DESIGN AND CONSTRUCTION OF PRACTICAL OPTICAL FIBRE HYDROPHONES

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1. INTRODUCTION

Optical fibre sensing is a subject which has attracted considerable attention in recent years. We have described elsewhere (Ref 1) the design of a reflectometric time-division multiplexed interferometric system for underwater acoustic sensing. This system consists of an array of encapsulated optical fibre coils, serially multiplexed together. The effect of an acoustic field acting on a coil is to cause a change in the optical path length of the coil, and this can be detected as a phase change in light passing through the coil. This system has obvious applications in such areas as seabed mounted surveillance, towed and hull mounted sonar arrays. This technique offers a number of advantages over conventional piezoelectric sensing techniques, including the self-multiplexing configuration of the system, and the fact that no electronics are required in the hydrophones or the array, coupled with the usual advantages of fibre optic data transmisssion, such as immunity to EMI.

Until recently, development has concentrated on proving and evaluating the system concept, leading to the deployment of a small demonstrator array off the English coast in 1986. The hydrophones used in this system were of the simplest possible form, consisting of an optical fibre coil encapsulated in a solid cylinder of epoxy resin. This design has the advantages of simplicity, reliability and robustness, but its inherent responsivity (defined as the phase change produced in one metre of fibre by a unit of acoustic pressure) is quite low. Recent work has addressed the problem of increasing the hydrophone responsivity, and hence enhancing the overall capabilty of the system. At the same time, consideration has been given to the practical design of hydrophones, bearing in mind the requirements for specific applications. The hydrophone improvement programme has thus been in two stages. Firstly, an analysis of fundamental operating principles was carried out, based on a simplified two-dimensional model. This led to the development of a computer model for the prediction of responsivity. In parallel with this, a number of different hydrophones were designed, constructed and tested, and the test results compared with the predictions of the model. The two aspects of the programme were complementary, as initially the experimental results were used to test the accuracy of the model, and when confident of its accuracy the model was used as an aid in the design of hydrophones with a high responsivity.

2. THEORETICAL MODELLING

2.1. The Basic Sensitivity Equation

The phase shift induced in a length of optical fibre by an applied pressure field is given by:

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$$\Delta \phi = K_0 n Ls_1 - \frac{1}{2} n^3 koL (s_1 p_{12} + s_3 p_{12} + s_2 p_{11})$$
 (1)

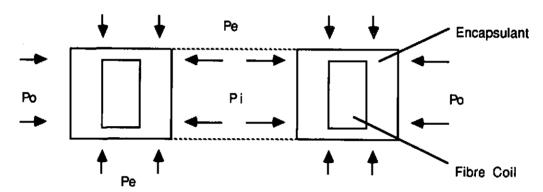
Where K_0 is the optical wave number, n is the refractive index of the glass, s_1 , s_2 and s_3 are the strains and pij are the Pockels (elasto-optic) coefficients.

This expression contains two main terms, which describe the first order effects contributing to the total phase shift (higher order effects have been neglected). The first term is a length change effect due to the axial strain acting in the fibre. The second term describes the elasto-optic effect, which is basically a strain induced change in the refractive index of the glass and is dependent on both the radial and axial strains. In general, for a glass the two terms are found to act in opposite senses, and their effects tend to cancel.

The simplest case to analyse is that of an infinitely long straight fibre with a single layer of coating material. The phase change in the fibre is found by calculating the strains induced in the fibre by an external pressure field, and then substituting these in equation (1) (Ref. 2.). In this case, the main contribution to the phase change is found to be the length change effect due to the axial strain, with a magnitude typically around twice that of the change produced by the elasto-optic effect. The total phase shift produced is strongly dependent on the material properties (specifically the Young's Modulus (E) and Poisson's ratio(σ)) and the thickness of the coating layer. If the layer is thick, and E is much smaller than that of the glass, the effect is to considerably increase the acoustic responsivity of the fibre. This is mainly attributed to the large increase in axial strain caused by the presence of the coating. To give a typical example, the phase responsivity of a bare fibre is found to be around -90 dB re 1 rad/Pa/m.i.l. Addition of a soft cladding with a thickness of 5mm increases the responsivity by around 20 dB.

2.2 Hydrophone Designs

The approach we have taken to modelling is similar to that described for the long straight fibre, but applied to a cylindrical geometry. Thus the basic hydrophone design that we have set out to model is a coil of fibre encapsulated in a cylinder of some other material, possibly with a central hole, as shown in Fig 1. The use of a coiled configuration is made necessary by the dual requirements for a large phase shift (which implies a fibre length of the order of 100m.) and small hydrophone size. The encapsulant serves a number of purposes. Firstly, it provides strength and rigidity to the fibre coil. Also, it acts to destroy coil resonances, which can have major effects on the frequency response. Finally, from the brief discussion of the long straight fibre, we have seen that the addition of coating to a "bare fibre" can have a large beneficial effect on the responsivity of the fibre: the encapsulant similarly acts to increase the responsivity.



Side view of simple hydrophone showing acting pressure fields Figure 1

We will denote the pressure acting on the outer and inner curved surfaces, and the flat end surfaces as P_0 , P_i and P_e respectively. Fig 1 shows the simplest possible configuration, where it can be seen that $P_0 = P_i = P_e$. This is essentially a hydrostatic configuration, and was the first case to be modelled. This basic design can however be modified to achieve a number of other potentially useful configurations. These are:-

- a) Pressure released centre $P_0 = P_e$, $P_i = 0$
- b) Pressure released centre, pressure released ends $P_i = P_e = 0$, $P_0 = 1$ (normalised)
- c) Pressure amplified ends, pressure released centre $P_i = 0$, $P_0 = 1$, $P_e > 1$

All these configurations were also modelled. The practical realisations of the various configurations are discussed later.

2.3 Basic Modelling Principles

2.3.1 Philosophy

The model was designed to provide an estimate of the effect on responsivity of varying certain parameters, in particular the dimensions and the mechanical properties of the encapsulant material, and was implemented on an IBM-AT computer. It was designed as an interactive design aid and in view of these requirements, it was decided that sophisticated modelling (such as finite element methods, for example) was not the most suitable approach. Fortunately, the specific cases that we wished to consider can be modelled using a simpler approach involving less numerical calculation. This approach was based on making the following assumptions:-

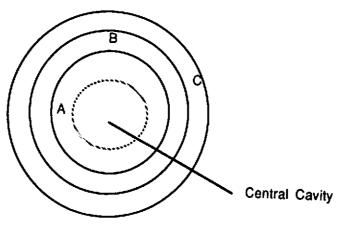
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- i) The hydrophone is exposed to a hydrostatic pressure field (ie the dimensions are small compared to the wavelength of the incident soundwave).
- ii) Within the hydrophone, stresses and strains are independent of axial position.
- iii) Stresses and strains are axially symmetric.

2.3.2 Description of Model

The calculation of hydrophone responsivity is carried out in two stages. Firstly, static elasticity theory is used to calculate the stresses and strains created within the bulk cylinder. These are then used to calculate the phase shift produced within the fibre. We will deal with the elasticity aspect first.

We consider the case of a cylinder composed of 3 symmetric layers of elastic material A, B and C (Fig 2). A, may be a solid cylinder, or may contain a cylindrical cavity.



Hydrophone cross-section Figure 2

For convenience, we first consider the plane-strain problem. This assumes that the ends of the composite cylinder are held a fixed distance apart, while the pressure Po acts on the outer curved surface and Pi acts on the inner surface. Although this fixed end case is not one of the favoured designs, it is mathematically the simplest case and so serves as a valuable illustration of the approach adopted.

By analysis of forces acting on a small element of the cylinder (assuming there is no shearing) it is readily shown that:

$$S_{rr} + r \frac{d}{dr} s_{rr} - S_{\theta\theta} = 0$$
 (2)

Where s** is the stress tensor in cylindrical coordinates.

The assumption is now made that each of the layers is an isotropic elastic material. Thus the following relationships hold:

$$e_{rr} = \frac{1}{E} (S_{rr} - \sigma(S_{\theta\theta} + S_{zz}))$$

$$e_{\theta\theta} = \frac{1}{E} (S_{\theta\theta} - \sigma(S_{rr} + S_{zz}))$$

$$e_{zz} = \frac{1}{E} (S_{zz} - \sigma(S_{rr} + S_{\theta\theta}))$$
(3)

Where e refers to the various strain coordinates in each of the layers, E is the Young's modulus and σ is the Poisson's ratio.

In terms of the radial displacement u = u(r), the strain Tensor components are:

$$e_{rr} = \frac{du}{dr}$$
 and $e_{\theta\theta} = \frac{u}{r}$ (4)

Substituting (3) and (2) yields the following:

$$r^2u'' + ru' - u = 0$$
 (5)

This homogeneous 2nd order ODE is readily solved by trial functions. The general solution is:

$$u = \alpha r + \frac{\beta}{r} \tag{6}$$

where α and β are constants.

We need therefore to determine the values of α and β in each of the 3 layers. This leads to six unknown variables for which we require six simultaneous equations. These are generated by imposing the following boundary conditions:-

- a. Normal stress at the inner faces matches applied pressure.
- b. Hoop strain at the A/B interface is continous.
- c. Normal strain at the A/B interface is continuous.
- d. Hoop strain at the B/C interface is continuous.
- e. Normal strain at the B/C interface is continuous.
- f. Normal stress at the interface matches applied pressure $(S_{ff} = -Po)$.

As an example, f gives rise to the following:

$$S_{rr} I_{r=a} = -P_0 \tag{7}$$

Thus substituting for Srr from (2a) and noting:

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$$e_{rr} = \alpha - \frac{\beta}{r^2}$$
, $e_{\theta\theta} = \alpha + \frac{\beta}{r^2}$ (8)

the boundary conditions become:

D,
$$\alpha_{C} \cdot \left(\frac{E}{1+\sigma}\right) \frac{1}{a^{2}} B_{C} = -P_{0}$$
 (9)

where the suffix c refers to the layer.

The other boundary conditions are expounded similarly.

The simple elasticity problem therefore reduces to the matrix equation:

$$\begin{bmatrix} b^2 & -1 & 0 & 0 & 0 & 0 \\ 1 & -1/r^2 & -1 & -1/r^2 & 0 & 0 \\ 1/DA & 1/rAr2^2 & -1/DB & 1/FBr2^2 & 0 & 0 \\ 0 & 0 & 1 & 1/r3^2 & -1 & 1/r3^2 \\ 0 & 0 & 1/DB & 1/FB^2r3^2 & -1/DR & -1/Fcr3^2 \\ 0 & 0 & 0 & 0 & d^2 & -1 \end{bmatrix} \begin{bmatrix} \alpha'A \\ \beta'A \\ \alpha'B \\ \beta'B \\ \alpha'C \\ \beta'C \end{bmatrix} \begin{bmatrix} -P_ib^2 \\ 0 \\ 0 \\ 0 \\ -P_0u^2 \end{bmatrix}$$

where $F = E/(1+\sigma)$ and a, r₃, r₂ and b are the radii of the interfaces in decreasing order and $\alpha' = D\alpha$, $\beta' = F\beta$.

Thus the elasticity problem is solved by solving the matrix equation. In the case of a non-hollow "A" region, the system reduces to a 5 x5 matrix equation obtained from the above by effectively removing the first row and the second column, together with the β entries in the column matrix.

So far we have only considered the plane strain case. The case of non-zero axial strain (ie as in Fig 1) is more physically relevant, although mathematically more complex. We will consider this case briefly.

In this case, we make a further assumption that the axial strain is constant throughout the structure. It is found that eqn 5 still applies, but the axial strain ezz (common to all layers) must be included. If ezz is specified, then the problem is again the solution of 6 linear simultaneous equations. However, more usually it is the end pressure Pe which is specified and ezz is then a further unknown, for which we require a new equation. This can be derived from a futher boundary condition, namely the balance of forces on the end of the hydrophone, written as:

$$\sum \int S_{ZZ} dA = -P_{e} \int dA$$
 (11)
all cylinder

This leads to a system of 7 simultaneous equations (and a 7 x 7 matrix) which may be solved by standard methods.

Thus solution of these equations completely specifies the stresses and strains at every point within the cylinder. Layers A and C are normally specified as containing only encapsulant, while layer B is a composite of optical fibre and encapsulant. In the simplest models, the presence of the fibre in the structure is usually ignored. However, the very high modulus of the glass in comparison with a typical encapsulant (typically 72 GPa compared to 1- 3 GPa) means that in practice the presence of the glass fibre has a major effect on the stiffness of layer B, and hence on the stresses and strains in the hydrophone. We have modelled Layer B as an isotropic elastic material with an effective E and σ which have been averaged on a volume proportion basis, (ie, the effective E and σ have been calculated by averaging the properties of the glass and the encapsulant).

The procedure is then to calculate the hoop strain and the radial and axial stresses at the nominal fibre radius. The hoop strain is readily identified as the axial strain of the fibre. The composite radial and longitudial stresses are identified as the stresses transverse to the fibre. Knowledge of these three quantities provides the transverse strains within the fibre, via the usual stress/strain relationship. The responsivity of the hydrophone is then calculated using eqn 1.

The responsivity will of course depend on a large number of parameters, including the dimensions of the cylinder, the position of the fibre coil within the cylinder, and the material properties of the encapsulants. The standard output of our model gives the responsivity as a function of E and σ for the encapsulant, with the hydrophone dimensions fixed.

The flexibility of the model is such that we can also consider the non-hydrostatic cases where one or other of the pressures is either zero or differing in magnitude from the others. In such cases the boundary conditions will be different and this will of course lead to different values of the stresses and strains. Without describing the detailed mathematical solutions for these cases, we will discuss some intuitive physical arguments which show how hydrophone responsivity varies in the different configurations. These arguments are in fact closely backed up by the predictions of the model.

2.3.3 Physical Arguments

If we first consider the hydrostatic case ($P_O = P_i = P_e$), the pressure P_O will tend to compress the coil inwards. The end pressure will produce a radial component of strain with a magnitude determined by the Poisson's ratio of the encapsulant, and with the opposite sign to P_O . In addition, the internal pressure P_i will have an outward radial component which acts in the same sense as the radial component of P_e (ie they will tend to expand the coil). Thus P_e and P_i will act together to oppose the compressive effect of P_O , but it is found that P_O is the dominant factor and the coil will compress under pressure. However, the effect of P_e and P_i will be to reduce the responsivity of the hydrophone. The effect of P_e is strongly dependent on the Poisson's ratio of the encapsulant, while the Young's modulus E will also be a major contributory factor to the responsivity. These two requirements are really a statement of the fact that in the hydrostatic case, the bulk modulus should be as low as possible to ensure maximum compression of the coil.

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If P_i is reduced to zero (ie. zero internal pressure), the difference between the compressive and expansive forces becomes larger, as the radial component of P_e is now the only factor opposing the compressing effect of P_0 . In this configuration, therefore, the sensitivity is somewhat enhanced compared with the hydrostatic case. If P_e is now also reduced to zero, the compressive effect of P_0 is unopposed, and this configuration will give the maximum possible sensitivity for a hydrophone operating in a compressive mode. In theory, the hydrophone can be made to operate in an expansive mode by maximising the effects of P_e and P_i , and/or minimising the effects of P_0 . However, this is not easy to achieve in practice, as will be explained in the next section.

3. HYDROPHONE CONSTRUCTION AND TESTING

In order to test the validity of the model, a series of hydrophones was constructed. These hydrophones were initially built to a single design, but were capable of modification to other designs covered by the model. In this section, we describe the practical realisation of these designs, discuss the practical results obtained and compare them with the predictions of the model.

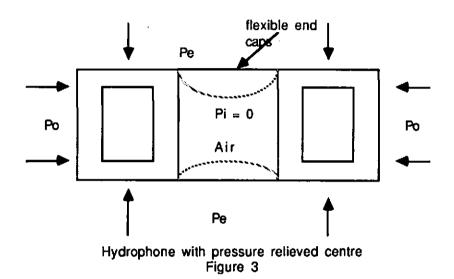
All the hydrophones were acoustically tested for responsivity in the same way. Each hydrophone was placed in a water tank (approximate dimensions 3 x 3 x 3 m) adjacent to a B&K 8100 reference hydrophone. The two hydrophones were insonified using a Gearing & Watson Hydrosounder, and their outputs were compared using a Hewlett Packard 3562A spectrum analyser, over the frequency range 5 Hz-2kHz.

3.1 Hydrostatic Design

This is the basic design, in which the acoustic pressure acts equally on all surfaces of the hydrophone, and is realised as shown in Fig 1. It is the simplest to construct, and was used in our original subsea demonstrator array. The hydrophone was 7.5 cm in diameter and 5 cm deep, and contained 200 m of fibre with an Araldite epoxy resdu encapsulant. The responsivity of this design has been measured for many hydrophones and has been found to be -67dB re 1 rad/Pa/m.i.l. This compares with a predicted responsivity of -68 dB re 1 rad/Pa/m.i.l. This is adequate for some applications, but where a high multiplexing gain, in conjunction with a low system noise floor is required, a higher responsivity is desirable.

3.2 Zero Internal pressure Design

In this design, the pressure acting outwards in the central cavity of the hydrophone is set at zero. This can most easily be achieved by sealing the ends of the cavity by flexible membranes, as shown in Fig 3 since P_i , the internal pressure, counteracts the compressing effect of P_0 (usually dominant), its reduction to zero increases the compressibility and hence the responsivity of the hydrophone.



The effect of reducing P_i to zero was explored using a number of hydrophones of identical external diameter but with differing central cavity diameters. When the cavities were sealed, trapping air in the hydrophone centre, it was found that the gauge sensitivity increased by an amount dependent on the size of the central cavity, as shown in Fig 5. It can be seen that the experimental results obtained agree very closely with the predictions of the model. For instance, an air-filled hydrophone with a central cavity diameter of 3 cm has a responsivity of -53dB re 1 rad/Pa/m.i.l., compared with the model prediction of -52dB. This is an improvement of 14dB over the hydrostatic case.

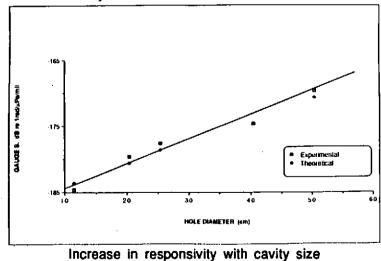
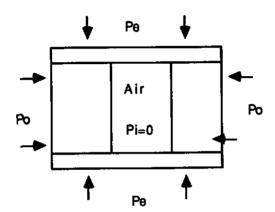


Figure 4

3.3 Zero Internal Pressure, Amplified End Pressure

This design can be realised by enclosing the central cavity with rigid end plates completely covering the ends of the hydrophone (Fig §).

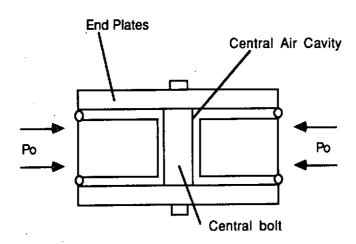


Hydrophone with pressure amplified ends Figure 5

As explained in Section 2, these have the effect of removing the internal pressure while causing an amplification of the end pressure, the magnitude of which is determined by the size of the central cavity. It is interesting to note that if the amplification factor is great enough, the effect of the end pressure can be as great as that of the outer pressure, but will act in the opposite sense, and the resultant strain on the fibre can be reduced to zero. Thus by carefully adjusting the size of the internal air cavity, the responsivity of the hydrophone can be made very low, although this has not yet been practically demonstrated. This has potentially useful applications in, for example, the construction of acoustically insensitive delay or reference coils. However, it should be noted that for a hydrophone where the air cavity diameter is typically one third that of the total hydrophone diameter, the end amplification factor is quite small, and the hydrophone can be treated as the zero internal pressure design discussed in 3.2.

3.4 Zero Internal Pressure, Zero End Pressure

This design can be realised as shown in Fig 6. The ends of the hydrophone are enclosed by rigid end caps which are prevented from touching the ends by a central bolt, while an o-ring is used between the end-plate and the hydrophone itself. This has the effect of reducing both the internal and the end pressures to zero. As these pressures normally work against the external pressure, removing them actually maximises the effect of the external pressure, so that a given acoustic pressure will produce the maximum compression of the coil. Hence if the hydrophone is to be used in a compressional mode, this design gives the maximum possible acoustic sensitivity. End plates of this design were fitted to several of the test hydrophones, and they produced the expected increase in sensitivity. For the case of the 3cm diameter cavity, the gauge responsivity measured was -50dB re 1 rad/Pa /m.i.l. This was an improvement of around 3dB over the zero internal pressure case.



Hydrophone with pressure relieved ends and centre Figure 6

3.5 Discussion

The responsivities measured for the various hydrophone designs are summarised in Table 1. It can be seen that the hydrostatic design produces an increase in responsivity of around 20 dB over the bare fibre.

Table 1 - Gauge Responsivities of Various Configurations

Configuration	Responsivity dB re 1 rad/Pa/m.i.l.
Bare fibre	- 90
 Hydrostatic hydrophone 	- 6 7
* Zero internal pressure	- 5 3
 Zero internal pressure, zero end pressure 	- 5 0

* With 3 cm central cavity

Reduction of the internal pressure to zero produces an improvement of 14 dB over the hydrostatic case, for a 3cm cavity. However, this is very dependent on the size of the cavity, and for larger cavities, improvements of 20 dB have been measured. Finally, removal of both the internal and the end pressures produces a further improvement of typically 3 dB.

In the case of the pressure amplified ends, the hydrophone can actually be made to act in an expansive mode, if the pressure amplification can be made high enough, but the sensitivity is critically dependent on the dimensions of the hydrophone. Little practical investigation has been carried out of this case.

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4. CONCLUSIONS

An optical fibre hydrophone research and development programme has been described. The purpose of this programme was to design and build practical fibre hydrophones with a high phase responsivity. A mathematical model was developed to predict hydrophone performance and implemented as a computer programme. At the same time, a number of hydrophones were built to several different designs and tested. It was found that the predictions of the model were in close agreement with the experimental results. Several designs providing very high responsivity were identified.

5. REFERENCES

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- 2. McMahon and Ciels "Fibre Optic Hydrophone Sensitivity for Different Sensor Configurations", Applied Optics, Vol 18, No. 22.

6. ACKNOWLEDGEMENTS

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