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SOURCES OF SOUND AND OF SILENCE, AND SOME CONSEQUENCES

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There have been very few attempts in the past to define what is meant by a "source of sound", and there is at present considerable confusion and controversy about the definitions that have been attempted. The principal reason for this would appear to be that "sources of sound" do not seem to exist as material entities in their own right, in the way that electric charges do as sources of electrical fields. Any material element of mass in suitable motion can be a "source of sound", or so it might appear.

In what follows, some definitions are offered and on the basis of these certain classifications of "sources of sound" are made, based essentially on consideration of the mechanical power that they are capable of producing in the form of acoustic waves. In making these definitions and classifications it becomes necessary to distinguish between "real physical sources" and those "equivalent sources" from which acoustic waves may appear to emerge, but which physically represent simply "reflection" processes, not "creation" processes.

For materials that are homogeneous and isotropic, and in small amplitude motion about a uniform internal equilibrium state of rest or of uniform motion, the acoustic (sound) motion is best defined as that associated with the fluctuations of the isotropic stress (hereinafter the (acoustic) "pressure") that are rapid enough to be adiabatic to a first order. The small amplitudes and first order adiabaticity have the consequences that the mass, linear momentum, and energy transport equations of the mechanical continuum model of the material can be linearised, and that the pressure, $p(x_k,t)$, and mass density fluctuations, $\rho(x_k,t)$, are related by $p(x_k,t) = c^2\rho(x_k,t)$, where c is the constant equilibrium state speed of sound. The material behaves to first order as a lossless one and can be characterised by c and the equilibrium mass density ρ. (This first order losslessness is of course that of a material with zero irreversible process parameters such as those of viscosity and thermal conductivity.) The local physical dynamics of this kind of motion is then simply that the pressure satisfies an inhomogeneous scalar wave equation with a "source" term $-q(x_k,t)$, representing whatever effects are due to other types of motion of the material (e.g., turbulence), and/or to external "body forces" (e.g. electromagnetic field forces). This term $q(x_k,t)$, being mathematically in this context a function of (x_k,t) which is specified independently of the pressure $p(x_k,t)$, is a "source of sound"; specifically it is a distribution of monopole moment per unit volume. Such an independent mathematical specification of q is in accord with the realities of the physics that is being modelled only if the shear stress and Reynolds stress fluctuations and the body force fluctuations actually are linearly independent of the acoustic pressure determined by the inhomogeneous scalar wave equation and the associated boundary and initial conditions. (The first order adiabaticity condition, equivalent here to setting irreversible mechanism parameters to zero, has already taken care of such influences as those of viscous stresses.) The source term $q(x_k,t)$ is both a mathematical and a physical "source of sound" only when this is true. It seldom is strictly true, physically, but often can be assumed true to a first order: that is, the acoustic motion and the other fluctuating motions are (nearly) linearly independent).

The second, and indeed the only other, "source of sound" can be identified in the following complete linear differential system, which is the general mathematical model for the forced physical linear acoustics of a material of the kind already specified in any specified open space-time region (V (boundary $S\notin V$), $T(t_1,t_2)$) ($\notin$ means "not in", $\in$ means "in"):

$$[\partial^2/\partial x_i^2 - (1/c^2)\partial^2/\partial t^2]p(x_k,t) = -q(x_k,t),(x_k,t)\in(V,T), \quad (1)$$

$$\alpha(x_k,t..)p(x_k,t) + \beta(x_k,t..)\partial p(x_k,t)/\partial n = \gamma(x_k,t) \text{ for } x_k(\in V) \text{ on } S, \quad (2)$$

$$p(x_k,t) = \partial p(x_k,t)/\partial t = 0 \text{ for } t=t_{1+} \text{ and all } x_k\in(V). \quad (3)$$

Here "$x_k(\in V)$ on S" means that x_k is not "at" S itself but lies "on" S just inside V, and similarly t_{1+} is a time infinitesimally larger than t_1. The material outside V may or may not be the same (ρ,c) material as in V, and V may be any part of a larger region filled with (ρ,c) material. Similarly (t_1,t_2) may be any part of a larger time interval of interest. In the general free plus forced motion problem p and $\partial p/\partial t$ would have non-zero specified values at the initial time t_{1+}. In the spatial boundary condition (2) on S the coefficients $\alpha(x_k,t..)$ and $\beta(x_k,t..)$ are specified linear algebraic-differential-integral operators: i.e., $\alpha(x_k,t..)$ means $\alpha(x_{ks},t,\partial/\partial x_{ks},\partial/\partial t,\int dx_{ks},\int dt)$, where x_{ks} is a position co-ordinate on S and the integrals are indefinite, and similarly for $\beta(x_k,t...)$.

It is the inhomogeneous term $\gamma(x_{ks},t)$ in the boundary condition (2) which is the second, and only other, "source of sound". This conclusion, that only q and γ are "sources of sound", is the first consequence of the analysis being presented; the specific evidence justifying the conclusion is to follow.

For the linear acoustic motion being considered, which is irrotational, the normal acoustic particle velocity on S, $v_n(x_{ks},t)$, is related to the pressure gradient by $\rho\partial v_n/\partial t = -\partial p/\partial n$. (Note that it is only this irrotational acoustic particle velocity that is involved here, which physically is not necessarily the total particle velocity in the material.) Thus, specification of $\gamma(x_{ks},t)$ represents specification on S of a certain linear combination of isotropic stress and rate of change of normal acoustic linear momentum density, or normal acceleration (since ρ is constant). Appropriate choices of α and β can give γ's representing pressure specification, normal acoustic velocity or momentum or acceleration specification, etc.. Condition (2) can be called a generalised Robin condition, and evidently includes as special cases the well known "passive" boundary conditions (when $\gamma=0$) such as the Neumann (hard wall), Dirichlet (pressure release), and general time and position dependent specific normal acoustic impedance specification of $Z = p/(-(1/\rho)\int dt(\partial p/\partial n)$. The $\gamma\neq 0$ conditions are "active", and it is only for these that power may need to be supplied to maintain the specified conditions on S. Both q and γ are externally maintained forcing functions, the first being applied to the material in V generally, and the second to the material surface on the boundary S. As a physical example γ may represent a specified, or experimentally determined, linear combination of the pressure and the normal acoustic particle velocity on a loudspeaker diaphragm in forced motion.

Because the system (1)-(3) is linear it is convenient, both mathematically and physically, to regard its response to point-impulse forcing as a fundamental one, with its response to any forcing then being simply an integral superposition of appropriate weighted responses of this kind to impulses delivered at different space-time points. The point-impulse volume source q_δ is the generalised function $\delta(t-t')\delta(x_k-x_k')$ (where $\delta(x_k-x_k') = \delta(x_1-x_1')\delta(x_2-x_2')\delta(x_3-x_3')$). It is evidently highly non-physical as it stands, since there can be no physically measurable quantity which is equal to zero at every space-time point but one, and has the value (infinity)^4 at that point! Also, a $p(x_k,t)$ proportional to $H(x_k-x_k')$, say, is non-physical as it stands because it would have a $\delta(x_k-x_k')$ gradient, etc.. Nonetheless, it is very convenient mathematically to use such generalised functions, with due regard of course to the rules for manipulation of such functions, in obtaining what are ultimately physically interpretable solutions of the system (1)-(3). No such functions can be regarded as actually physically existing as such at their points of non-analyticity, but in keeping with the basic concept of generalised functions they can be interpreted as limits of equivalent functions that are actually smooth and bounded in a small but finite neighbourhood of such a point of non-analyticity, the equivalence being that of the average behaviours of the respective non-analytic and smooth functions over such a neighbourhood. The neighbourhood is well-defined physically as the smallest space-time "volume" that can be "geometrically" identified with the three metre sticks and clock available to the observer - the finite resolving power limit of measurement.

In this context then, non-analytic functions, including both bounded ones such as the Heaviside step function $H(x)$ and its indefinite integrals, and unbounded generalised ones such as the Dirac delta function $\delta(x)$ (the derivative of $H(x)$) and its derivatives, are mathematically admissible in the analysis of system (1)-(3) and physically interpretable as their appropriately smoothed analytic equivalents.

The solution of system (1)-(3) can now be considered.

When, in the general Green formula for the problem (1)-(3), the adjoint Green function is taken to be the particular integral $p_{\delta\infty}(x_k,t|x_k',t') = \delta(t-t'-|x_k-x_k'|/c)/4\pi|x_k-x_k'|$, it is found that

$$\left\{\begin{array}{ll}\text{for } (x_k,t)\in(V,T), & p(x_k,t)\\ \\ \text{for } (x_k,t)\notin(V,T), & 0\end{array}\right\} = \int_V \frac{q(x_k',t-|x_k-x_k'|/c)}{4\pi|x_k-x_k'|}\,dx_k'$$

$$+\int_S\left[\frac{(\partial p(x_k',t')/\partial x_\ell')(x_k',t-|x_k-x_k'|/c)}{4\pi|x_k-x_k'|}+\frac{\partial}{\partial x_\ell}\left[\frac{p(x_k',t-|x_k-x_k'|/c)}{4\pi|x_k-x_k'|}\right]\right]dS_\ell(x_k') \qquad (4)$$

The $p(x_k,t)$ given by this expression for $(x_k,t)\in(V,T)$ obviously exists if the integrals do, and if they do then it is the unique solution there, because of the invariance of the Green formula to choice of the complementary function part of the Green function. Also, the formula similarly gives a unique $p(x_k,t)$ of zero for all space-time points outside

(V,T). Note that the solution exists and is unique for points at which $q \neq 0$ as well as for those at which $q=0$: i.e., there is "sound" inside the volume source distribution q.

For realistic modelling of many physical problems the volume source distribution q is zero. A non-zero q is physically realistic (i.e., directly physically interpretable) only in problems such as those of the Lighthill acoustic analogy theory of sound generated aerodynamically, in which case it would be $(1/c^2)\partial p^2/\partial t^2 - \partial^2\rho/\partial t^2 + (\partial^2/\partial x_i \partial x_j)(\rho v_i v_j + p_{ij} - p\delta_{ij})$, where p_{ij} is the stress tensor, it being assumed that this q can be prescribed independently of knowledge of $p(x_k,t)$, at least to the extent necessary to give reasonable estimates of p in the otherwise quiescent (ρ,c) region outside the disturbed turbulent flow. Obviously, if by an exceptionally good guess, q were the physically correct one then the solution (4) would give the physically correct total pressure p everywhere, both inside and outside the source distribution, (V,T) here being assumed to be all space-time.

The surface integral of equation (4) can be interpreted as a superposition of fields of surface source distributions of monopole moment per unit area $\partial p/\partial n$ and dipole moment per unit area $-\hat{n}_i p$, respectively. (Here $\hat{n}_i$ is the surface outward normal unit vector). Mathematical equivalents to these are volume source distributions of moments $(\partial p/\partial n)\delta(n-n_{S-})$ and $-\hat{n}_i p\delta(n-n_{S-})$, respectively, where n_S is a point on S and n is distance in the outward normal direction there (n_{S-} is a point in V approaching n_S). Again it is obvious that if the physically correct p and $\partial p/\partial n$ were to be specified on S, then the physically correct, exact, value of the surface contribution to p to all points in (V,T) would be given by the Green formula (4).

These monopole/dipole surface source distributions and their volume source equivalents are not necessarily "sources of sound". They certainly are not when $\gamma=0$ on S and consequently the boundary is passive. When a complementary function part is added to the adjoint Green function, determined by requiring the Green function to satisfy the homogeneous form of condition (2), so that the adjoint Green function is then the "actual" point-impulse response $p_{\delta I}(x_k,t|x_k',t')$ of the system (1)-(3) (this response has the particular integral part $p_{\delta\infty}$ as previously defined and its complementary function part can be denoted as $p_{\delta Ic}(x_k,t|x_k',t')$), then the general Green formula produces the alternative forms

$$\left\{\begin{matrix}\text{for } (x_k,t) \in (V,T), & p(x_k,t) \\ \text{for } (x_k,t) \notin (V,T), & 0\end{matrix}\right\} = \int_V \frac{q(x_k',t-|x_k-x_k'|/c)}{4\pi|x_k-x_k'|}\,dx_k'$$

$$+ \int_V\int_T q(x_k',t')p_{\delta Ic}(x_k,t|x_k',t')dx_k'\,dt'$$

$$+ \int_S\int_T \left\{\begin{matrix}[\gamma(x_k',t')/\beta(x_k',t')]\;p_{\delta I}(x_k,t|x_k',t') \\ -[\gamma(x_k',t')/\alpha(x_k',t')]\;\partial p_{\delta I}(x_k,t|x_k',t')/\partial n'\end{matrix}\right\} dS(x_k')\,dt'. \quad (4a,b)$$

Here the passive reflections of waves from q by the surface S that were included in the surface integral in equation (4) now appear separately in the second volume integral of equations (4a,b). Since $p_{\delta Ic}$ satisfies the homogeneous scalar wave equation as a function of (x_k,t) for $(x_k,t) \in$

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(V,T), this integral is a complementary function part of $p(x_k,t)$ for $(x_k,t)\epsilon(V,T)$ and thus has no mathematical (or physical) sources there. Its contribution to $p(x_k,t)$ for $(x_k,t)\epsilon(V,T)$ can be interpreted as due to mathematical "image sources" in the region outside (V,T), these being like the "image source" seen in a mirror, physically non-existent in that outside region. The surface source integral now, in either of its (4a,b) forms, vanishes when $\gamma=0$, demonstrating that it indeed, together with the volume source q, are the only "sources of sound" in the system (1)-(3). With $p_{\delta I}$ in these expressions written as $p_{\delta\infty}+p_{\delta Ic}$, it is evident that in the first form (4a) $(\gamma/\beta)p_{\delta\infty}$ is interpretable mathematically as the contribution of a surface distribution of monopole moment per unit area (γ/β), and similarly in the second form (4b) $-(\gamma/\alpha)\partial p_{\delta\infty}/\partial n'$, that of a directed dipole moment per unit area of a surface layer of dipoles oriented in the inward normal direction, both of these interpretations being exactly analogous to the corresponding ones for equation (4). Now, however, one has exactly the same contribution given by either the monopole layer or the dipole layer. There is no real ambiguity about this, as it is γ, appearing in both forms, which is the "source of sound" and not either the monopole layer or dipole layer as such. Again, both of these layers have volume source layer mathematical equivalents. However, for both forms (4a,b) the terms depending on $p_{\delta Ic}$ in the surface integral are <u>not</u> interpretable as monopole/dipole surface layer contributions, since $p_{\delta Ic}$ is analytic on S, its functional form there not being comprised of members of the $H(x)$ non-analytic family. As for the $p_{\delta Ic}$ volume integral, they are image source terms.

From the considerations so far, it is clear that since the system (1)-(3) is linear, and a unique solution exists, albeit expressible in different forms, the causal "sources of sound" q and γ for a given solution are unique. This is the second consequence of the analysis. The third consequence follows immediately. Since these causal "sources of sound" are unique, any other source distributions capable of producing the same fields in (V,T) must be non-causal.

The fourth consequence has already been mentioned obliquely in passing. It is that the solution (4), with the equivalent monopole and dipole layer volume representations $\partial p/\partial n\delta(n-n_{s-})$ and $-\hat{n}_t p\delta(n-n_{s-})$, shows clearly that the composite source distribution consisting of q plus these layers, if placed in a (ρ,c) material of infinite extent, would produce no sound outside the surface S of V. It is therefore a "source of silence". It exists within the region V because the equivalent monopole/dipole layers are "on" S just inside V. It includes the effects of the volume (q) and surface (γ) "sources of sound" for the interior problem, which in general require a supply of power to be maintained. However, since p is zero on the outer side of S, this composite source radiates no power to any region outside V, and thus requires no power from outside V for its maintenance. The composite source distribution and the internal sound field it produces comprise an internally equilibrated "rest state" as far as the outside (ρ,c) world is concerned (like that of the ground state of an atom?), this state being maintained by power supplied to the internal sources q and γ from somewhere else. As equation (4) is causal, this internal equilibrium state is causal, and is a causal "source of silence" for the exterior region. Without the singular surface source layers, however, it would not necessarily be a "source of silence". For future reference it is convenient to call the total composite volume source (i.e., q plus the singular volume source equivalents of the monopole and dipole surface layers) for such a free field external problem Q.

A simple example of a Q illustrates that at least one of the singular layers is necessary. One would not expect both to be necessary as they are linearly related. The pressure spectral density for the case of a point source at $r=0$ and a spherical monopole layer at $r=a$ has the form, for $r>a$ and where $k=\omega/c$,

$$p(r,\omega) = q_o(\omega)\frac{e^{-ikr}}{4\pi r} + q_a(\omega)\frac{\sin ka}{ka}\frac{e^{-ikr}}{4\pi r},$$

and this is zero if $q_a(\omega)=-(ka/\sin ka)q_o(\omega)$. For this $q_a(\omega)$, the field in $0<r<a$ is $q_o(\omega)\sin k(r-a)/\sin ka$. In principle, a zero field in $r>a$ can be obtained causally in real time. The pressure spectral density measured by a microphone at $r_M(0<r_M<a)$ is $p(r_M,\omega)=q_o(\omega)\sin k(r_M-a)/\sin ka$. The convolution formula for the desired $q_a(t)$ is then $q_a(t)=\int_{\infty}^{\infty}p(r_M,t')K(t-t')dt'$, where $K(t)$ is the inverse Fourier transform of $-(\omega a/c)/\sin[\omega(r_M-a)/c]$, which of course is predeterminable from the given geometry. If this convolution integral can be done in a time less than $(a-r_M)/c$, then $q_a(t)$ is known in time to use it to control a spherical lousdpeaker array on $r=a$ so that the wavefronts $q_o(t-r/c)4\pi r$, as they arrive at $r=a$ and pass through it are joined by outward going waves from the loudspeaker array which exactly cancel them out in $r>a$. Note from the expression for $p(r,\omega)$ in $r<a$ that infinite simple harmonic responses are predicted at the frequencies of resonance of a sphere with a pressure release condition at $r=a$. With a dipole loudspeaker array on $r=a$ as well as the monopole one these can be avoided, and it can be arranged that the outgoing waves from q_o are completely absorbed in the monopole/dipole array, so that the field is zero for $r>a$ and $q_o(t-r/c)/4\pi r$ in $r<a$. An interior field can be reduced to zero in a similar way. Given a primary spherical source shell at $r=b(>a)$, a spherical source shell on $r=a$ can causally and in real time, in principle, be controlled so that the field is zero throughout $0<r<a$.

This is the fourth consequence: the existence of this infinite variety of causal "sources of silence". The mathematical silence is complete. Approximating this silence physically depends on being able to devise suitable microphone and loudspeaker arrays, and fast enough controllers.

The fifth, and next to last, consequence has to do with the uniqueness or otherwise of source distributions in free field: specifically on whether or not two different source distributions can produce the same near and/or far fields. It is already known from the proved uniqueness of the Green formula result for $p(x_k,t)$ that a given q produces a unique p. The question here is the inverse: given p, does a q_1 exist such that $p=\int_V q\,dV=\int_V(q+q_1)\,dV$. Before integration over time with $q+q_1$ as the source distribution, the Green formula for $p(x_k,t)$ is

$$p(x_k,t) = \int_V\int_T [q(x_k',t')+q_1(x_k',t')]\,\frac{\delta(t-t'-|x_k-x_k'|/c)}{4\pi|x_k-x_k'|}\,dx_k'\,dt'.$$

By the hypothesis, $p=\int_V\int_T q\,dx'\,dt'$, so the integral over q_1 in this expression must be zero. Operating on this integral by $\partial^2/\partial x_i^2-(1/c^2)\partial^2/\partial t^2$ gives $0=\int_V\int_T(-)q_1(x_k',t')\delta(t-t')\delta(x_k-x_k')dx_k'\,dt' =-q(x_k,t)=0$. This proves

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that the source distribution q is unique. Furthermore, the integral over q in the first of the two preceding expressions gives the unique p at all (x_k, t) in the infinite space-time region: i.e., for all points inside q and all points outside q. This conclusion, at least for points outside q, is at variance with the non-uniqueness of q claimed, and proved mathematically, in section 7.2 of reference [1]. The difference in the conclusions is not due to any errors in mathematical logic in either of the proofs, but to the different premises. In reference [1] p is defined (as existing) only outside q; here p is defined (as existing) everywhere. It has been well known for over a century, of course, that there is an infinite variety of mathematical volume and surface sources inside any given volume V that can produce the same field p outside V, and this explains the reference [1] result. The present result shows that of these it is only the causal unique q which can also produce the corresponding p inside V.

A further aspect of this uniqueness question concerns the possibility of determining q from far-field information. In respect to this question, the "physical interpretation" given in section 7.2 of reference [1], namely that a source distribution cannot be identified uniquely from such information, is unhelpfully misleading, especially to those with source identification interests, if not wholly wrong unless properly qualified. The facts of the matter are as follows, with brief, outline explanations; full proofs are too lengthy for inclusion here.

First, only sources of finite spatial extent have far fields, by definition, as will be explained later. Second, sources of finite spatial extent comprise two mutually exclusive classes: (i) the "sources of silence" Q which have zero fields everywhere outside the source region, and can be called more briefly "reactive", these self-evidently having zero far fields; (ii) sources which have non-zero far fields, which can be called "active". Third, given this classification of sources of finite extent, sources of infinite extent evidently can be regarded as belonging, as limiting cases, to the Q class of "sources of silence"; their "far fields", detectable only by observers "at" infinity, are also zero. Fourth, for an active distribution $q_A(x_j, t)$, the four-fold frequency-wavenumber Fourier transform $q_A(\kappa_j, \omega)$ does not vanish for $\kappa_j = -kx_j/|x_j|$ and the far field frequency spectral density of the pressure $p_{AFF}(x_j, t)$ is given by [1, see equation (10.7)][2, see section 1.5 especially pp. 37-41]

$$p_{AFF}(x_j, \omega) = [\exp(-ik|x_j|/4\pi|x_j|]q(\kappa_j = -kx_j/|x_j|, \omega).$$

Hence, from the existence/uniqueness theorem for Fourier transform pairs, it follows that, with the far field directivity function spectral density defined as $D(kx_j/|x_j|, \omega) = 4\pi|x_j|\exp(ik|x_j|)p_{AFF}(x_j, \omega)$,

$$q(x_j, t) = \iint_{-\infty}^{\infty}\int D(-\kappa_j, \omega)\exp(i\kappa_j x_j + i\omega t)d\kappa_j\, d\omega$$

exists and is unique.

A general source distribution, in view of the linearity of the problem, can be expressed as a sum of a member each of the two mutually exclusive classes, active (subscript A) and reactive (subscript R): $q = q_A + q_R$. If both q_A and q_R are of finite extent then q_A can be determined uniquely from the far field directivity information. Since q_R requires no power from the acoustic region outside itself, it cannot be identified by any acoustic probing from outside: it will produce a zero scattered field in response to

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any such probing. The only acoustic observations by means of which it can be determined therefore must be made in its interior; it is, of course, in a unique, one-to-one, relationship with the interior field which it produces. Since all q_A, by definition, are of finite extent, this covers all cases except those in which q_R is of infinite extent. In such cases the field of q_R remains present as $|x_k| \to \infty$, as well as the far field of q_A. Hence the total field as $|x_k| \to \infty$ is $p_\infty = p_{AFF} + p_{R\infty}$, where the subscript ∞ denotes the approximation to first order in $1/|x_k|$ as $|x_k| \to \infty$. (Note that one cannot logically write p_{RFF} because the definition of "far field", i.e., the result when the integrand in the integral for p is approximated to first order in $x_j'/|x_j|$, is not applicable; $p_{R\infty}$ must be obtained as the limiting form for large $|x_j|$ of the original integral, without this approximation of the integrand.) In these circumstances it is possible in principle to distinguish between p_{AFF} and $p_{R\infty}$ from knowledge of the total pressure p_∞ on two distinct large concentric spherical surfaces, not just the one required when any q_R that may exist is of finite extent. The p_{AFF} contributions to p_∞ are radially outward travelling waves of the form $D(\theta,\phi)f(t-r/c)/4\pi r$, carrying power towards infinity, and the $p_{R\infty}$ contributions are of a standing wave type, carrying no power towards infinity (on time average, of course). Given this, it is sufficiently evident that pressure measurements on these two spheres give enough information, when suitably analysed, to determine each of the two contribution, p_{AFF} and $p_{R\infty}$, on, say, the inner sphere. The active source distributions q_A can then be determined from this information. Furthermore, as a representation of the frequency spectral density of the reactive field p_R everywhere in space is available in the form of absolutely convergent series of linearly independent spherical wave functions of which the angle dependence factors are a complete orthonormal set of basis functions on the surfaces of concentric spheres of any radius, it is possible in principle, on the basis of this two sphere information, to determine p_R and q_R everywhere inside the smaller of the two spheres.

It is such an expansion in spherical wave functions that can be used to prove that any reactive source q_R contained in a region of finite extent produces zero field everywhere outside this region, and hence is a Q, a "source of silence". The proof consists of first determining that the field of any source distribution with zero far field has zero field on the surface of any sphere containing q_R in its interior (this follows almost trivially from the orthogonality of the angle dependence factors of the spherical wave functions). Second, by considering various such spheres and using an analytical continuation process it is shown that the field vanishes everywhere outside the source region.

The sixth consequence is, in brief, that information on at most two large concentric spheres is sufficient to determine the unique source distribution everywhere within the smaller sphere, when q_R is of infinite extent.

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