STATISTICS OF SURFACE SCATTERED SIGNALS

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ABSTRACT The Fresnel corrected physical optics approximation has been used to examine the statistical properties of waves scattered from a time-varying Gaussian random surface. With certain restrictions, the first and second order distribution functions of the scattered wave field have been obtained. Close analogies with the problem of wave propagation in a random medium have been stressed.

1. INTRODUCTION

Because it is a perennially popular subject in underwater acoustics, there is an extensive literature on the theory of reflection and scattering of waves by a randomly rough surface. Many of the treatments have been concerned with deriving the appropriate forms to be used for the boundary conditions [1] and with justifying approximate evaluations of the Helmholtz integral in either the Fraunhofer [2] or the Fresnel [3] limits. In dealing with the statistical description of the scattered wave, most investigations [4] have been limited to derivations of the first two moments of the field received at a point, or to evaluation of joint second moments of the scattered field at temporally or spatially separated points. In addition, many of the results obtained have been restricted to one of two limits corresponding to slightly rough (small Rayleigh parameter) or very rough (large Rayleigh parameter) surfaces. Hence, while much has been accomplished, important problems remain. Thus for example, in a signal processing context, a calculation of the variance at the output of a nonlinear device whose input is a surface scattered signal requires knowledge of fourth order moments of the surface transfer function.

In the present paper the Fresnel corrected physical optics approximation has been employed to derive, with certain restrictions, first and second order distribution functions of the field scattered from a time-varying Gaussian random surface. Furthermore, the formulation is sufficiently general to encompass the entire range of Rayleigh parameter values.

In studying this problem it has been observed that there exist extremely close analogies between the small and large Rayleigh parameter regimes for scattereing from a rough surface and the corresponding Rytov (weak scattering) and saturated (strong scattering) regimes for wave propagation in a random medium. Indeed, much of the impetus for the present investigation had its source in recent work on wave propagation in a random medium [5,6]. The similarities between these problems are not accidental but signify an underlying unity in the physical ideas involved. Because the surface scatter problem is, in many regards, the conceptually simpler, it can serve profitably as a pedagogical introduction to a study of wave propagation in a random medium.

2. SCATTERING REGIMES

Consider the scattering geometry depicted in Fig. 1. It is imagined that the surface is illuminated by a wave of frequency $\omega = ck$ (where c is the wave propagation speed and k the wave number) emitted by an omnidirectional point source (S), and that the scattered field observation point is 0. Denoting the deformation of the surface by $z = \zeta(\underset{\sim}{x}, t)$, it is assumed that ζ is a zero mean, stationary and homogeneous Gaussian process with space-time correlation function

$$<\zeta(\underset{\sim}{x}, t)\ \zeta(0, 0)> = <\zeta^2>\ \Psi(\underset{\sim}{x}, t), \tag{1}$$

characterized by a suitably defined correlation length L.

All the qualitative features of the scattered field are determined by two parameters Λ and Φ, which are identical in meaning to analogous parameters describing the character of wave propagation in a random medium [5, 6]. The parameter Λ is a size scale defined by

$$\Lambda^{-1} = \frac{kL^2 \sin^2\psi}{2}\left(\frac{1}{r_1} + \frac{1}{r_o}\right). \tag{2}$$

Λ is essentially the square of the semi-major axis of the Fresnel ellipse on the surface in units of the surface correlation length. The parameter Φ is a strength measure well known in the surface scattering literature as the Rayleigh parameter, and is defined as

$$\Phi = 2k<\zeta^2>^{\frac{1}{2}} \sin\psi\ . \tag{3}$$

The Fresnel approximation can be considered as an evaluation of the Helmholtz scattering integral by the method of stationary phase. When the surface is smooth there is a single point of stationary phase located at the origin for the coordinate system in Fig. 1. When a rough surface is present the point of stationary phase becomes perturbed away from the origin if $\Phi\Lambda < 1$, although the scattered field remains substantially coherent. However, when $\Phi\Lambda > 1$ multiple points of stationary phase will occur. When this happens, the multiple contributions to the scattered field interfere weakly if $\Phi < 1$, and there is a dominant coherent component; if $\Phi > 1$ the multiple contributions to the field interfere strongly and the scattered field becomes incoherent.

3. SUMMARY OF RESULTS

Denote by $p(\underset{\sim}{r}_o, \underset{\sim}{r}_1, t)$ the complex envelope of the field at 0, and let $p_o(\underset{\sim}{r}_o, \underset{\sim}{r}_1, t)$ be the corresponding quantity for a mirror surface. Then when $\Lambda >> 1$, the normalized intensity $I = |p/p_o|^2$ can be shown to obey Rice statistics with moments

$$<I^n> = n!\left[1 - \exp(-\Phi^2)\right]^n\ {}_1F_1(-n;\ 1;\ \frac{\exp(-\Phi^2)}{1-\exp(-\Phi^2)}). \tag{4}$$

It is noted that this result contradicts an assertion in [7] that the incoherent component of the received field obeys Hoyt statistics. The difficulty can be traced to the neglect of Fresnel corrections in the development in [7].

In the limit $\Phi << 1$ an expansion of eqn. (4) indicates that the lower order moments of I approach those of a log-normally distributed variable; this situation corresponds, therefore, to the Rytov regime

Table 1

Quantity	Power of dependence on f_o	B	E
Piston velocity	2	1	1
Force on piston	0	1	1
Total radiation impedance	-2	0	0
Radiation impedance/ρcA	0	0	0
Radiated power	2	2	2
Radiated power/piston area	4	2	2
Input current	2	2	1
Input impedance	-2	-2	0
Input power	2	2	2

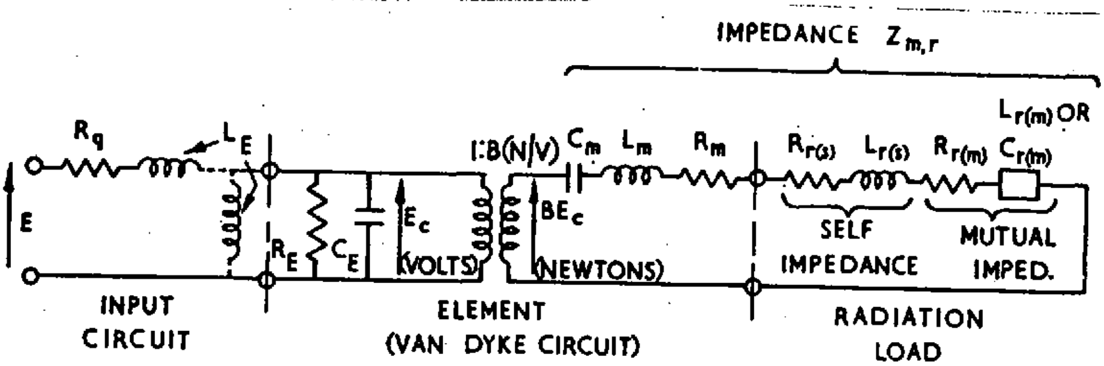
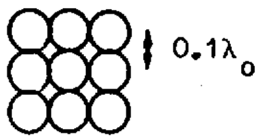


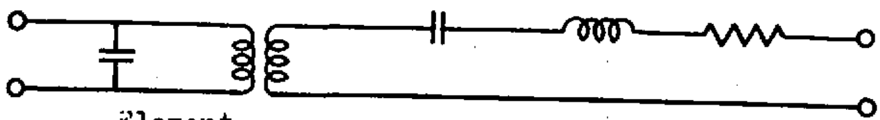
Fig 1 Equivalent circuit of a piezo-electric element, its radiation load and the assumed input components.



Geometry for each array

Element	f_o(kHz)	a/λ_o	η_{ma}	Q_w	B(N/V)	k	tan δ
A	1	0.05	0.90	6	1	0.28	0
B	1	0.05	0.90	12	1	0.28	0

	μF	V : N	m/N	kg	N/m/s
A	0.213	1 : 1	18.1×10^{-9}	0.28	146
B	0.107	1 : 1	9.1×10^{-9}	1.67	146



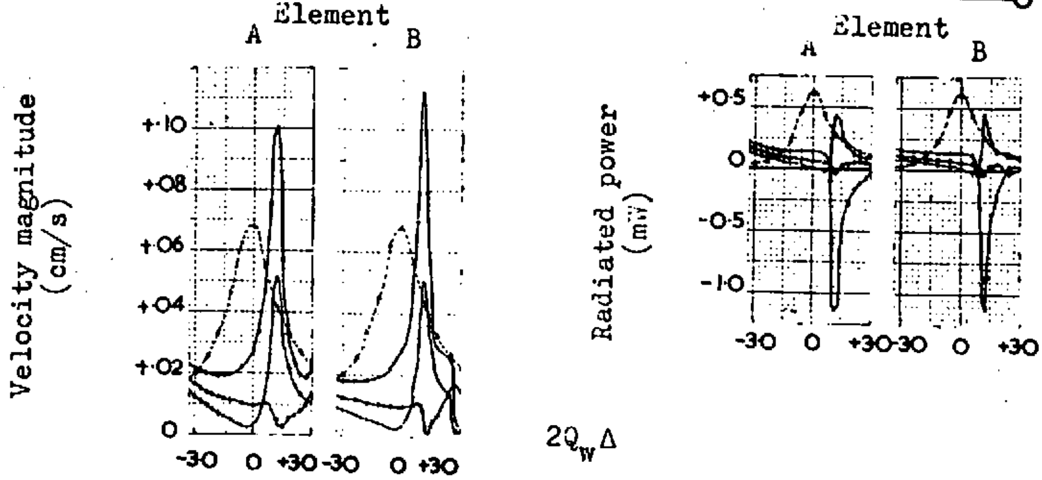
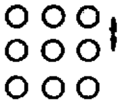
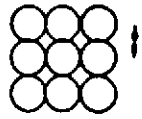


Fig 2 Example of scaling law 3a

Array using element B



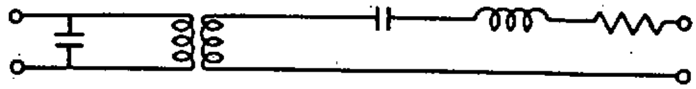
Array using element C



Element	f_o(kH)	a/λ_o	η_{ma}	Q_w	B(N/V)	k	tan δ
B	1	0.05	0.90	12	1	0.28	0
C	1	0.10	0.90	12	1	0.28	0

	μF	V : N	m/N	kg	N/m/s
B	0.107	1 : 1	9.1×10^{-9}	1.7	146
C	0.007	1 : 1	0.6×10^{-9}	34.3	2230

Constant voltage. No steering or tapering.



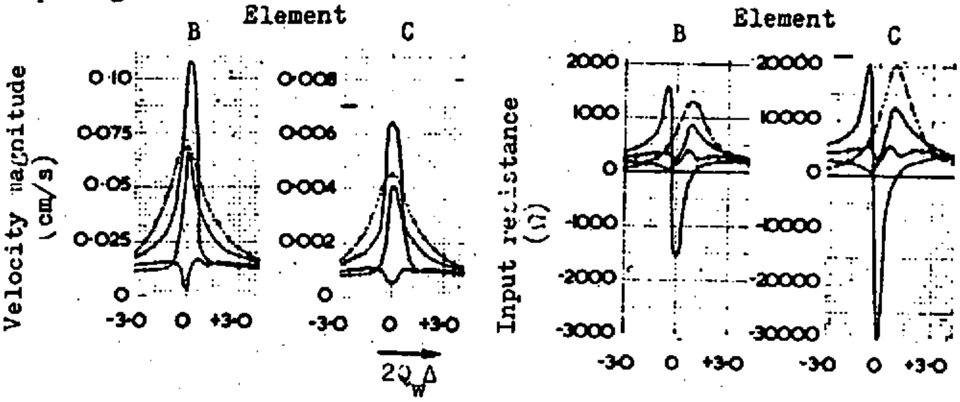
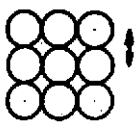


Fig 3 Example of scaling law 4

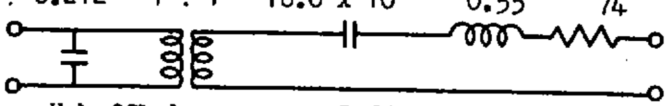


Array in infinite, rigid baffle uses element B. Unbaffled array uses element D.

Element	f_o(kH)	a/λ_o	η_{ma}	Q_w	B(N/V)	k	tan δ
B	1	0.05	0.90	12	1	0.28	0
D	1	0.05	0.90	12	1	0.28	0

	μF	V : N	m/N	kg	N/m/s
B	0.107	1 : 1	9.1×10^{-9}	1.67	146
D	0.212	1 : 1	18.0×10^{-9}	0.55	74

Constant voltage. No steering or tapering.



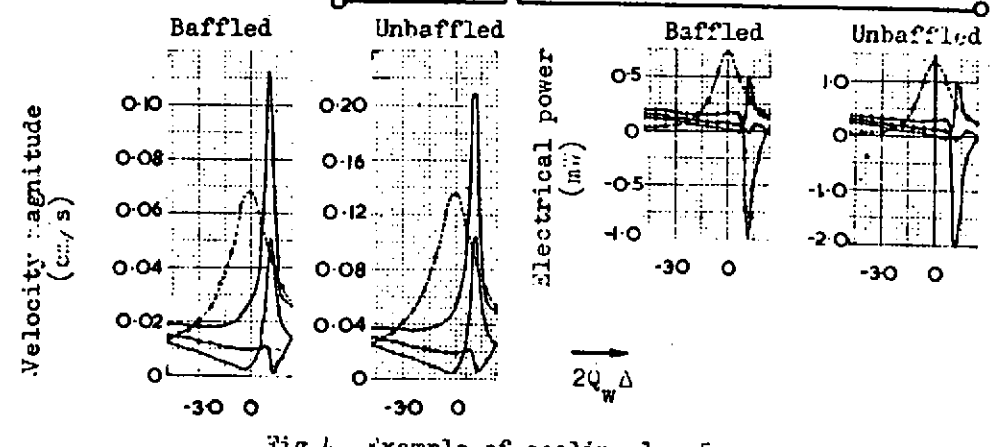


Fig 4 Example of scaling law 5