taken during a pump sample and these were digitized and recorded in a similar manner to the backscattered data.

The material used for the suspension was solid glass spheres, known as ballotini. The particles were nominally spherical with a minimum of 80% being true spheres The ballotini was sieved and found to have a peak in the size distribution at 160μm diameter.The results are shown in Fig2.

It was crucial to the experiment that the suspension was uniform over the length and crossection of the tower and a number of measurements to examine this were taken on several occasions, covering a concentration range 0.1-0.8Kgm^{-3}. The results of these observations are presented in Fig3a. The values obtained from the pump sample show no significant trend in the suspended concentration profile over the usable length of the sediment tower. Although there was some variability between measurements there was no discernible systematic variation. The standard error in the pump sample measurement of the concentration was calculated to be 10%. Profiles of the acoustic backscattered signal taken during pump sampling are shown in Fig3b and these yield very consistent results which reaffirmed that no substantial change in concentration occurred over the period of an acoustic measurement. It was therefore concluded that the homogeneity of the concentration profile in the tower was adequate for a critical examination of the theoretical predictions.

As mentioned in the INTRODUCTION, to establish the scattering properties of the ballotini a series of measurements were conducted to obtain the form function. These were carried out using a number of different particle sizes between 0.68-6.34mm diameter, and covering a frequency range 0.8-3.5Mz. The experimental arrangement was similar to that used in the tower with the suspension replaced by a single glass sphere. The particle was suspended in the water using a fine nylon line. The transceiver insonified the sphere from below using a pulse of typically twenty or more particle diameters in length. The pressure at the sphere was measured and the backscattered signal recorded was normally averaged over 256 returns to improve the signal to noise ratio.

ANALYSIS AND MEASUREMENTS

To investigate the process of scattering by ballotini a series of primary measurements focused upon an examination of the form function over a broad range of ka in the backscatter direction. k is the wave number in water and a is the particle radius. The definition of the form function is given by,

$$f_m = \frac{2r\, P_R\, e^{-i(kr - \omega t)} e^{\alpha_w r}}{a\, P_T}$$

Where P_T is the transmitted pressure incidence on a sphere located on the acoustic axis, P_R is the received pressure at the transceiver, r is the range from the sphere, and α_w is the attenuation of sound due to the water. The results of the measurements are presented in Fig4 where the dependence of $|f_m|$ on ka in the backscatter direction is shown. The main feature of the form function is seen to be its high variability associated with the complex interactions of the sound with the sphere. Calculations for the form function were computed using

$$f_m = \frac{-2}{ka} \sum_{n=0}^{\infty} (2n+1)\, (-1)^n\, b_n \qquad (2)$$

Where b_n is the solution to the boundary conditions at the sphere and can be found in Ref14. Comparison of the theoretical predictions with the observations show good agreement over most of the ka range. There is some degree of departure for ka<2, however, in this region the signal to noise ratio of the system was relatively poor, due to the small spheres employed. There was also some concern on the influence of the sphere mounting. Further measurements are required to resolve these differences. However, the results do show that we are justified in using the spherical description of scattering for investigating the interaction of sound with a suspension of ballotini.

To obtain the backscattered signal from a suspension of spheres an integral needs to be carried out over the insonified volume. For the present analysis the spheres are treated as uniform in size and of sufficient dilution that multiple scattering is not significant. Initially we start with the single sphere expression and from equation (1) we have

$$P_R = \frac{a\, P_o r_o\, D^2}{2r^2}\, |f_m|\, e^{-2\alpha_w r}\, e^{i(2kr-\omega t)} \qquad (3)$$

P_o is the is the far field reference pressure at range r_o, usually calculated to be at r_o=1m and D is the transceiever directivity function. For a monostatic configuration the ensemble average rms backscattered pressure, $<P_{rms}>$, from a suspension can be written as

$$<P_{rms}> = \frac{a\, P_o r_o}{2r^2}\, |f_m|\, e^{-2r(\alpha_w+\alpha_s)} \left\{ N \int_0^{\pi}\int_0^{2\pi}\int_r^{r+\tau c/2} D^4\, r^2\, \mathrm{Sin}\theta\, d\theta\, d\phi\, dr \right\}^{1/2} \qquad (4)$$

$$\alpha_s = \frac{1}{r}\int_0^r \xi\, M(r)dr \qquad \xi = (3/2a^3k^2\rho_s) \sum_{n=0}^{\infty} (2n+1)\, |b_n|^2$$

The attenuation due to the sediments is given by α_s, which for the present analysis is linearly related to the concentration profile through ξ[25], ρ_s is the density of the suspended sediment, N is the number of particles per unit volume, τ is the pulse length and c is the speed of sound in water. For the usual disc transceiver employed the directivity function is a first order Bessel function, j_1, also rewriting in terms of mass concentration, M(r), equation (4) can be expressed as

$$<P_{rms}> = \frac{P_o r_o}{r} |f_m| \left(\frac{3\tau c\, M}{16a\rho_s}\right)^{1/2} e^{-2r(\alpha_w+\alpha_s)} \left[\int\left\{\frac{2j_1(ka_t \mathrm{Sin}\theta)}{ka_t \mathrm{Sin}\theta}\right\}^4 \mathrm{Sin}\,\theta\, d\theta\right]^{1/2} \qquad (5)$$

The transceiver radius is given by a_T. The backscattered pressure is seen to be a function of a number of variables and those specifically considered here are the dependence on $\sqrt{M}$, $\sqrt{(\tau c)}$ and r. Absolute measurements of the pressure were obtained by calibrating the transmitting and receiving systems using a PVDF hydrophone, and predictions were computed by evaluating equation(5)

Fig5 shows the results for the variation of $<P_{rms}>$ with $\sqrt{M}$. For low values of $\sqrt{M}$ there is a linear dependence, however, as the concentration gradually increases the relationship becomes strongly non-linear. This divergence from linearity is due to the influence of the suspended sediment attenuation on the backscattered signal. It can be seen in Fig5 that a situation is being reached where further increases in concentration would result in a reduction in the backscattered pressure for the range, 0.5-0.6m, shown in the diagram. The form of the curve is a function of range, due to the effect of the sediment attenuation, with the departure from linearity occurring at lower values of $\sqrt{M}$ for increasing distance from the transceiver. Two theoretical curves are shown in Fig5, the linear curve with $\alpha_s = 0$ and the solution where the sediment attenuation is accounted for. The agreement between prediction and measurement is good, with the form of the curve and the absolute level in close agreement. There is a degree of over prediction at the highest concentrations investigated, and further observations are required to determine the source of this discrepancy.

Measurements of the backscattered pressure level were obtained at the same range, 0.5-0.6m, for a fixed concentration as the pulse duration was extended. The results of these observations are shown in Fig6. There is seen to be a linear increase in the backscattered signal with $\sqrt{(\tau c)}$. This is due to the number of simultaneously received backscattered signals at the transceiver, from the scatterers within the insonified volume, being given by $A(\tau c/2)$ where A is the insonified crossection. This linear dependence reaffirms the incoherency of $\langle P_{rms} \rangle$. The theoretical line is in close agreement with the data showing that the absolute level of the backscattered pressure is being correctly predicted.

The dependence of the backscattered pressure on range for three concentrations is presented in Fig7. There is a steady decrease in signal with distance due to the combined attenuation of the water and the suspension and the inverse dependency with range. Comparison of the predictions with the measurements shows favourable agreement for the three curves over the farfield range. There is some discrepancy at the highest concentration with an increasing divergence as the range is increased. The most likely cause of this is an underestimation for the value of α_s, however, experimental measurements of attenuation were unable to confirm this due to the limited concentration range covered to date. An interesting feature and one remarked upon when looking at the $\langle P_{rms} \rangle$ dependency on $\sqrt{M}$, is the variation in backscattered signal with range due to the combined influence of the signal increase owing to the linear influence of the $\sqrt{M}$ term and the signal reduction arising from the M dependency in the argument of the exponent. Backscattered pressure levels of the same value can be obtained for concentrations having different orders of magnitude, albeit though for a limited distance from the transceiver. This does make it essential to know accurately the value for ξ since at high concentrations small uncertainties in this value will yield significant errors in the predicted backscattered pressure

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CONCLUSIONS

A number of measurements of the ensemble average rms backscattered pressure have been taken. Attention has focused on the dependence with mass concentration, pulse duration and range from the transceiver. Comparison of the theoretical predictions, based on an integral evaluation over the insonified volume with the backscattered pressure described using the form function, has shown good agreement with the data. The assumption of incoherent backscattering and insignificant multiple scattering is seen to be valid for the concentrations investigated here. Some degree of discrepancy was noted at high concentration levels and this was consistent with an underestimate of the attenuation. Further observations are required in this region to resolve precisely the origins of the differences.

REFERENCES

1. Rayleigh. The Theory of Sound. Dover Publications Inc. (1945).
2. Faran, J.J. JASA 23 405-418 (1951).
3. Hickling, R. JASA 34 1582-1592 (1962).
4. Neubauer, W.G., Vogt, R.H. and Dragonette, L.R. JASA 55 1123-1129 (1974)
5. Hampton, L.D. and McKinney, C.M. JASA 33 664 (1961)
6. Rudgers, A.J. JASA 45 900-910 (1969).
7. Morse, P.M. and Ingard, K.U. Theoretical Acoustics. McGraw-Hill (1968).
8. Dragonette, L.R., Voght, R.H., Flax, L. and Neubauer, W.G. JASA 55 1130-1137 (1974).
9. Vogt, R.H. and Neubauer, W.G. JASA 60(1) (1976).
10. Flax, L., Dragonette, L.R. and Uberal, H. JASA 63 723-731 (1978).
11. Flax, L., Gaunaurd, G.C. and Uberall, H. Phys. Acoust. 15 191-294 (1981)
12. Numrich, S.K., Dragonette, L.R. and Flax, L. in Elastic Wave Scattering and Propagation by Varadan, V.K and Varadan, V.V Ann Arbor Science 149-175 (1982)
13. Marston, P.L., Williams, K.L. and Hanson, T.J.B. JASA 74(2) (1983).
14. Gaunaurd, G.C. and Uberall, H. JASA 73(1) 1-12 (1983).
15. Ahuja, A.S. JASA 66(3) (1979).
16. Varadan, V.K, Varadan, V.V, Su, J.H and Pillai, T.A.K. JASA 71(6) (1982)
17. Waterhouse, R.V. JASA 75(3) (1984).
18. Williams, K.L. and Marston, P.L. JASA 78(3) (1985).
19. Perdigâo, J.M., Ferreira, A., Lefebre, J.E. and Bruneel, C. Ultrasonics 26 (1988).
20. Werby, M.F. and Gaunaurd, G.C. IEEE J. Oceanic Engng. 14(4) (1989).
21. Hay, A.E. and Mercer, D.G. JASA 78(5) 1761-1771 (1985).
22. Ma, Y., Varadan, V.K., Varadan, V.V. and Bedord, K.W. JASA 74(2) (1983)
23. Sheng, J. and Hay, A.E. JASA 83 598-610 (1987).
24. Libicki, C. and Bedord, K.W. JASA 85(4) (1989).
25. Hay, A.E. and Schaafsma, A.S. JASA 85 (3) (1989).

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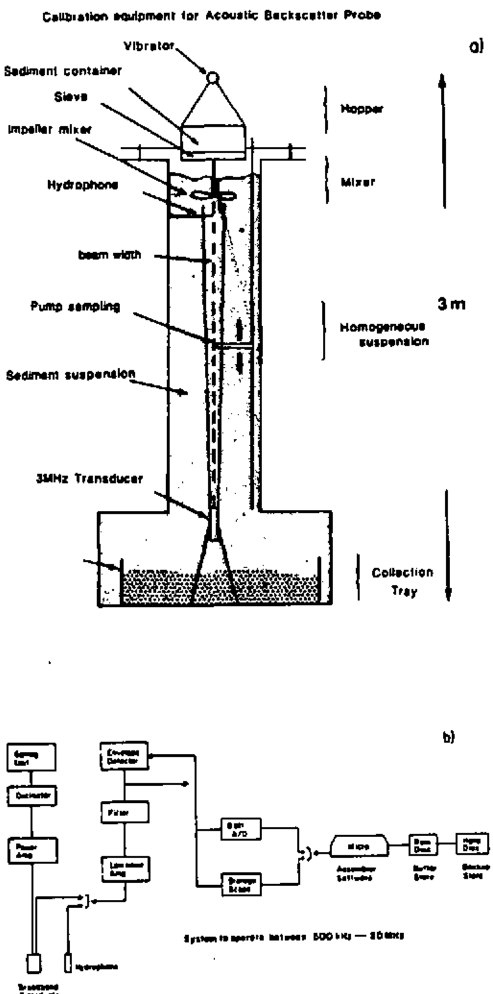

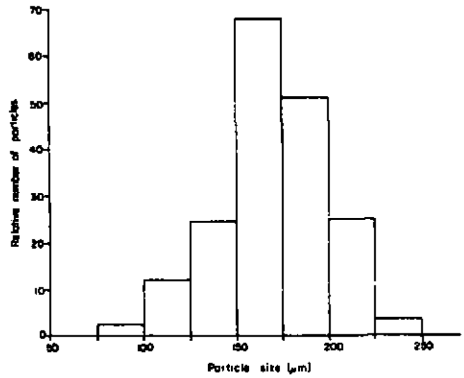

Fig 2 Particle size distribution

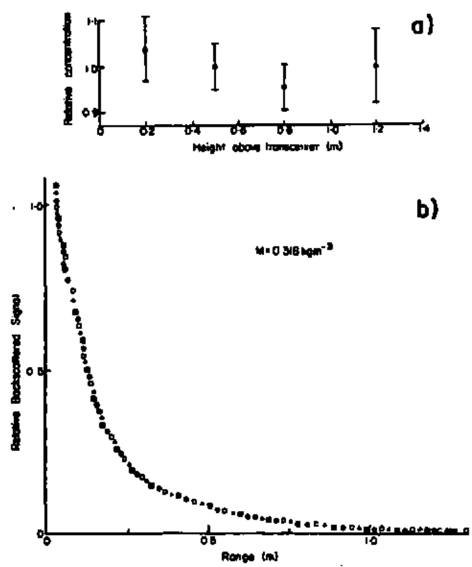

Fig 1 Experimental arrangement

Fig 3 Pump sample and acoustic measurements of the suspension

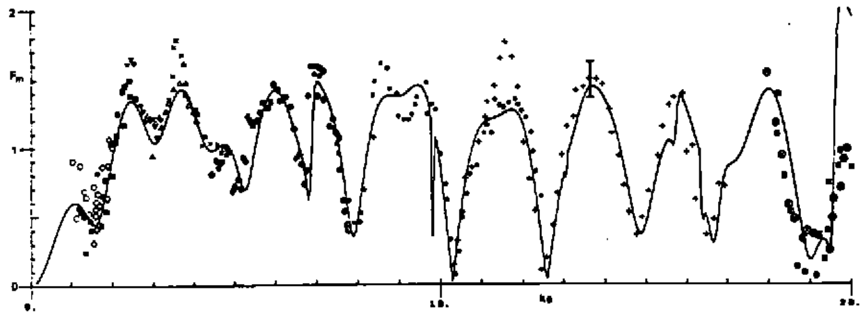

Fig4 FORM FUNCTION measurements of ballotini

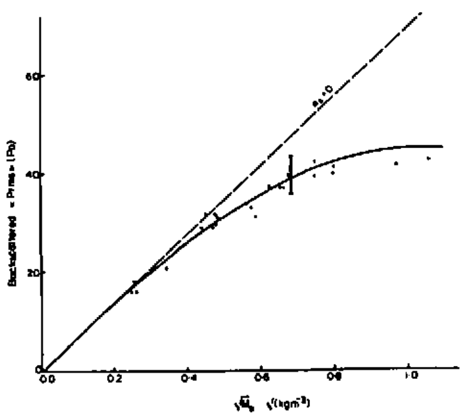

Fig5 Variation of backscattered pressure with concentration

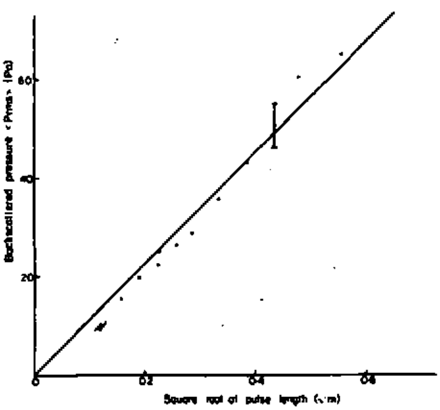

Fig6 Dependence of <Prms> on √(tc)

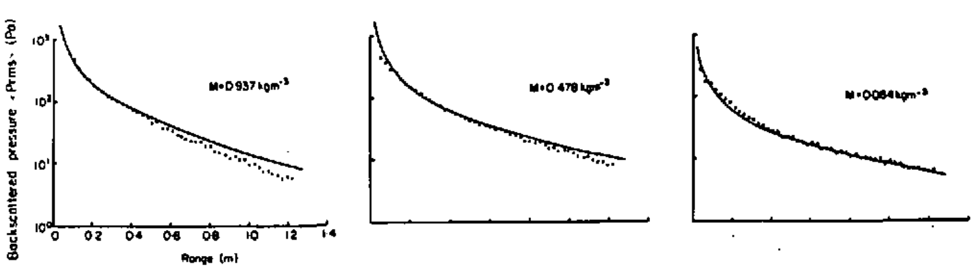

Fig7 Variation of <Prms> with range