Proceedings of The Institute of Acoustics

DIFFRACTION OF A SPHERICAL HARMONIC WAVE BY A POROUS LAYER: A NEW THEORETICAL APPROACH AND EXPERIMENTAL RESULTS

P.J.T. Filippi and D. Habault, C.N.R.S., L.M.A., Marseilles

J.S. Bolton, I.S.V.R., University of Southampton

1. Introduction

We consider here the diffraction of a spherical harmonic wave by a homogeneous isotropic porous layer, bounded on one side by a perfectly rigid plane, and on the other by a second homogeneous isotropic medium occupying the half-space. An exact solution of this problem has been derived in reference [1] for either conservative or dissipative media, and is expressed in such a way as to avoid the branch integrals which normally appear. It is composed of contributions from an image source, and layer potentials whose strength is expressed in terms of mode series. This form of solution facilitates analytical approximations and numerical calculations.

A formal statement of the problem is given in section 2, while the particular case of source and receiver on the surface is dealt with in section 3. Finally in section 4 experimental results are presented and compared with the theory.

2. Statement of the Problem

Let Ω_1 be the half-space in $R^3 (z > h > 0)$ containing a fluid of density ρ_1 and speed of sound c_1. The space Ω_2 is a constant thickness porous layer $(0 < z < h)$ characterised by a complex density ρ_2 and speed of sound c_2. The plane at $z = 0$ is assumed perfectly rigid. A point source generating a simple harmonic source $(e^{-i\omega_0 t})$ is located in Ω_1 at $s(0,0, s > h)$. The sound pressures p_1 and p_2 in Ω_1 and Ω_2 respectively satisfy the usual Helmholtz equations and continuity conditions [1].

3. The Particular Case of the Source and Receiver on the Plane z = h

The derivation of both the exact general solution and the solution for source and receiver on the surface of the porous layer may be found in reference [1]. The latter solution may be approximated for large source-receiver separations (large enough to ensure the decay of any surface wave present), but since the development is somewhat lengthy it will be presented elsewhere [2]. The final result is

$$p_1(s,x) = 2i \frac{k_1}{k_2^2 - k_1^2} \frac{\rho_2^2}{\rho_1^2} \frac{\cos^2(\sqrt{k_2^2 - k_1^2}\, h)}{\sin^2(\sqrt{k_2^2 - k_1^2}\, h)} \frac{e^{ik_1\rho(s,x)}}{4\pi \rho^2(s,x)} .$$

In the far field, note that p_2 decays as ρ^{-2}. This expression clearly shows the effect of layer thickness and can be compared to the case of reflection of a wave by a semi-infinite homogeneous isotropic porous medium [3]. Indeed, when $h \to \infty$

$$\frac{\cos^2(\ )}{\sin^2(\ )} \rightarrow -1$$

and

$$p_1(s,x) \simeq -2i\frac{k_1}{k_2{}^2 - k_1{}^2}\frac{\rho_2{}^2}{\rho_1{}^2}\frac{e^{ik_1\rho(s,x)}}{4\pi\rho^2(s,x)}$$

which is identical with equation (29) of reference [3].

4. Experimental Results

Figure 1 shows the results of recent grazing incidence propagation tests conducted in the large anechoic chamber of the L.M.A., C.N.R.S., Marseilles. Attenuation versus distance was measured using first one, then two layers of 3 mm carpet for a source flush with the surface and a receiver approximately 0.5 cm above it. It may be seen that by 1.5 m the far field (defined here as the region displaying a $1/r^2$ pressure dependence) has been reached in both instances and that in this region there is a constant difference of 10 dB between the one and two layer cases. This is consistent with the results of the last section which demonstrate that, all else remaining constant, the far field pressure should increase for decreasing h.

In order to make a quantitative comparison of theory and experiment, the propagation constant for the carpet material was inferred from measurements of normal specific impedance. (Using the technique of Ferrero and Sacerdote [4] the propagation constant may be found graphically if the ratio of the normal impedances of one and two layers is known.) At 8000 Hz, k_2 was estimated to be $k_2 = 237 + i\ 96.7\ [m^{-1}]$ while $k_1 = 148\ [m^{-1}]$. Knowing this, the far field level difference may be calculated as

$$20 \log \left| \frac{\cot^2\left(\sqrt{k_2^2 - k_1^2}\ h\right)}{\cot^2\left(\sqrt{k_2^2 - k_1^2}\ 2h\right)} \right|$$

where h = 3 mm. The calculated level difference is 11.5 dB which is in reasonable agreement with the experimental results.

6. Conclusions

A new theory of reflection from a homogeneous porous layer has been developed which has the advantage of avoiding branch integrals. As an illustration of its usefulness, it has been approximated for the special case of source and receiver on the surface, in which case a simple relation relating layer thickness to farfield pressure results. This theory is different from previous solutions for the near-grazing case [5] as the index of refraction c_1/c_2 is not required to be large. Thus near-grazing propagation above homogeneous layers may be accurately predicted if the propagation constant of the material is known, a quantity which may be determined from measurements of the normal specific impedance of the layer. Finally, the predictions of the theory have been shown to be in reasonable agreement with propagation experiments conducted over a thin carpet.

References

1. D. HABAULT and P.J.T. FILIPPI 1978 Journal of Sound and Vibration 56, 87-95. On the resolvent of the Pekeris operator with a Neumann condition.

2. P.J.T. FILIPPI and D. HABAULT 1979 Journal of Sound and Vibration (to appear).

3. P.J.T. FILIPPI and D. HABAULT 1978 Journal of Sound and Vibration 56, 97-103. Reflexion of a spherical wave by the plane interface between a perfect fluid and a porous medium.

4. M.A. FERRERO and G.G. SACERDOTE 1951 Acustica 1, 137-142. Parameters of sound propagation in granular absorbent material.

5. S.-I. THOMASSON 1977 Journal of the Acoustical Society of America, 61, 659-674. Sound propagation above a layer with a large refraction index.

Figure 1: Attenuation vs. distance at 8000 Hz over 1 and 2 layers of 3mm carpet. (Source height = 0.0, Receiver height = 0.5 cm.)