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COMPARISON OF LEAST SQUARES ESTIMATION AND IMPULSE RESPONSE TECHNIQUES FOR ACTIVE CONTROL OF FLAME NOISE

P. J. DINES

CAMBRIDGE UNIVERSITY ENGINEERING DEPARTMENT

To control a random disturbance a measure of the noise is necessary. In flames the light emission, at C_2 or CH wavelengths, provides such a measure [1]. The coherence of two signals measures the amount of information of one held in the other. Typically the coherence for the light and sound of a flame is high but nonperfect. In the active control of flame noise there is no time delay between detecting the light emission and introducing the antisound. This limits the upper frequency of control and requires the controller to calculate the antisound as fast as possible. The controller is a digital filter and the accuracy of two methods of calculating the filter weights is discussed below.

The basic model is of a sampled system with series for input {u} and output {y}. The latest output sample is thought of as the aggregate of weighted previous outputs (the autoregressive (AR) part of the model) and weighted inputs (the moving average (MA) part).

$$y_k = b_0 u_k + b_1 u_{k-1} + \dots + b_m u_{k-m} - a_1 y_{k-1} - \dots - a_n y_{k-n}$$

In z-transform notation: $A(z)Y(z) = B(z)U(z)$
where A(z) and B(z) are polynomials in z and $X(z) = \sum_{-\infty}^{+\infty}(x_i z^{-i})$.

Once the unknown weights {a} and {b} are estimated the system can be modelled by a digital filter. Two methods of estimating the weights have been investigated: minimum least squares (the ARMA method), which estimates both {a} and {b} and is implemented by a recursive digital filter; and by using the impulse response (the MA method), which assumes $A(z)=1$ and which is implemented by a nonrecursive digital filter.

For systems amenable to active control two series can be found which represent a sample input and output of the desired controller and the ARMA method uses these to find the weights [2]. The basis of this method is to recognise that we do not have true input and output samples but close approximations; in which case the time series may be written in vector form as:

$$\underline{Y} = \Phi\underline{\theta} + \underline{\varepsilon}$$

Φ contains the samples, $\underline{\theta}$ the weights, $\underline{\varepsilon}$ is the error.
The ARMA method considers the best, or optimal, solution for the weights, $\hat{\underline{\theta}}$, to be that for which $\sum_{i=q}^{N}(\varepsilon_i^2)$ is a minimum. This may be found by differentiating and setting to zero:

$$\hat{\underline{\theta}} = (\Phi^T\Phi)^{-1}\Phi^T\underline{Y} \qquad \text{(T denotes transform)}$$

For lack of bias $E[\underline{\hat{\theta}}-\underline{\theta}]\to 0$ as $N\to\infty$.

For least squares estimation this means $E[(\Phi^T\Phi)^{-1}\Phi^T\underline{\varepsilon}]\to 0$. The bias will be zero if Φ and $\underline{\varepsilon}$ are independent, and if $E[\underline{\varepsilon}] = 0$ as well.

If we assume that the current output can be found from values of the input only, the MA model, then the output due to an impulse ($u_0=1$; $u_i=0$ for $i\neq 0$) will be: $y_0=b_0$; $y_1=b_1$; ...; $y_n=b_n$; ...
The weights {b} are the values of the impulse response {h} at the respective sampling times. The impulse response, {h}, of a system and its frequency response, $H(\omega)$, are a Fourier transform pair so that once $H(\omega)$ is estimated {h} and hence the weights, can be found easily via discrete Fourier transform algorithms.

In general an ARMA model uses fewer weights and this is important in flame noise control because each weight means a longer calculation time. The choice of order is a problem because to underparameterize the system means error in estimation. Eykhoff [3] describes a criterion for judging whether the residual sum of squares is significantly reduced by increasing the order of the ARMA model. In practice the order is selected by the speed of implementation. The order of the MA model is selected once the impulse response dies down to quantization levels. Truncating {h} will effect the frequency response, becoming $H'(\omega)$, of the filter but the difference from $H(\omega)$ can be calculated by Fourier transformation. Forming {h}' need not result in a useless estimation, see below. The filter response to a limited input must be limited too otherwise the filter is unstable.
This means $\sum_{-\infty}^{+\infty}|h_n| < \infty$. Since the MA model is a finite impulse response method the stability criterion will always be satisfied. For the ARMA method the stability criterion is related to the poles of $H'(z)$. If the roots of $B(z)$ are inside the unit circle of the z-plane then the filter is stable. This test is easy to apply in retrospect.

In order to discuss the problems of implementing the two models in more detail a simple system was identified by both approaches. The system consisted of a Hewlett Packard Mechanical Transfer Function Simulator. This was connected to a Computer Automation LSI-2 minicomputer through antialiasing filters, Fig 1. $H(\omega)$ of the simulator is shown in Fig 2. The sampling frequency was 8kHz throughout the tests. The impulse response of the system is very short, as we would expect considering the damping of the frequency response. If {h} is truncated to the first 32 points then $H'(\omega)$ has a response virtually unchanged from $H(\omega)$. If the input/output samples from which $H(\omega)$ was derived are used for least squares estimation then a series of ARMA models may be found, each of differing order. If an estimation of order 32 is taken and $H'(\omega)$ calculated then it also is found to be a close approximation to $H(\omega)$ (only the dip at ~1100Hz is not exactly matched).

Flame noise control, however, concerns input and output signals of nonperfect coherence. So the coherence between the input and the output of the simulator was detrimented by the addition of an independent noise source on the output. In order to retain good coherence over part of the frequency range the additional noise was low-pass filtered at 500Hz before being added to the output, Fig 1. The nonperfect coherence is reflected in the frequency response shown in Fig 3. $\tilde{H}(\omega)$ is not time invariant and so spectral averaging methods do not converge to $H(\omega)$ over the frequencies where the signals are not coherent. An attempt can be made to model {h} of the system but a Fourier transform of $\tilde{H}(\omega)$ leads to considerable disturbance before the time origin of the impulse response. However $\tilde{H}'(\omega)$ retains most of the detail of $\tilde{H}(\omega)$ if $\{\tilde{h}\}$ is truncated after point 32. This shows that the important information is held in these first points. An ARMA model of the system has the correct characteristics but much of the detail is lost. This is again a 16-pole/16-zero model. The low frequency noise leads to bias of the ARMA estimate. Although the residual errors had zero mean ($E[\underline{\varepsilon}]=0$) they were correlated with the output.

Nonlinearity in the measurements manifests itself as uncertainty in the estimate of $H(\omega)$. In the active control of flame noise $\tilde{H}(\omega)$ is calculated using Ross's method [2] and a typical result is shown in Fig 4. This has an impulse response which has a considerable disturbance in the noncausal part. If only the first 32 points of $\{\tilde{h}\}$ are taken and $\{\tilde{h}\}'$ transformed the result shows the modelling to be inadequate. The significant part of {h} has not been retained. This is not surprising for the part holding the maximum in $\{\tilde{h}\}$ is in the noncausal half. If the impulse response is shifted by 16 points, to make it causal, the delay introduced through the filter is of 16 sampling periods. However $\tilde{H}'(\omega)$ is now adequate in amplitude but not in phase. In the case of the flame such a delay cannot be countenanced. This means that $\{\tilde{h}\}$ must have its significant part after time zero and this in turn implies a limit to the coherence or the amount of noise tolerable. How does the ARMA method fare when trying to model $\tilde{H}(\omega)$? The bias leads to a false estimation. It is worse than in the simulator with noise because there is nonperfect coherence throughout the frequency range for the flame.

In the active control of flame noise time is at a premium. Calculation time in the digital filter controller must be kept as low as possible. This study shows that an impulse response method performs better than least squares estimation under the same nonperfect coherence conditions. The key to which method to use lies in the coherence between the light and sound from the flame.

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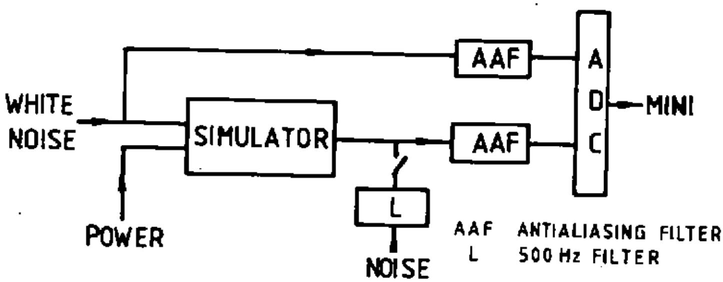

FIG 1. BLOCK DIAGRAM OF SIMULATOR SYSTEM

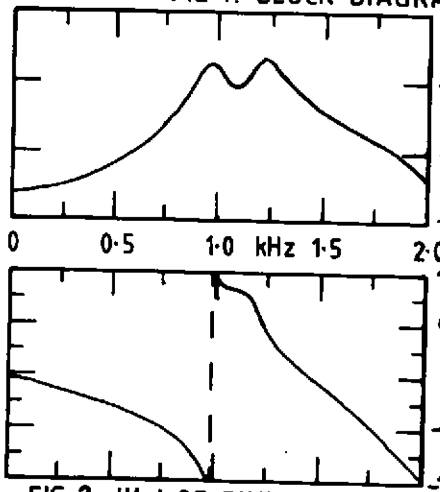

FIG 2. H(ω) OF SIMULATOR

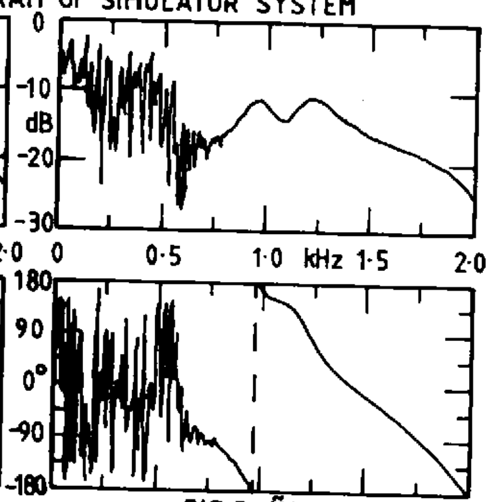

FIG 3. Ĥ(ω)

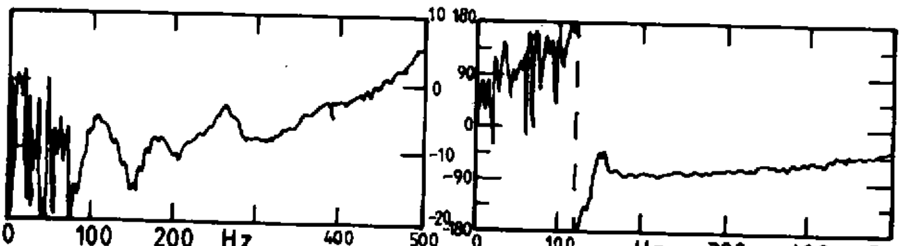

FIG. 4 H(ω) OF FLAME