inter-noise 83

WAVE PROPAGATION ABOVE LAYERED MEDIA

Paul J.T. Filippi

Laboratoire de Mécanique et d'Acoustique (CNRS), B.P. 71,
13277 Marseille cedex 9, France.

INTRODUCTION

It has been shown that the sound field reflected by the plane boundary of a layered ground can always be described by a specularly reflected wave and layer potentials. Despite of its generality, this representation is not quite suitable for engineering purposes. Dealing with a very simple case, Weyl /1/, and later on Ingard /2/ and Thomasson /3/, proposed a representation of the solution in which the layer potential terms are replaced by the sum of a surface wave and a Laplace type integral.

In this paper, it is shown that this kind of representation can be obtained for a very wide class of stratified grounds : locally reacting surface (Thomasson's solution) ; finite thickness layer of porous medium ; porous medium with depth varying porosity ; thin elastic plate; etc. Furthermore, the Laplace integral representation is quite convenient for the numerical identification of the acoustic parameters of a ground model from wave propagation measurements ; this method is illustrated by outdoor experiments.

LAYER POTENTIAL AND LAPLACE'S TYPE INTEGRAL REPRESENTATION

Let us consider the following simple layer potential :

$$(1)\quad \begin{cases} \varphi(M) = i/16\pi \int_{z=0} H_0^1(\alpha\rho)\, e^{ikr}\, r^{-1}\, dx'\, dy' \\ \rho^2 = x'^2 + y'^2 ,\quad r^2 = (x-x')^2 + (y-y')^2 + z^2 ,\quad \operatorname{Im}\alpha > 0 \end{cases}$$

Assume that (R, Θ, Π/4) are the spherical co-ordinates of M. It can easily be shown /4/ that φ is given by the Fourier integral :

$$(2)\quad \begin{cases} \varphi(M) = (2\pi)^{-3} \int_0^{2\pi} dv_1 \int_0^{\infty} u_1\, du_1 \int_{-\infty}^{+\infty} \tilde{\varphi}(u_1, v_1, \zeta_1)\, e^{i\zeta_1 R}\, d\zeta_1 \\ \tilde{\varphi} = \left[-(u_1^2+\zeta_1^2)+k^2\right]^{-1} \left[-u_1^2 - \zeta_1^2 + (u_1 \cos(v_1 - \pi/4)\sin\theta + \zeta_1 \cos\theta)^2 + \alpha^2\right]^{-1} \end{cases}$$

The integration with respect to ζ_1 is first performed, using the residues theorem. Then, an integration with respect to v_1 is made, and we are left with an expression of the form :

(3) $$\varphi(M) = (2\pi)^{-1}\int_0^\infty f(u_1)\left[e^{ig_1R}/2ig_1\right] u_1\, du_1 \qquad \text{+surface wave terms}$$

$g_1^2 = k^2 - u_1^2$, $\operatorname{Im} g_1 > 0$

Finally, use is made of the steepest descent contour, leading to :

(4) $$\varphi(M) = e^{ikR}\int_0^\infty h(\theta,\alpha,t)\, e^{-kRt}\, dt \qquad \text{+surface wave terms}$$

The expressions of h and of the surface wave terms are given in details in ref./4/. Such an expression is very well-adapted to numerical computation. Indeed, the Hankel functions can be computed by Padé approximants, as given in /5/. If only a rough accuracy is needed, the Laplace's type integrals can be computed with a Lobotto's formula using Laguerre polynomials. If a given accuracy is needed, a Gauss method, with automatically adjusted intervals, is one of the most efficient techniques.

DIFFRACTION OF A SPHERICAL WAVE BY AN INFINITE LAYER OF POROUS MEDIUM, WITH EXPONENTIALLY VARYING POROSITY

Various theoretical models for grass-covered grounds have been compared by R.J. Donato /6/. It is shown that the best fit with experimental data is obtained for an infinite layer of porous medium, the porosity of which decreases exponentially with depth. For a harmonic wave ($e^{-i\omega t}$) the sound velocity in the porous medium is definied by :

(5) $$c_2^2 = e^{-aZ}\,\Omega^2/\omega^2$$

Let ρ_1 be the density of air, ρ_e the equivalent density of the porous medium, and α_j^2 the zeros of the function :

(6) $$D = (k^2 - \xi^2)\,\rho_e^2\, J^2_{2\xi/a}\left[\frac{2\Omega}{a}\right] + \Omega^2 \rho_1^2\, J'^2_{2\xi/a}\left[\frac{2\Omega}{a}\right]$$

It can be shown /4/ that the sound pressure in air is gigen as the sum of : a/ a direct wave ; b/ a specularly reflected wave ; c/ a series of simple layer potentials with density $-iH_0(\alpha_j\rho)/4$; d/ a series of z-derivatives of the same simple layer potentials. Using the results described in section II, the two series can be replaced by a series of surface waves and of Laplace's type integrals.

ACOUSTIC GROUND PARAMETERS IDENTIFICATION

Let $L(M_i)$ be the sound pressure levels at n points M_i lying on the ground, due to a point source located at the ground level too. Let ζ_j be the q(<n) parameters defining an "a priori" given ground model. One of the best fits is obtained for the minimum of :
$Q = \Sigma[L(M_i) - L_{th}(M_i\ \zeta_j)]$, where $L_{th}(M_i\ ;\ \zeta_j)$ is the theoretical

sound pressure level.
The corresponding values of the ζ_1 are computed by a Marquardt's algorithm, which requires to compute a lot of values of the theoretical solution. Thanks to the Laplace's type representation of the solution, the numerical identification of the ζ_1 can be performed on a small computer, and is rather fast. Furthermore, the value of the minimum of Q gives an idea of the model validity.
As an example, the local reaction model has been used for a grassy ground. The theoretical sound pressure deduced from (4) is that already given by S.I. Thomasson. The value of the normal specific impedance has been deduced from the experimental data plotted on figure 1. Then, the theoretical excess attenuation has been computed for a point source 22 cm above the ground level, and a microphone moving along a horizontal line, 22 cm above the ground: fig. 2 shows that the previously determined normal specific impedance enables an excellent prediction of the experimental results.

CONCLUSION

The solution of many other problems of wave propagation above or within layered media can be represented in terms of surface waves and Laplace's type integrals. The main interest of such representations is that their numerical evaluation is very fast. It is so quite reasonable to look at more complicated problems. For example, the influence of a barrier above an absorbing ground can be accounted for by a layer potential, using the ground Green's function as kernel. This will lead to an integral equation on the boundary of the diffracting obstacle, only. Solving it numerically will not require a very important computation time.

REFERENCES

1 H. Weyl,"Ausbreitung elektromagnetisher Weller über einen ebenen Leiter", An. der Phys., Leipzig, 60, 481-500, (1919)

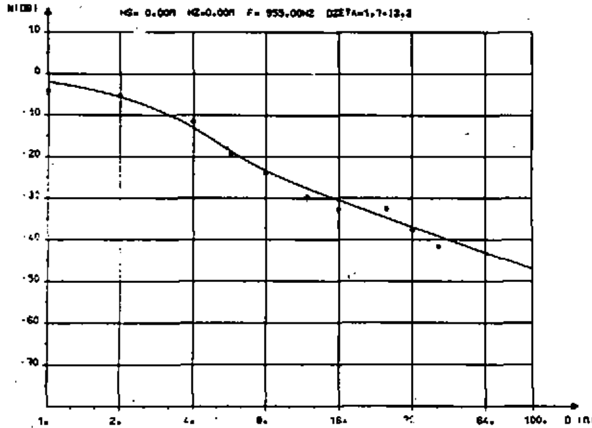
2 U. Ingard, "On the reflection of a spherical sound wave along a boundary", J.A.S.A., 23, (3), 329-335, (1951).

3 S.I. Thomasson, "Reflection of waves from a point source by an impedance boundary", J.A.S.A., 59, 780-785, (1976).

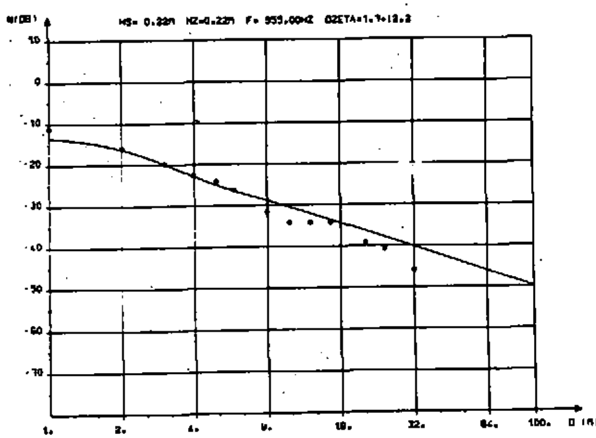
4 P.J.T. Filippi, "Extended sources radiation and Laplace's type integral representation : application to wave propagation above and within layered media", J.S.V., 91, (1), to appear.

5 Y.L. Luke, "Mathematical functions and their approximations", Academic Press.

6 R.J. Donato, "Impedance models for grass-covered ground", J.A.S.A. 61, 1449-1452, (1977).



-Figure 1 -



-Figure 2 -