SOME PRACTICAL APPLICATIONS OF NONLINEAR ACOUSTICS

AN EXPERIMENTAL DETERMINATION OF THE WAVE DRAG COEFFICIENT ON SPHERES

Introduction

The measurements of the average force exerted on small spheres in a plane wave acoustic field are described in this chapter. Due to basic limitations imposed by the available instruments, the measurements were restricted to spheres having a radius of about 100 microns (10^{-2}cm). Most of the equipment associated with the orifice experiments was used in these measurements.

Experimental Technique

All measurements of force were obtained by observing the deflection of a pendulum which was suspended in the acoustic field. This pendulum was fashioned out of spheres and cylinders. Fig. 15 illustrates how a spherical particle was suspended so as to hang on the axis of the horizontal three inches in diameter. A loudspeaker is fixed to one end of the tube about 1 meter from the sphere. The opposite end of the tube is determined with a 1-meter long ρc fiberglass wedge whose tip was 1 meter from the sphere.

The whole tube assembly is tightly sealed with modelling clay. Sound pressure measurements were obtained using one section of a Brush rochelle salt cell placed in the tube about two feet from the sphere. Before each measurement, this microphone was calibrated in situ at each frequency for which measurements were to be obtained. The calibration was a comparative procedure utilizing a Western Electric 640-AA having its diaphragm flush with the inside walls of the tube.

The deflection of the pendulum was observed through a plane glass port in one section of the tube. The magnitude of the deflection was ascertained by viewing the foot of teh pendulum through a low powered microscope equipped with an eyepiece which included a scale.

The basic equipment is illustrated in the Block Diagram of Fig. 15. Two loudspeakers, each with their separate amplifiers, were sometimes used instead of one; this was usually done in measurements involving two harmonics.

In most of the experiment the relative phase of the second harmonic was adjusted so that $\phi = \pm \pi/2$; this was done by an electronic phase shifting circuit. In all experiments the inertia of the spheres was sufficient to prevent the sphere from following the oscillatory motion of the medium.

In computing from the deflection of the pendulum the force exerted on the sphere from the deflection of the pendulum, it was necessary to correct for the weight of the supporting fiber as well as for the force exerted on the fiber by the acoustic wave.

For the measurements in a standing wave, the pendulum was maintained at a fixed distance from the hard termination which replaced the ρc cone. It was found necessary to adjust the frequencies slightly of the tube resonance, which reduced the steady circulations in the tube to a point where they did not cause disturbing forces on the pendulum.

The Variation of $-\log|d_3|$ with f and ϕ

Detailed measurements were undertaken to determine the dependence of the partial drag coefficient, d_3, on the fractional harmonic content f and on the phase ϕ. In Fig. 16 the experiments with f as variable are compared with the

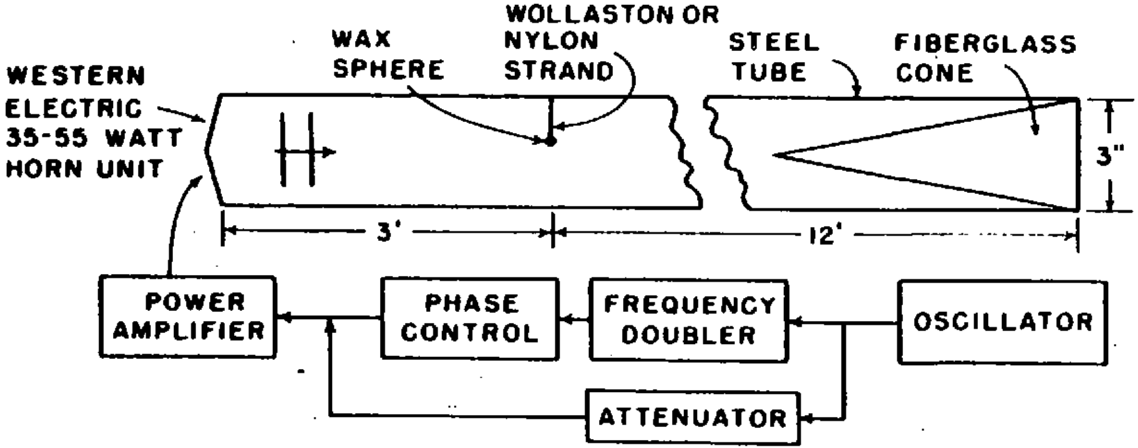

WESTERN ELECTRIC 35-55 WATT HORN UNIT
WAX SPHERE
WOLLASTON OR NYLON STRAND
STEEL TUBE
FIBERGLASS CONE
3"
3'
12'
POWER AMPLIFIER
PHASE CONTROL
FREQUENCY DOUBLER
OSCILLATOR
ATTENUATOR

FIG. 15

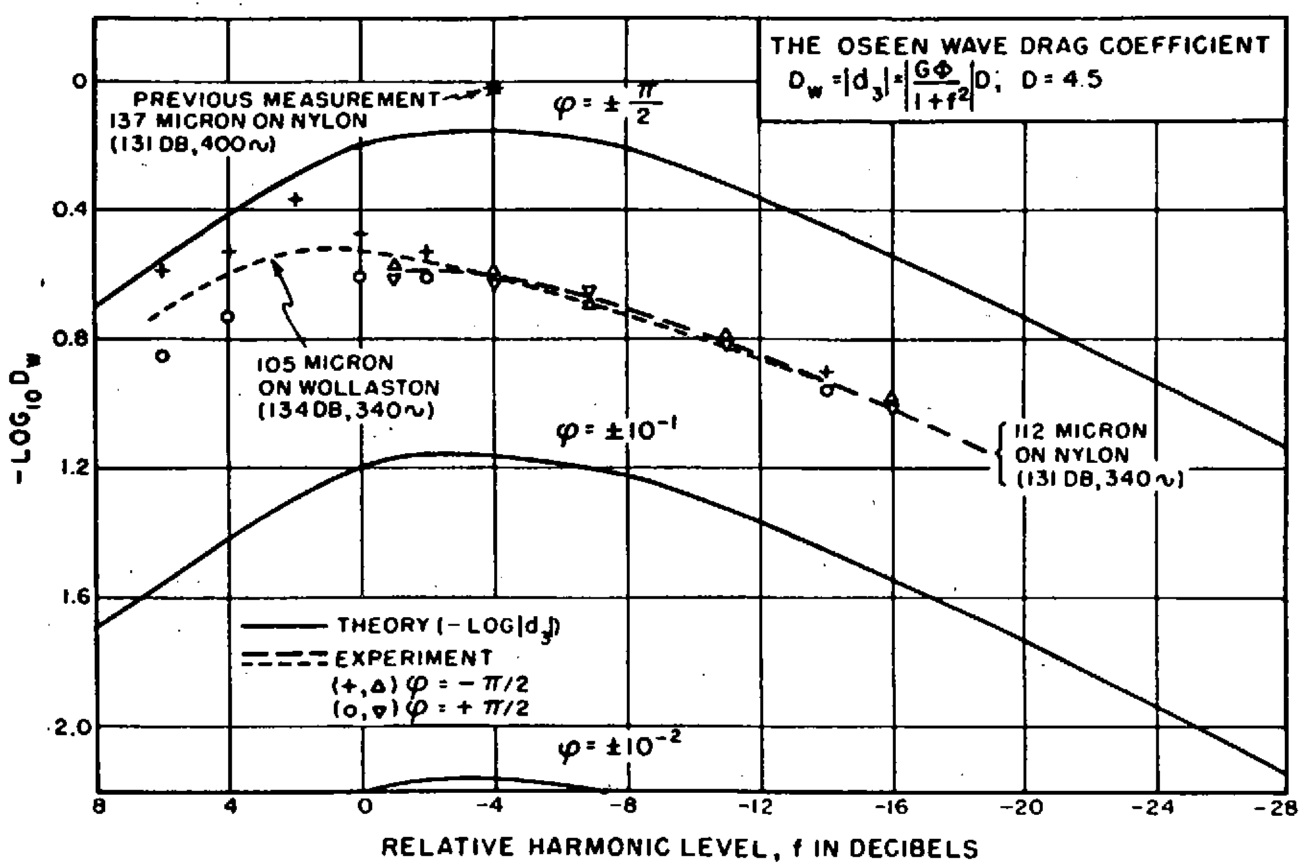

THE OSEEN WAVE DRAG COEFFICIENT
$D_w = |d_3| = \left|\frac{G\Phi}{1+f^2}\right| D$; D = 4.5
PREVIOUS MEASUREMENT
137 MICRON ON NYLON
(131 DB, 400 ∼)
φ = ± π/2
105 MICRON
ON WOLLASTON
(134 DB, 340 ∼)
φ = ± 10⁻¹
112 MICRON
ON NYLON
(131 DB, 340 ∼)
THEORY (− LOG|d3|)
EXPERIMENT
(+, Δ) φ = − π/2
(o, ∇) φ = + π/2
φ = ± 10⁻²
−LOG10 Dw
RELATIVE HARMONIC LEVEL, f IN DECIBELS

FIG. 16

theory developed in the preceding chapter. The general shape of the experimental curve agrees with theory in having a broad maximum. The experimental maximum however comes at too large a value of f. Furthermore for f values less than -4db, the experimental values appear to be less than the theoretical curve by a factor of about three.

d_3 has been found to vary both with the frequency and with the sound pressure level. In view of this fact the results can be considered to constitute fairly good agreement with the simplified theory which does not itself explain why d_3 should vary with either the frequency or the intensity of the wave.

In Fig. 17 some experimental data are given holding f constant and varying ϕ. Theoretical curves are plotted for varying percentages of the second harmonic distortion. The results of two experiments have been plotted. Data plotted as circles and plus marks were obtained with f = -6db while the squares and crosses refer to data taken with f = -13db. The plus and cross marks refer to data obtained with ϕ positive while the circles and squares signify negative phase angles. All the data plotted in Fig. 17 were obtained using a 340 cps fundamental sound pressure level of 134 decibels. As with the previous results these experimental points again show that the measured force is less than that predicted by theory. However, the shape of the experimental curve agrees reasonably well with the theoretical curves.

At this point it is well worth remembering that the scale in Figs. 16 and 17 is considerably expanded compared to the scale in Fig. 14. The complete extent of the abscissa of Figs. 16 and 17 corresponds to about two divisions in Fig. 14. We see from this fact that the magnitude of the Oseen-type force will not depend critically on the phase angle ϕ. It is also possible for the Oseen force to be relatively strong even for waves having as low as 1 percent second harmonic distortion.

The Wave Drag Coefficient Versus Sound Pressure Level and Frequency

The measurements of the wave drag coefficient discussed in this, and subsequent sections, were taken as a function of the sound pressure level of the fundamental wave component. Three discrete fundamental frequencies were used; the frequencies being generally in the neighborhood of 200, 340, and 600 cps. The measurements involving second harmonic distortion were performed with a constant f of -3 decibels, that is the second harmonic distortion was 71 percent. The data presented were evaluated by averaging the wave coefficient measured at $\phi = +\pi/a$ with that measured at $\phi = -\pi/2$.

The experiments illustrating the interaction of steady flow with sound, were made by holding the steady flow velocity constant; this means that the flow fraction, f, is a function of the sound pressure level. The theoretical drag coefficient, therefore will be in this case a function of the sound pressure level.

Special attention should be paid to the way in which data obtained in standing waves has been presented. d_3 is here evaluated for standing waves by dividing the force, not by the energy density but by the local kinetic energy density multiplied by two. Furthermore by sound pressure level we mean the sound pressure level of an equivalent progressive wave which has the same average kinetic energy density as the standing wave at the position of the object. These conventions permit us to represent on the same figure the measurements obtained in stationary as well as progressive waves.

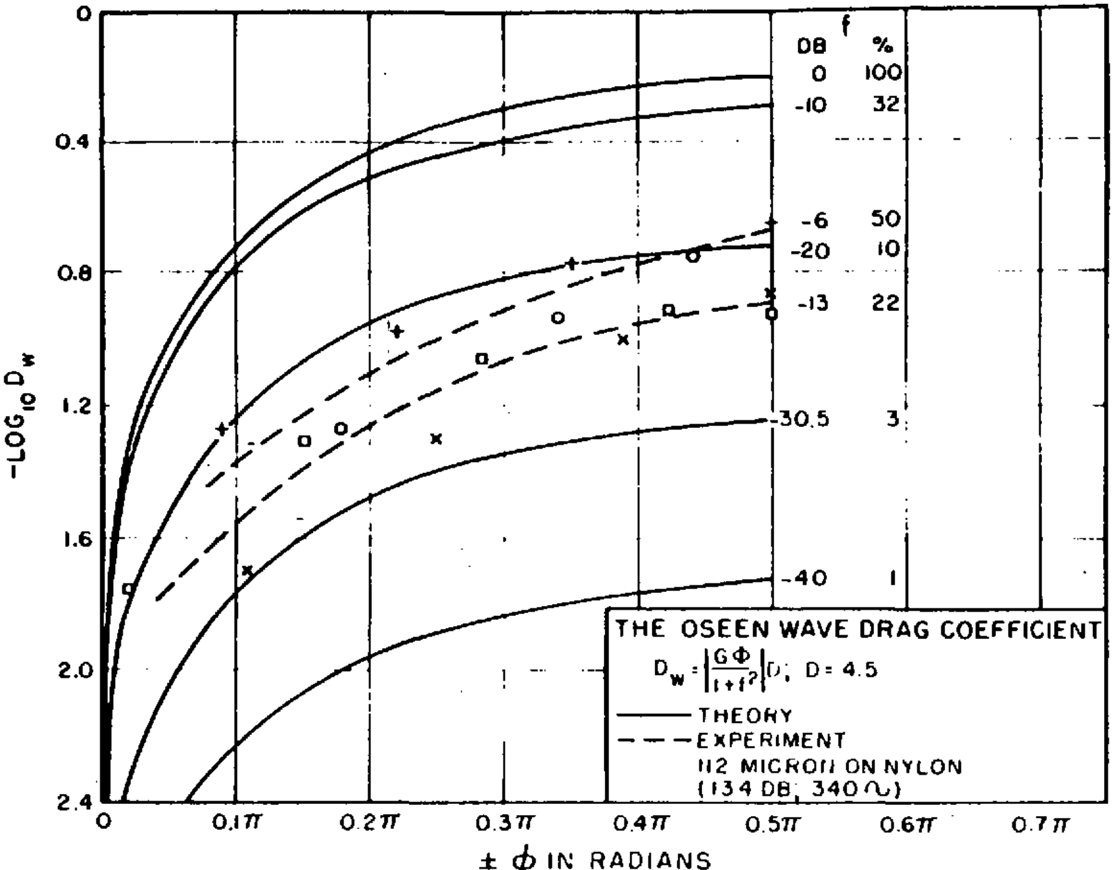

f
DB %
0 100
-10 32
-6 50
-20 10
-13 22
-30.5 3
-40 1
THE OSEEN WAVE DRAG COEFFICIENT
$D_W = \left|\frac{G\Phi}{1+f^2}\right| D$; D = 4.5
THEORY
EXPERIMENT
112 MICRON ON NYLON
(134 DB, 340 ∿)
$-LOG_{10} D_W$
0
0.4
0.8
1.2
1.6
2.0
2.4
0
0.1π
0.2π
0.3π
0.4π
0.5π
0.6π
0.7π
± φ IN RADIANS

FIG. 17

In Fig. 18 results are given for measurements taken in a wave containing a fundamental and a second harmonic component. The data pertain to a sphere 105 microns in radius supported on a 1 micron diameter platinum Wallaston wire. The results have not been corrected for the fiber force. Such a correction would add about 0.1 to the experimental curves.

A characteristic of these results is that $-\log D_W$ appears to be much greater than theory predicts (force too small) unless the medium displacement amplitude exceeds about 3 sphere diameters; in which case, within the limits imposed by the dynamic range of the experiment, $-\log D$ appears to level off to a value which depends on frequency. This constant value increases with increasing frequency. A linear extrapolation of $-\log D$ to zero frequency gives .45, a value about .3 greater than the theory (drag too small by a factor of 1/2). This extrapolation was made for a sound pressure level of about 140 db.

The differential force due to the superposition of a sound wave on a steady flow field has been measured, these results appear in Fig. 19 in terms of the negative logarithm of D_W (which might in this instance be called a differential drag coefficient indicative of the fact that it represents the difference in the force acting with and without sound). Theoretical curves are plotted for uniform and parabolic d-c flow distributions. Notice that the experimental values bear a relation to the theoretical curves which is very similar to that found in Fig. 18. A correction for the fiber force cannot be applied to the data of Fig. 19 unless it is assumed that the same correction is applicable which pertains to the data of Fig. 18. No direct measurement of the differential drag coefficient for the supporting fiber was made.

SIGNIFICANCE OF THE RESULTS

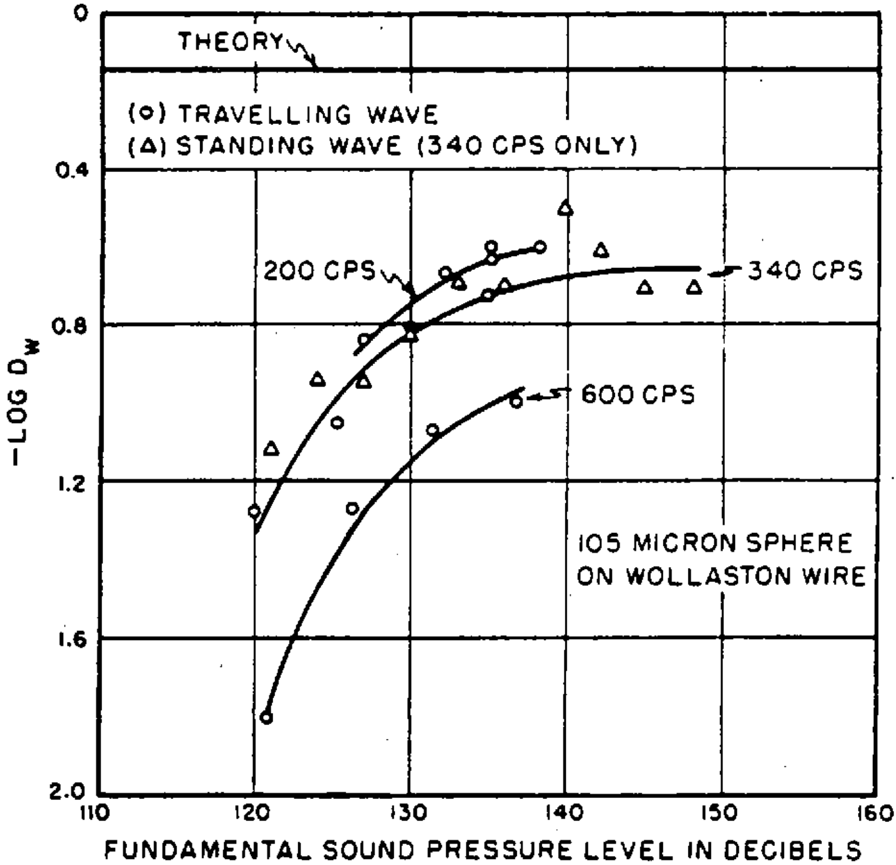
The Virtual Mass of an Oscillating Sphere

In this and the following sections we shall draw some conclusions about the oscillating sphere by comparing the non-linear behavior of the sphere with that of the orifice.

In Fig. 20 we have plotted on the same curve quantities proportional to the fractional reduction in the orifice as well as a quantity proportional to the drag coefficient for the sphere. The data are given for experiments carried out both with and without d-c flow velocity. The ordinate is a dimensionless particle displacement parameter obtained by dividing the effective displacement amplitude by the effective length of the object. In the absence of d-c flow the effective displacement amplitude is the amplitude of the fundamental component of the particle displacement. (This definition would need to be modified whenever the second harmonic exceeded the fundamental). When there is steady flow, the effective displacement amplitude is defined either as above, or as the ratio of the average distance travelled by the medium in one period of the wave, depending on which of the two definitions leads to the larger value. The effective length of the orifice is the sum of the diameter and the thickness of the orifice. The effective length of the sphere is its diameter.

The quantities plotted in Fig. 20 are:

$\frac{\delta_{NL}}{d}$, the ratio of the reduction in the orifice mass $_{NL}$ expressed as an effective end correction, divided by the diameter of the orifice;



$-\text{LOG}\,|d_3|$ FOR f = −3 DB

FIG. 18

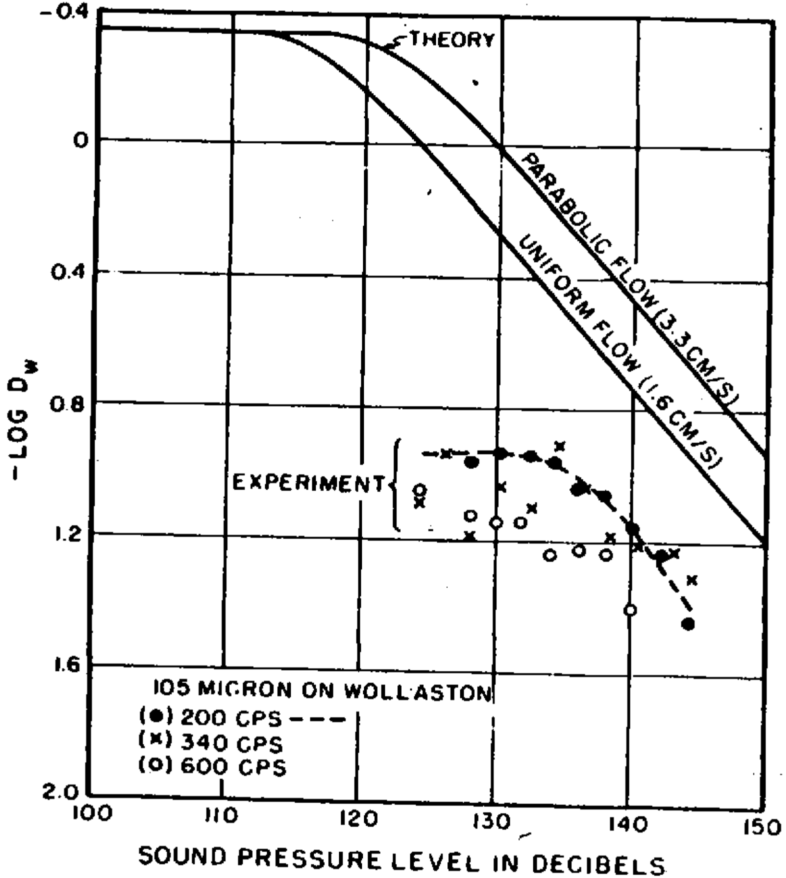


FIG. 19

WAVE DRAG COEFFICIENT FOR A SIMPLE HARMONIC WAVE WITH A SUPERPOSED STEADY FLOW VELOCITY

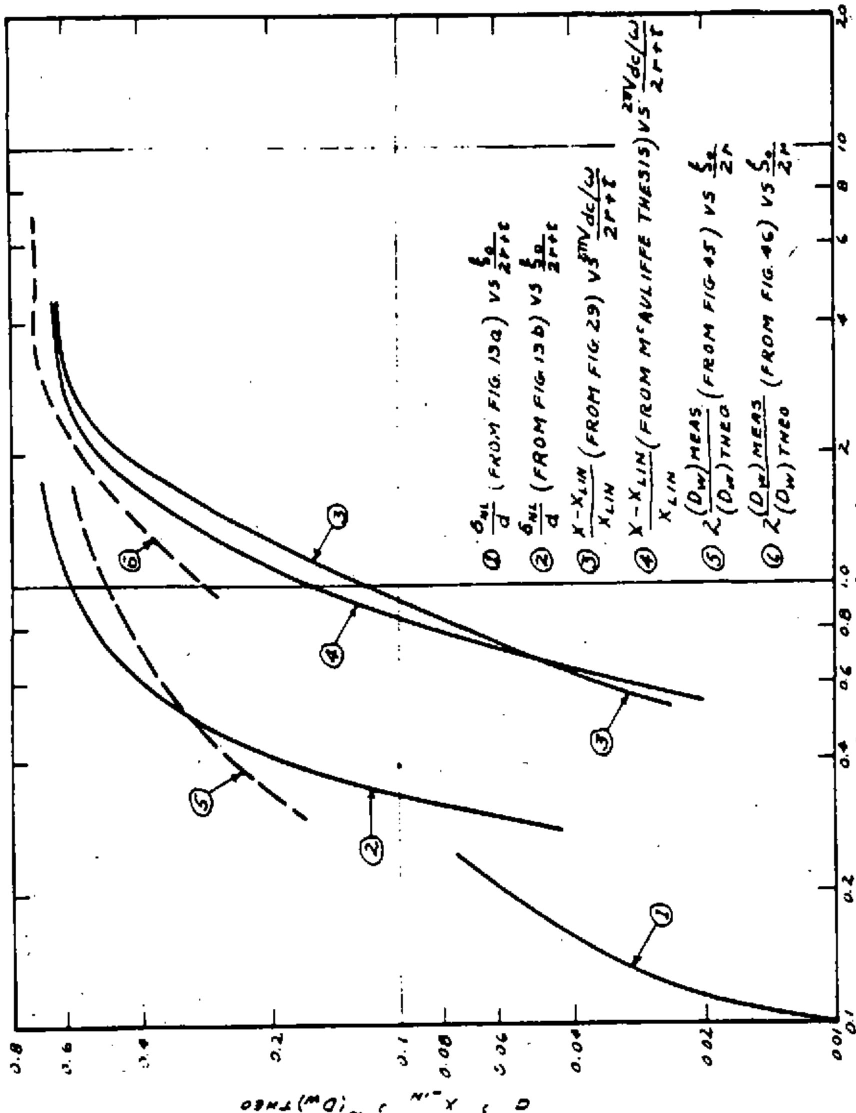


FIG. 20 — A COMPARISON OF THE INCREMENTAL REACTANCE FOR ORIFICES AND THE DRAG COEFFICIENT RATIO FOR SPHERES

$\frac{X - X_L}{X_L}$, the ratio of the reduction in the orifice reactance, from its linear value X_2, divided by X_L;

$\frac{(2\ D_W)_{meas.}}{(D_W)_{theo.}}$, the ratio of twice the measured wave drag coefficient for the sphere divided by the theoretical value for this quantity obtained in Chapter V.

For the rough comparison we wish to make, it is justifiable to consider

$\frac{\delta_{NL}}{d}$ and $\frac{X - X_L}{X_L}$ as being a measure of the same quantity.

From the results plotted in Fig. 20 we see that the fractional reduction of the orifice reactance and quantity $2\ \frac{(D_W)_{meas.}}{(D_W)_{theo.}}$ for the sphere, vary in a similar way with the particle displacement parameter. Both these quantities tend to approach a constant value which is reached approximately when the effective particle displacement is the same order of magnitude as the effective dimension of the object.

Since the reactance of the orifice is proportional to its kinetic mass, we see as mentioned earlier that the kinetic mass of the orifice is considerably reduced at large amplitudes.

We may now ask whether the virtual mass of the sphere is reduced at large amplitudes by the same mechanism which is responsible for reducing the kinetic mass of the orifice. There are two considerations which make this supposition plausible. We first examine the general hydrodynamic equation:

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u - \nu\nabla^2 u - \frac{1}{3}\nu\nabla(\nabla \cdot u) = \frac{-\nabla p}{\rho} + \vec{F}/\rho \qquad (32)$$

where ν is the kinematic viscosity assumed to be constant, ρ is the density, and u the velocity. We assume all body forces, $\vec{F}$ are zero and we assume that the fluid is incompressible; hence

$$\nabla \cdot u \equiv 0$$

and Eq. (32) becomes

$$\frac{\partial u}{\partial t} + (u \cdot \nabla)u + \frac{\nabla p}{\rho} - \nu\nabla^2 u = 0 \quad . \qquad (33)$$

If the first two terms of Eq. (33) are omitted the remaining equation leads to Stokes' solution when applied to a sphere. The omission of the first term implies the steady state; while the omission of the second term linearizes the equation.

Oseen's extension of Stokes' solution provides a rough approximation of the time independent solution of Eq. (33). Oseen's solution includes for a sphere the influence of the second order term. No solution in the literature has been found for the oscillatory case which includes the effects of the second order term.

The acceleration forces are represented by the term $\frac{\partial u}{\partial t}$. This term is also responsible for the radiation reactance of an oscillating sphere.

If it is true that at large amplitudes of oscillation the radiation reactance of the sphere is diminished (just as for the orifice), one would conclude that the influence of the local acceleration term, $\frac{\partial u}{\partial t}$ had diminished correspondingly. Thus a treatment which neglected altogether the local acceleration forces, might be expected to agree approximately with experiments performed at large amplitudes, even though such a treatment represented poorly the results of experiments performed at low amplitudes. We have seen that our experimental results bear out this point of view.

It is possible to apply the coherence criterion previously used for the orifice to the virtual mass of the sphere. The kinetic mass of a sphere which is small compared with the wavelength, is 1/2 mass of the fluid displaced by the sphere. This mass is for the most part concentrated in the vicinity of the sphere. When the particle displacement amplitude approaches the diameter of the sphere, it is reasonable to assume that the coherence of this mass will be diminished. For flows having a low peak Reynold's number the reduction in the mass results from the irreversible nature of Oseen's solution. The wake formed periodically on either side of the sphere plays a role similar to the jet in the case of the orifice. The kinetic mass in the region of the wake will be diminished. Just as with the jet, the coherent reaction of the fluid which is undisturbed by the wake can be considered to take place across the free boundary separating the wake from the rest of the fluid; for this reason the radiation reactance of the sphere should not vanish completely as the particle displacement amplitude increases at least not until compressibility effects become important.

The Non-Linear and Differential Absorption Cross-Section of a Small Sphere

In addition to the usual viscous and thermal boundary losses, other losses at the sphere will occur either at large acoustic amplitudes (non-linear absorption) or at small amplitudes provided the sphere drifts through the medium (differential absorption).

By methods analogous to those employed to ascertain the non-linear losses in orifices, we find that the non-linear absorption cross-section for a sphere, based on the Oseen force is

$$\delta_{NL} = 9a^2 u_o / c \qquad (34)$$

In the above formula a is the radius of the sphere, u_o the particle velocity amplitude of the wave and c the velocity of sound.

The differential absorption cross-section for steady flow in the same direction as the wave vector is

$$\delta_D = 14.1\ r^2\ u_{dc}/c \tag{35}$$

provided the drift velocity exceeds the alternating velocity amplitude. The differential cross-section is proportional to the ratio of the drift velocity u_{dc} to the sonic velocity.

Now it must be emphasized that Eqs. (3) and (4) are based on Oseen's theory and we have seen that the application of this theory to periodic flows is approximately valid only for low frequencies and for large particle displacements. In the absence of direct measurements a better approximation to δ_{NL} and δ_D might be based on experimentally determined value of the wave drag coefficient. One might modify Eqs. (3) and (4) by multiplying them by the ratio of the experimental value of the wave drag coefficient to its theoretical value.

ACKNOWLEDGEMENTS

The final half of this lecture dealing with the parametric array and with the absorption of sound by sound will appear separately. The work reported in this paper was done at the M.I.T. Acoustics Laboratory in 1950-51 and comprised part of the author's Ph.D. dissertation under the guidance of Professor Richard H. Bolt who initiated the original investigations of the nonlinear properties of orifices at M.I.T. In addition to Professor Bolt's vital support and interest, much valuable assistance in the experimental work was provided by Mr. Peter W. Sieck and Mr. Keith Hoyt.

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