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BRITISH ACOUSTICAL SOCIETY - SPRING MEETING

5th/7th April 1972 at Loughborough University.

"ULTRASONICS IN INDUSTRY" SESSION.

ULTRASONIC WAVE PROPAGATION THROUGH A MULTILAYERED MEDIUM

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Introduction

In the field of non-destructive testing there exists a particular problem of examining bonded multilayered structures for bonding defects and voids.

Ultrasonic methods are well suited to the detection of such voids because of the high acoustic mismatch between solids and gases. However, the combination of thin sheets and many interfaces makes the testing of layered structures a difficult task by normal ultrasonic means due to the formation of multiple echoes in the first layer encountered.

A survey of the existing techniques of testing multilayered structures[1] indicated that there is, as yet, no method which can determine not only the size but more importantly the depth location of any non-bond or defect.

The Proposed Solution

The masking effect of these multiple echoes in the first layer can be avoided by applying the well-known resonance technique. By this method acoustic "transparency" of the first layer can be achieved by choosing the test frequency such that the wavelength is exactly twice the layer thickness. However, at the second interface the problem of multiple echoes arises again and flaw echoes beyond the second layer are masked.

It is proposed to extend the resonance testing system to a multilayered medium. In the general case, in Figure 1, non-bond 'A' would be detectable by forming a resonance of the first layer and non-bond 'B' by resonance of the first (n-1) layers.

Hence, it is necessary to be able to predict the resonance phenomena associated with the passage of an ultrasonic wave at any frequency through a multilayered medium.

Acoustic Wave Propagation through a Layered Medium

The resonance phenomena of a multilayered medium can be studied conveniently by means of a reflection coefficient which can be derived in two ways[2];

a) by solving the wave equation with the boundary conditions being the continuity of pressure and particle velocities normal to the surface,

b) by use of the input impedance concept analogous to transmission line theory.

It is possible to write down the pressure and particle velocity in matrix form and obtain a "characteristic" matrix for each bounded layer i.e. except for the two outer media[3]. The overall reflectivity of a general system of layers can then be found by taking the product of all the characteristic matrices.

Using the notation of Figure 1, the input impedance, to the nth layer, $Zin_{(n)}$, can be expressed as;

$$Z_{in(n)} = \rho_n c_n \left\{ \frac{Z_{in(n+1)} + j\rho_n c_n \cdot \tan k_n d_n}{\rho_n c_n + j Z_{in(n+1)} \cdot \tan k_n d_n} \right\}$$

where, $\rho_n c_n$ - characteristic impedance of the nth layer.

k_n - wave number for nth layer.

d_n - thickness of nth layer.

From this the reflection coefficient, R, can be written,

$$R = \left| \frac{Z_{in(2)} - \rho_1 c_1}{Z_{in(2)} + \rho_1 c_1} \right|$$

Both methods yield identical results and it is then possible to examine the reflection coefficient for any system of layers at any frequency.

Graphical Representation of the Reflection Coefficient

In the testing situation the layer thicknesses and material constants are predetermined constants. Figure 2 shows the variations in the reflection coefficient over a limited range of frequencies for a system of nine, thin, alternate layers of steel and resin. From this frequency spectrum several important observations can be made. The higher resonances (i.e. around 5.5 MHz and 10.3 MHz) are due to the fundamental resonances of the individual layers and are of very high 'Q'. For the proposed testing system the lower resonances are of most interest due to the relatively low 'Q's involved and the lack of higher harmonics.

In order to investigate these lower resonances in more detail it became necessary to return to the simple case of two transition layers. The reflection coefficient now becomes;

$$R = \left| \frac{(\rho_2 c_2 \alpha - \rho_1 c_1 \gamma) + j(\rho_2 c_2 \beta - \rho_1 c_1 \delta)}{(\rho_2 c_2 \alpha + \rho_1 c_1 \gamma) + j(\rho_2 c_2 \beta + \rho_1 c_1 \delta)} \right|$$

where,
$$\alpha = \rho_4 c_4 (\rho_3 c_3 \cos k_2 d_2 \cos k_3 d_3 - \rho_2 c_2 \sin k_2 d_2 . \sin k_3 d_3)$$
$$\beta = \rho_3 c_3 (\rho_2 c_2 \sin k_2 d_2 \cos k_3 d_3 + \rho_3 c_3 \cos k_2 d_2 \sin k_3 d_3)$$
$$\gamma = \rho_3 c_3 (\rho_2 c_2 \cos k_2 d_2 \cos k_3 d_3 - \rho_3 c_3 \sin k_2 d_2 \sin k_3 d_3)$$
$$\delta = \rho_4 c_4 (\rho_3 c_3 \sin k_2 d_2 \cos k_3 d_3 + \rho_2 c_2 \cos k_2 d_2 \sin k_3 d_3)$$

The reflection coefficient vanishes if;

$$\left.\begin{array}{l} (\rho_2 c_2 \beta - \rho_1 c_1 \delta) = 0 \\ (\rho_2 c_2 \alpha - \rho_1 c_1 \gamma) = 0 \end{array}\right\}$$

These simultaneous equations give rise to an interesting solution;

$$\begin{cases} \tan k_2 d_2 = -\dfrac{\rho_2 c_2 \left(\rho_1 c_1 \rho_4 c_4 - (\rho_3 c_3)^2\right)}{\rho_3 c_3 \left(\rho_1 c_1 \rho_4 c_4 - (\rho_2 c_2)^2\right)} \cdot \tan k_3 d_3 \\ \tan k_3 d_3 = \sqrt{\dfrac{(\rho_3 c_3)^2 (\rho_1 c_1 - \rho_4 c_4)\left(\rho_1 c_1 \rho_4 c_4 - (\rho_2 c_2)^2\right)}{\left(\rho_4 c_4 (\rho_2 c_2)^2 - \rho_1 c_1 (\rho_3 c_3)^2\right)\left(\rho_1 c_1 \rho_4 c_4 - (\rho_3 c_3)^2\right)}} \end{cases}$$

Thus, provided the expression within the square root is positive, total transmission (i.e. no reflection) is possible for suitable layer thickness-wavelength ratios (other than quarter or half wavelengths). In the general case the above equations are only partially satisfied and some reflection is apparent as is evident in Figure 2 at frequencies of about 600 kHz and 1.1 MHz.

Experimental Work and Conclusions

Experimental work has been carried out on a simple sandwich structure consisting of two thin stainless steel sheets separated by a thin (0.16 mm) bonding layer of resin to test the limitations of the theory. Even though the theoretical derivation assumes plane wave propagation at normal incidence in loss-less layers, close agreement was found with the experimental results.

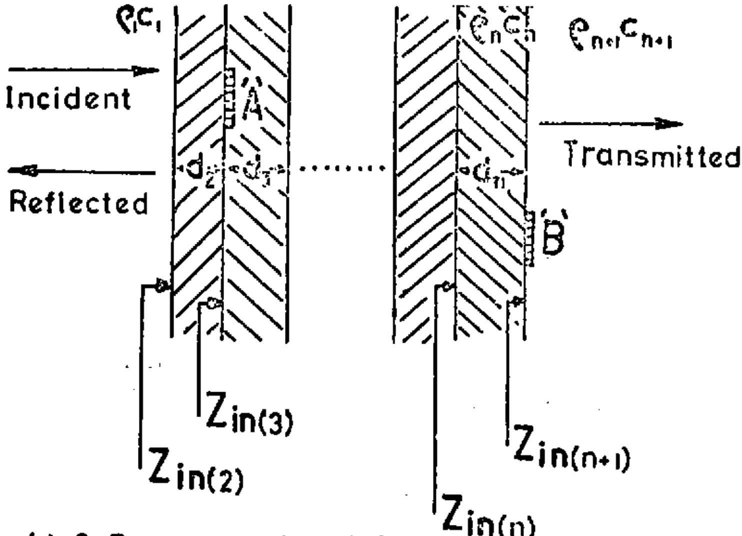
Work is now proceeding on more complicated structures containing non-bonded areas at known depths and results so far indicate the feasibility of the proposed testing system.

References

1. G. Curtis. 'Evaluation of Adhesion by Ultrasonic Techniques'. Ultrasonics for Industry Conference paper 1970.
2. L.M. Brekhovskikh. 'Waves in Layered Media' Academic press.
3. R.D. Ford. 'Introduction to Acoustics' Elsevier.

Acknowledgements

The author wishes to thank Professor J.W.R. Griffiths and Dr. J. Szilard for their help and guidance and also the Procurement Executive, Ministry of Defence, for supporting the work.



(A & B are non-bonds)

Figure 1. Notation for the general system of layers.

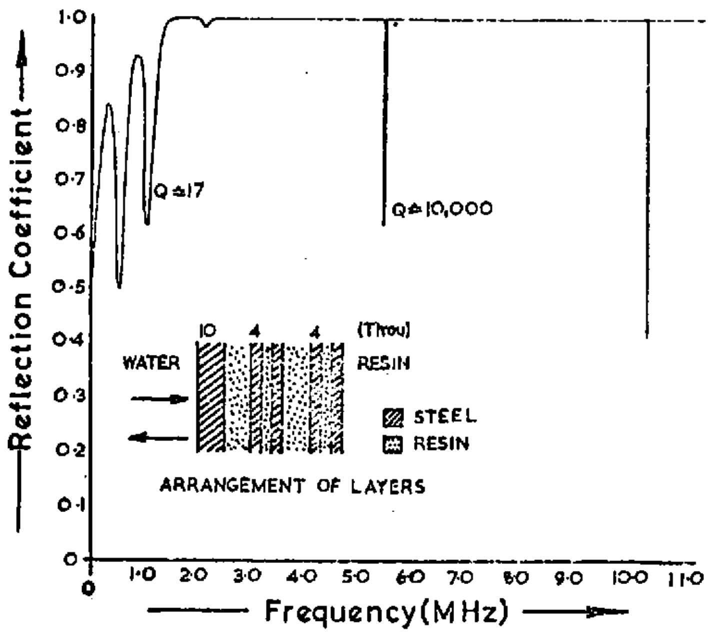


Figure 2. Reflection characteristics for a multilayered medium.