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A BOUNDARY INTEGRAL AND FINITE ELEMENT COUPLED FORMULATION FOR THE MODES IN A THREE-DIMENSIONAL CAVITY WITH A FLEXIBLE PANEL

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The primary aim of this paper is to communicate only the basic theory used in estimating natural frequencies of firstly a fluid-filled cavity, and secondly a combination with a flexible panel; comparing the latter with Pretlove's results |1|. It is also intended to just hint that the new numerical method of Boundary Elements, already applied in maritime research, acoustic scattering, geotechnics, has a tremendous future use (e.g., in sonar, and for other underwater structures |2|).

1. NOTATION

First representing the deflection $d(s)$ at a point s on the panel, by a scalar product of nodal components d_i and shape functions $n_i(s)$

$$d(s) = (\underline{n}(s))^T\underline{d} \qquad (1)$$

in the typical FEM mesh. At angular frequency ω, the structural behaviour, under loads $\underline{e}$, is described as

$$(K_s - \omega^2 M_s)\underline{d} = \underline{e} \qquad (2)$$

where K_s and M_s are structural stiffness and mass, respectively, and the nodal forces $\underline{e}$ arise (in this case) from the variation $p(s)$ in fluid loading from the cavity, which contains no acoustic sources; otherwise included as an extra vector $\underline{e}_x$ so that (strictly M_f for fluid)

$$\underline{e} = M_f\underline{p} + \underline{e}_x \qquad (3)$$

The fluid of ambient density ρ_o has a potential $\phi(s)$ giving the displacement normal to the panel $d_n(s)$ by differentiation, and the normal load $p_n(s)$

$$d_n(s) = \frac{\partial}{\partial n}\phi(s) \quad \text{and} \quad p_n(s) = \rho_o\omega^2\phi(s). \qquad (4)$$

2. OUTLINE OF THE METHOD

Henceforth $\tilde{\omega}$ will be one of the unknown natural frequencies of the cavity, and $\tilde{\phi}(y)$ the mode shape at y in the cavity V. With the aid of Helmholtz's integral formula $\tilde{\phi}(y)$ is easily expressed as

$$\tilde{\phi}(y) = \int_A \left(\frac{\partial\tilde{\phi}}{\partial n} G - \frac{\partial G}{\partial n}\tilde{\phi}\right)dA + \frac{\tilde{\omega}^2 - \omega^2}{c_o^2}\int_V \tilde{\phi}G\, dV \qquad (5)$$

using $G(s, y)$, which is a monopole acoustic source of angular frequency ω at y. The panel has area A.

In particular for $\tilde{\omega} = \omega$, once broken into elements, equation (5) determines the surface acoustic impedance matrix $Z_n(\omega)$, which is (symmetric by construction, ref. |2|) defined here so that

$$i\omega Z_n(\omega)\underline{d}_n = \rho_o\omega^2 M_f\underline{\phi} \qquad (6)$$

One can seek the eigenvalues λ^α of equation (6) such that

$$\lambda^\alpha\underline{d}^\alpha = M_f\underline{\phi}^\alpha \quad \text{and} \quad (\underline{d}^\alpha)^T\underline{d}^\beta = \delta^{\alpha\beta} \qquad (7)$$

the corresponding vectors $\underline{d}^\alpha$ (dropping the subscript n and using the Kronecker δ) are orthonormal.

For the special case G = N, where in equation (5) N now has the property $\partial N/\partial n = 0$, one may set

$$N(s, y) = \sum_\alpha ((\underline{n}(s)^T\underline{\phi}^\alpha.(\underline{n}(y))^T\underline{c}^\alpha)/\lambda^\alpha \qquad (8)$$

where $\underline{c}^\alpha$ are just Fourier coefficients; determined to be exactly $\underline{\phi}^\alpha$, using orthogonality of the $\underline{d}^\alpha$ and equation (5) again (with $\tilde{\phi} = \phi^\alpha$, $\tilde{\omega} = \omega$). In passing, note that for x, y in V the relationship with G = D, where D is zero on the panel is

$$N(y,x) - D(y,x) = \sum_\alpha (\phi^\alpha(y)\phi^\alpha(x))/\lambda^\alpha. \qquad (9)$$

Finally in terms of (5), this time for a rigid plate, the mode shape $\tilde{\phi}(s)$ is

$$\tilde{\phi}(s) = \frac{\tilde{\omega}^2 - \omega^2}{c_o^2}\int_V \tilde{\phi}N\, dV \qquad (10)$$

Now N has been found independently, but in practice $\phi^\alpha(y)$ are not known in V, only on A. If Green's theorem is applied to (10) as it stands, the argument is then circular; unless another angular frequency ω_2 is chosen. The same mode denoted now by $\tilde{\phi}_2(y)$ is then given in terms of a second function $N_2(s, y)$ at the new frequency

$$\tilde{\phi}_2(s) = \frac{\tilde{\omega}^2 - \omega_2^2}{c_o^2}\int_V \tilde{\phi}_2 N_2 dV \quad \text{and} \quad \omega < \tilde{\omega} < \omega_2 \qquad (11)$$

By interchange of $\tilde{\phi}$, $\tilde{\phi}_2$ inside equations (10) and (11), all that remains, in order to obtain a simultaneous solution to the equations, is an (approximate) expression for $\tilde{\phi}(y)$

$$\tilde{\phi}(y) = \sum_\alpha \phi^\alpha(y)a^\alpha/\lambda^\alpha, \quad \text{where} \quad \tilde{\phi}(s) = \sum_\alpha \phi^\alpha(s)a^\alpha/\lambda^\alpha \qquad (12)$$

the latter expansion over the panel area being complete; when one (or more) λ^α in equation (7) becomes infinite near $\tilde{\omega}$, say λ^0, the corresponding coeffi cient a^{α} nears unity.

The approximation $\tilde{\phi}_2(y)$ has a similar expression with coefficients b^{β}/μ^{β} (μ^{β} are eigenvalues at ω_2 and correspond to displacements $\underline{c}^{\beta}$, say). Substitution, in (10) and (11), after repeated use of (7), gives

$$a^{\alpha} = k \sum_{\beta} b^{\beta}(1 - \lambda^{\alpha}/\mu^{\beta})(\underline{c}^{\beta})^T \underline{d}^{\alpha}$$
$$b^{\beta} = k_2 \sum_{\alpha} a^{\alpha}(1 - \mu^{\beta}/\lambda^{\alpha})(\underline{d}^{\alpha})^T \underline{c}^{\beta} \qquad (13)$$

where $k = (\tilde{\omega}^2 - \omega^2)/(\omega_2{}^2 - \omega^2)$, and $k_2 = (\omega_2{}^2 - \tilde{\omega}^2)/(\omega_2{}^2 - \omega^2)$.

The solution of equations (13), by modification of previous iterative methods (especially |3|) is not discussed here; but in Table 1 a single frequency is presented, which is the result of k and k_2 converging from above and below. The important parameter is $\Delta^2 = \omega_2{}^2 - \omega^{2}$ (radian^2) representing the sampling interval.

3. THE COUPLED SYSTEM

The combination of plate and cavity gives

$$(K_s - i\omega Z(\omega) - \omega^2 M_s)\underline{d} = \underline{e}_x \qquad (14)$$

where in this case, comparing equations (14) and (6), the eigenvalues λ^{α}, which the method requires, must be defined by analogy with (7) as

$$\rho_o \omega^2 \lambda^{\alpha} \underline{d}^{\alpha} = \underline{e}_x^{\alpha} \qquad (15)$$

The resulting frequencies are presented in Table 2; although they do not agree with Pretlove, he states that his calculations using only four modes of vibration are incomplete.

TABLE 1. APPROXIMATE NATURAL FREQUENCIES OF RECTANGULAR CAVITY (A), DIMENSIONS 2.05" x 6" x 18" (c_o = 344 ms^{-1}), BY BOUNDARY ELEMENTS USING 9-NODE QUADRATIC ELEMENTS. Δ^2 = 0.6 x 10^6 (Hz^2). PLANES OF SYMMETRY Y = 0, Z = 0.

Modal-type	(0,0,1)	(0,0,2)	(0,0,3)	(0,1,0)	(0,1,1)	(0,1,2)
Exact solution (Hz)	376.2	752.4	1,128.6	1,128.6	1,189.7	1,356.4
BEM (Hz)	372.8	748.6	1,120.2	1,123.4	1,180.0	1,359.0
Modal-type	(0,0,4)	(0,1,3)	(0,1,4)	(0,0,5)	(0,1,5)	(0,0,6)
Exact solution (Hz)	1,504.8	1,596.1	1,881.0	1,881.0	2,193.6	2,257.2
BEM (Hz)	1,495.3	1,590.7	1,877.1	1,878.6	2,205.0	2,258.3

TABLE 2. PRETLOVE'S COUPLED SYSTEM WITH CAVITY (B), MODES (M,N) (SYMMETRIC IN Y = 0, Z = 0), DIMENSIONS 2.05" x 10.0709" x 18.127", PLATE THICKNESS 0.018" (Y-DIMENSION 6" PUBLISHED IS INCORRECT). 12-DEGREES OF FREEDOM PLATE ELEMENTS; MATCHED TO 9-NODE BOUNDARY ELEMENTS $\Delta^2 = 0.2 \times 10^5$ (Hz^2).

Approx. Plate Mode	(1, 3)	(1, 5)	-	-	(1, 1)
Pretlove's solution (Hz)	58.4	135.85	148.67	-	227.61
FEM/BEM (Hz)	59.2	131.48	155.78	209.06	239.52

Two typical modes are illustrated in Figures 1 and 2.

FIGURE 1. CAVITY (A), MODE NO. 2, FREQUENCY = 748.6 Hz.
FIGURE 2. COUPLED SYSTEM (B), MODE NO. 1, FREQUENCY = 59.2 Hz.

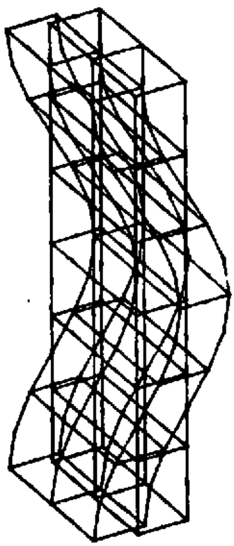

Fig. 1

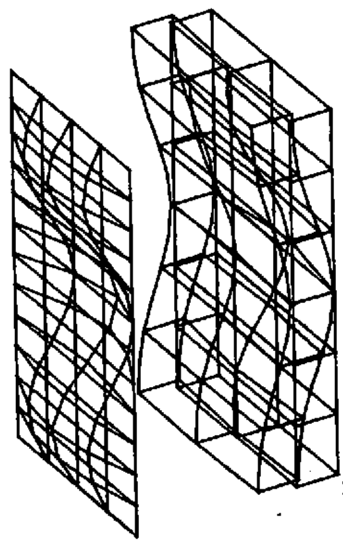

Fig. 2

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