

# AN ITERATIVE DECONVOLUTION TECHNIQUE FOR IMPROVING THE RESOLUTION OF 2-D PASSIVE SONAR IMAGES

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## 1. INTRODUCTION

In optical imaging it is generally recognised that, due to diffraction, the resolution is affected by the size of the aperture. The image of two separated incoherent sources is the superposition of two displaced Airy discs, each of which has the radial-intensity distribution shown in Fig.1a.

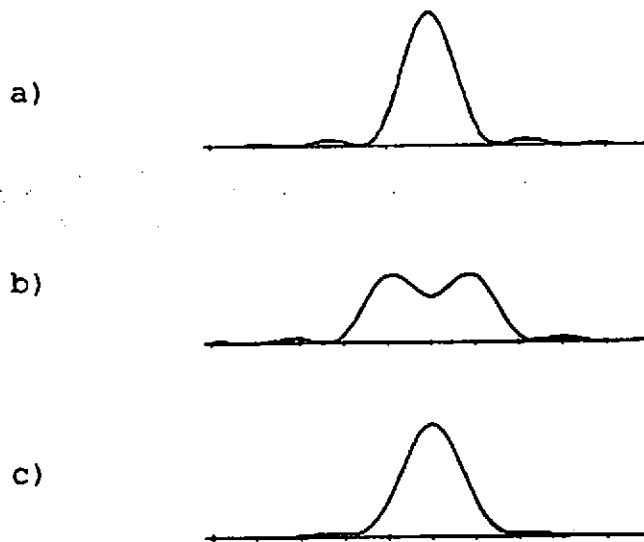


Fig.1

It was supposed for a long time that the image would require a small dip between the two peaks if they were to be resolved (e.g. Fig.1b) and, based upon this, the Rayleigh criterion of resolution was developed. This says that, if  $D$  is the diameter of the pupil, the minimum angular separation of the two sources must be  $1.22\lambda/D$  radians if they are to be resolved. In 1955 however this assumption was questioned by Toraldo di Francia [1] who pointed out that the image of the two points is different from that of one point, however close these points are together (e.g. Fig.1c).

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In radar and sonar, the beamwidths of an aperture whose dimensions are  $a \times b$  are approximately  $\lambda/a$  radians and  $\lambda/b$  radians in the two dimensions respectively, and these values are often taken to be the angular resolutions, in approximate equivalence to the Rayleigh criterion. A parallel occurs in the spectral analysis of time series where the frequency resolution is commonly taken to equal  $1/T$ , where  $T$  is the length of the time series. Recently there have been many attempts to improve on these resolution predictions, and superresolution techniques include autoregressive (maximum entropy), maximum likelihood, Pisarenko harmonic decomposition, and Prony's method. Kay and Marple [2] have produced good review papers on these and other techniques.

In 1984 Wagstaff and Berrou [3] described a technique for high resolution beamforming and spectral analysis which was conceptually and computationally less demanding than most of these. Their specific application was to improve the angular resolution of a sonar line array used for passive listening. From the outputs of 64 overlapping pre-formed beams, obtained by applying an FFT beamformer to the narrowbanded outputs of a 40-element horizontal line array, they applied their technique to produce a high-resolution estimate of the acoustic spatial power spectral density. They described their algorithm as the  $WB^2$  (Wagstaff-Berrou Broadband) technique.

This paper extends the technique to two-dimensional imaging involving only seven beams. Seven splayed sonar transducers point upwards at the sea surface. The time varying output of each beam arises from the noise radiated by ocean waves as they move across the intercepted area. Using the seven measurements only, the technique is applied to produce a time-varying two-dimensional image of noise intensity that contains  $64 \times 64$  pixels.

### 2. SUMMARY OF $WB^2$ TECHNIQUE

The basic principle of the technique is to make a high resolution estimate of incoming acoustic power and then to test its compatibility with the measurements. Based upon errors between the measurements and the beam outputs calculated on the basis of the estimate, a correction is then made to the estimate, to make it more compatible. The art of the method is to derive corrections which cause the estimates to converge to one which gives a small error, preferably rapidly and in a manner that is robust to noise. The procedure for deriving these corrections is a direct extension to two dimensions of the procedure given by Wagstaff and Berrou for one dimension.

## 3. EXPERIMENTAL CONFIGURATION

Our algorithm was applied to experimental data kindly provided to us by Marconi Underwater Systems Ltd. In their experiment seven directive receivers point upwards at the sea surface such that the beam centres intersect the sea surface in a pattern that corresponds to the centre and six corners of a hexagon, as shown in Fig.2. Each beam centre is at an angle of  $3.63^\circ$  to its neighbour.

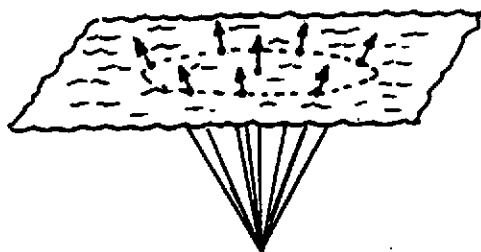


Fig.2

Power measurements are made in a narrow band centred at 24kHz, and the angular response functions are very close to the  $[2J_1(x)/x]^2$  intensity distribution expected of an ideal piston transducer, with the half-power width of each beam being approximately  $6.5^\circ$ . Precise details of the transducer responses are available as power sensibilities for a given position  $[x,y]$  in 2D space, and that of the central symmetrical beam, for example, is given by

$$D(x,y) = \frac{2.972E-4(x^2+y^2)^2 - 0.02812(x^2+y^2) + 1}{1.205E-5(x^2+y^2)^3 + 2.777E-5(x^2+y^2)^2 + 5.61E-4(x^2+y^2) + 1}$$

The power sensibilities of the outer beams are close to being a displaced version of this, but with modifications included to allow for asymmetry introduced by the inclination angle, such that each constant intensity contour is now an ellipse.

The time-varying output of each beam is a power measurement of the noise radiated by the turbulence etc. of ocean waves. The measurements are available at 0.1s intervals. They change with time, due to the breaking of the waves and their movement across the intercepted area.

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## 4. 2-D IMAGING ALGORITHM AND RESULTS

An example of measurement data from the seven transducers, extending over 28s, is shown in Table 1.

m	$B_m(1)$	$B_m(2)$	$B_m(3)$	$B_m(4)$	$B_m(5)$	$B_m(6)$	$B_m(7)$
1	0.2520	0.7491	0.5050	0.3074	0.2391	0.2844	0.3796
2	0.3185	1.1063	0.7508	0.3544	0.2537	0.2777	0.5012
3	0.4475	1.6433	0.7672	0.3905	0.3357	0.4137	0.6980
4	0.5676	1.7615	0.9151	0.4246	0.4032	0.4342	0.7490
5	0.7289	2.0562	1.1499	0.4887	0.3867	0.6085	1.1377
6	0.9414	2.0157	1.0561	0.4836	0.3806	0.7100	1.2891
7	1.1618	2.2081	0.9545	0.4936	0.4164	0.7332	1.5985
8	1.5275	2.2714	1.1914	0.5397	0.4481	1.1111	1.9060
9	2.1064	2.5626	1.4466	0.6805	0.4991	1.3444	2.0164
10	2.5199	2.9003	1.3535	0.8672	0.7032	1.3444	2.0164
11	2.6311	2.1319	1.4982	1.2206	0.9829	1.3320	1.8061
12	2.8753	1.7220	1.4621	1.4536	1.4467	1.7697	1.3419
13	2.4625	1.3958	1.1669	1.5561	2.0264	2.3498	1.3113
14	2.5226	1.0905	1.5331	2.2016	2.4967	2.2021	1.1976
15	2.9682	1.0093	1.8819	3.4493	2.7959	2.2540	0.8916
16	3.2584	0.8974	1.8819	6.0246	4.2482	2.5308	0.8112
17	3.1786	0.7613	1.6499	7.0873	6.0486	2.3087	0.7778
18	2.5720	0.6725	1.4200	5.5774	5.7091	2.4630	0.6478
19	1.8609	0.7090	0.7832	4.2830	4.7491	2.3300	0.4087
20	1.2745	0.7194	0.5847	3.3242	4.6209	1.6969	0.4087
21	0.8304	0.6029	0.5872	2.5123	3.9719	1.3399	0.3699
22	0.6205	0.5414	0.4460	1.8379	2.7815	0.8273	0.3394
23	0.4313	0.3750	0.2727	1.3802	2.1518	0.5772	0.2818
24	0.2951	0.2826	0.2091	0.9875	1.9653	0.5515	0.2172
25	0.2296	0.2089	0.2091	0.6673	1.3075	0.3979	0.1965
26	0.1757	0.1839	0.1820	0.5021	0.8505	0.3205	0.1826
27	0.1604	0.1594	0.1561	0.3868	0.7715	0.2799	0.1804
28	0.1477	0.1382	0.1708	0.3073	0.6166	0.1966	0.1629

Table 1

Fig.3 displays the 1st set as an isometric plot, where the units of the x and y scales are arbitrary except that they are in accordance with the transducers being 189 units below the sea surface.

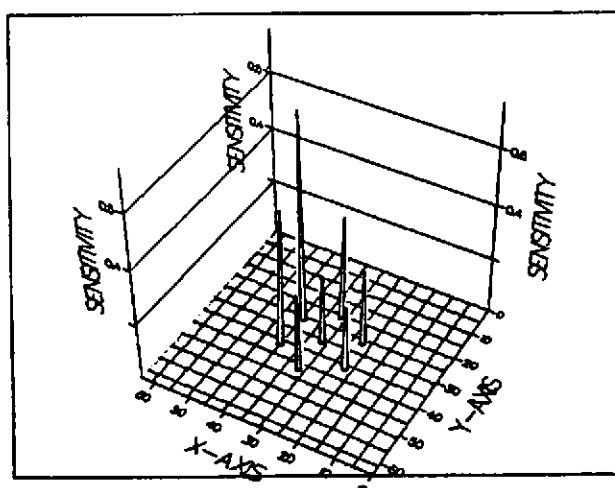


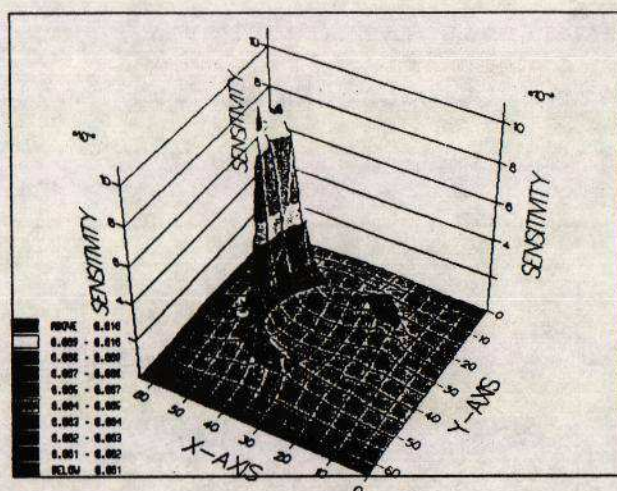
Fig.3



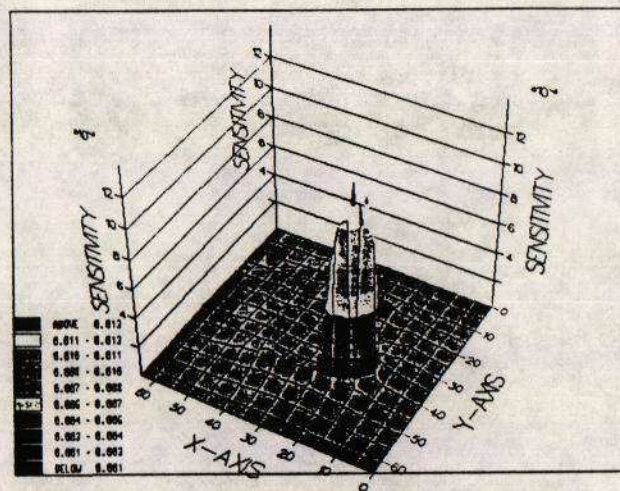
## IMPROVING THE RESOLUTION OF 2-D PASSIVE SONAR IMAGES

The area of the sea surface shown is divided into  $64 \times 64$  pixels for the purpose of processing, and the initial guess is obtained simply by smoothing the seven measurements over this area. This first estimate is multiplied with the power sensibility of each beam and the volume integrals are evaluated to give "calculated beam outputs". These are compared with the measurements to produce seven errors. Each error is multiplied by the top 6dB of the product of power sensibility and estimate to produce a correction contribution. The seven contributions are added, and divided at each position by the number of contributions that are non-zero at that position. This is equivalent to the procedure described by Wagstaff and Berrou except that it extends over  $x$  and  $y$  dimensions. The total correction is applied to the existing estimate to produce a new and improved estimate.

Fig.4a shows the estimate arising from this first set after 42 iterations. Fig.4b shows the estimate arising from the 28th set of measurement data after 42 iterations.



(a)



(b)

Fig.4

It is noticed in Fig.4 how the noise source has moved across the transducers. Estimates from in-between sets of data show that the noise source changes its size and deviates from a straight line path, but is generally consistent with what might be expected of a breaking wave.



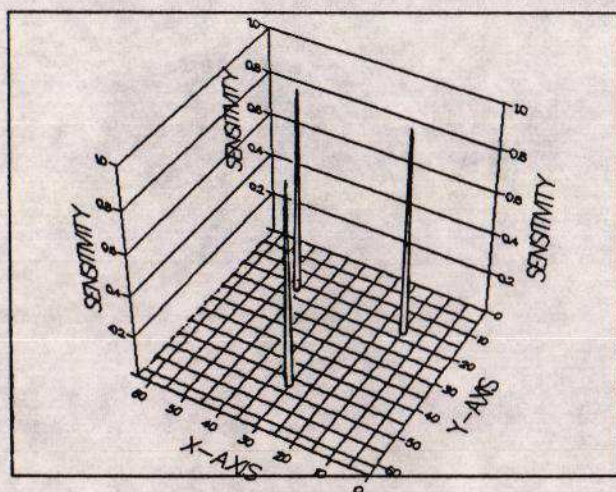
# IMPROVING THE RESOLUTION OF 2-D PASSIVE SONAR IMAGES

A major difficulty of acoustic imaging, as compared to optical imaging, is that of assessing the validity of the outcome. One way is to compare the seven measurements with the seven calculated beam outputs associated with the final estimate. This is done in Table 2 for the 28th set. It is seen that the discrepancies are generally less than 1dB.

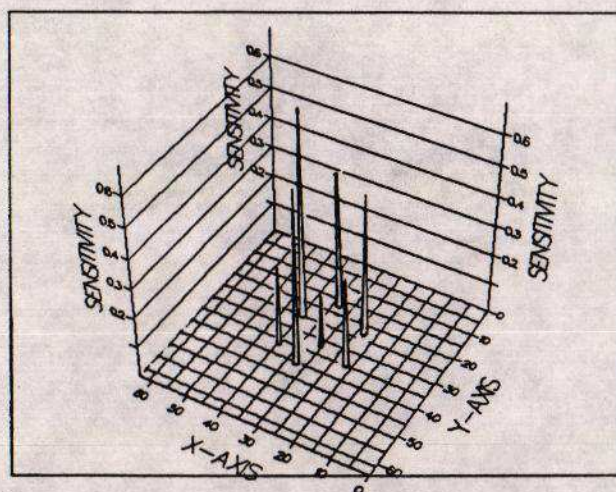
Beam Number	Measurement	Calculated Beam Outputs after 42 iterations
1	-8.31dB	-7.87dB
2	-8.59dB	-8.54dB
3	-7.67dB	-7.72dB
4	-5.12dB	-4.77dB
5	-2.10dB	-3.27dB
6	-7.06dB	-6.53dB
7	-7.88dB	-7.92dB

Table 2

As another means of gaining confidence in the algorithm, simulations have been undertaken to check its effectiveness. Fig.5a shows three discrete simulated targets, and Fig.5b the corresponding seven measurements.



(a)



(b)

Fig.5



## IMPROVING THE RESOLUTION OF 2-D PASSIVE SONAR IMAGES

Fig.6 shows the image resulting from an application of the algorithm. It is seen to agree well with the true target distribution of Fig.5a and provides additional credibility to the images obtained using real data.

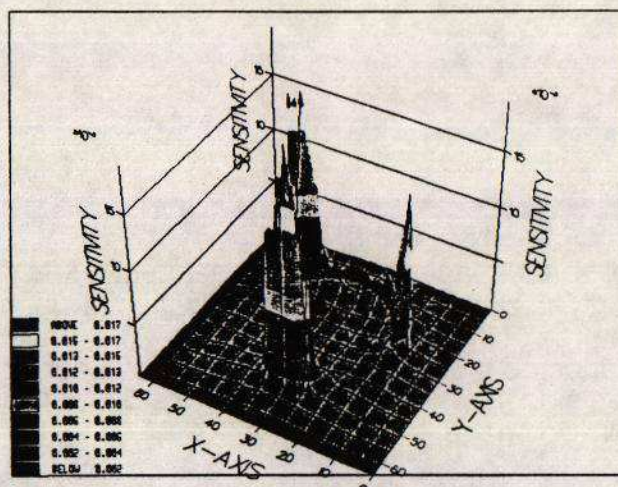


Fig.6

## 5. CONCLUSIONS

Using only one measurement from each of seven transducers a meaningful two-dimensional image of the noise radiated by the sea surface has been produced by a deconvolution technique that relies on precise information of the transducer beam patterns. The resolution is very much better than that expected from the transducer beamwidths. The algorithm is conceptually simple and computationally efficient. Support for the validity of the results has been provided by successful images produced using simulated data.

## 6. REFERENCES

- [1] G TORALDO DI FRANCIA, 'Resolving Power and Information', J Opt Soc Amer, 45(7) pp497-501 (1955)
- [2] S M KAY & S L MARPLE, 'Spectrum Analysis - A Modern Perspective', Proc IEEE 69(11) pp1380-1419 (1981)
- [3] R A WAGSTAFF & J BERROU, 'A Fast and Simple Non-linear Technique for High-resolution beamforming and spectral analysis', J Acoust Soc Amer, 75(4) pp1133-1141