OPTIMISATION OF ACTIVE CONTROL INPUTS FOR RADIATING STRUCTURES

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1- INTRODUCTION

This paper describes the application of a numerical approach to determine the optimum actuator positions in active structural-acoustic control. In this type of application control forces applied to the structure are optimised to reduce radiated noise. Most work that has been published in this area has concentrated on minimising the sound pressure level at specific points with relatively simple structures. The use of a numerical method permits the extension of the analysis to complex structures and enables the total radiated energy to be predicted provided that the sources characteristics are known.

The technique used in this case is based on a boundary element software package developed by STRACO in association with the University of Compiègne (RAYON). The basis of this software is the use of a variational formulation coupling finite and boundary element methods. For a given structure and known inputs the acoustic radiation efficiency, the vibration response and the complete description of the radiated field can be calculated using this software package.The technique adopted to deal with the specific problems associated with active control is to resolve the response of the structure to the known primary inputs and to optimise the secondary inputs to minimise the total radiated energy. This process can be carried out for any acceptable secondary input positions and the absolute optimum positions are those giving maximum noise reduction. This approach can be applied to any active structural acoustic control system.In this case the technique was used to define the optimum actuator positions for a structure to reduce radiated noise. The structural control was then implemented on the experimental system using force inputs, accelerometers to provide error signals and a control system based on the filtered X-LMS algorithm. In order to validate the method a rectangular plate was used as the structure and the primary force and secondary control forces were applied using point force inputs. The control and measurement systems were developed in association with Virginia Polytechnic Institute and ACOVIB.

2- THEORETICAL FORMULATION OF THE COUPLED PROBLEM

Consider a thin elastic shell driven by a harmonic vibration force $f(x,t) = Re(e^{-i\omega t} f(x))$ acting on its surface S. This results in a distributed shell displacement $U(x,t)$ $Re(e^{-i\omega t} U(x))$ which radiates an acoustic pressure field $p(x,t) = Re(e^{-i\omega t} p(x))$ inside and outside the shell. Using a variational formulation coupling Finite and Boundary element method proposed by HAMDI [1,2] and using a mass normalised mode shape expansion, the equation of the structure S after discretisation is,

$$(\mathbf{Z}_m - \mathbf{Z}_a)\,\mathbf{d} = \mathbf{g} \tag{1}$$

. $Zm = \Omega^2 + 2i\omega\,\xi\Omega - \omega^2\,\mathbf{I}$, is the diagonalized shell mechanical impedance matrix where, Ω and ξ are respectively the matrices containing the natural frequencies Ω_k and damping factors ξ_k associated with the mode W_k of the shell S. I is the identity matrix.

. $\mathbf{Z_a}$ is the modal acoustic impedance matrix of the shell surface S.
. **d** is the modal coordinate vector of the displacement U
. **g** is the modal coordinate vector of the force f(x).

It can be shown that radiated acoustic energy is given by,

$$Er = 1/2 \; Im\,(\mathbf{d}^H \, \mathbf{Za} \, \mathbf{d}) \tag{2}$$

In practice, the effectiveness of the active control of the vibrating structure is highly dependent on the location of the actuators, especially when their number is limited. A general numerical approach is proposed,which leads automatically to the solution of the optimum actuator postions to minimise the radiated acoustic energy.
A controlled zone Sc containing N nodes is defined on the surface S of the structure. To control the vibrating structure, actuators A_j are fixed at locations x_j in the controlled zone S_c. These actuators induce secondary loads v_jF_j on the nodes nj which permit reduction of the noise generated by the primary force F_0 (x) on S. Then the total load T acting on S is given by,

$$T = F_0 + v_jF_j \quad j= 1, n \text{ (number of actuators)} \tag{3}$$

where v_j is the complex control amplitude associated with actuator A_j.
The modal component $\mathbf{d_0}$ of the primary load and the modal coordinate $\mathbf{d_j}$ of the displacement vector U_j resulting from the force distribution $F_j = g_j{}^k\, W_k$ induced by the actuator A_j satisfy equation (1); the modal component of the total displacement is now given by,

$$\mathbf{d} = \mathbf{d_0} + v_j\,\mathbf{dj} \quad (j=1,n) \tag{4}$$

$$\text{or} \quad \mathbf{d} = \mathbf{d_0} + \mathbf{D}.v$$

The radiated acoustic energy, $Er = 1/2\; Im\,(\mathbf{d}^H\,\mathbf{Za}\,\mathbf{d})$ is a quadratic form of v_j and can be written in the more explicit form,

$$Er = (E_{00} + 2E_{0c} + E_{cc}) \tag{5}$$

where,

$E_{00}=1/2\,(\mathbf{d_0}^H\, \mathbf{Z_2}\, \mathbf{d_0})$ is the energy radiated by the primary source

$E_{0c}= 1/2\, \mathrm{Re}(\mathbf{d_0}^H\, \mathbf{Z_2}\, \mathbf{D}\, \mathbf{v})$ is the coupling energy between primary and secondary sources

$E_{cc}=1/2\, (\mathbf{v}^H(\mathbf{D}^H\, \mathbf{Z_2}\, \mathbf{D})\, \mathbf{v})$ is the energy radiated by the the secondary sources

and $\mathbf{Z_2}$, is the imaginary part of the modal acoustic impedance matrix $\mathbf{Z_a}$ which is symmetric. The control amplitude which minimise the radiated energy verifies the following linear algebraic equation,

$$(\mathbf{D}^H\, \mathbf{Z_2}\, \mathbf{D})\, \mathbf{v} = - \mathbf{D}^H\, \mathbf{Z_2}\, \mathbf{d_0} \tag{6}$$

and the corresponding optimum value of the radiated acoustic energy is given by,

$$E_r=E_{00} - E_{cc} \tag{7}$$

The radiated energy E_r is equal to zero, if the energy E_{cc} radiated by the secondary sources equals the energy radiated by the primary sources. This situation is possible if the displacement U_0 is a linear combination of the displacement U_j induced by the actuators. In practice this ideal situation does not occur because the primary and secondary loads are generally in different vector subspaces.The optimum value of the control amplitude vector **v** is obtained for a given position vector **x** of the n actuator which coincide with **n** nodes of the controlled area Sc of the shell FEM model. Then $C^n{}_N$ values of the acoustic radiated energy for the **n** actuators are obtained, where N is the number of nodes of the controlled area Sc. Since Er is not a convex function of the vector position **x**, the optimum value of **x** is that which gives the absolute minimum of the radiated energy for the $C^n{}_N$ possible positions in Sc.

To compute the optimal position **x** of the **n** actuators a simple numerical strategy was developed using RAYON software by performing the following steps:

1. solve equation (1) for the primary and all possible secondary source positions,
2. compute the amplitude vectors from equation (6)
3. compute local optimums and determine the absolute minimum of radiated energy
4. represent acceleration, acoustic pressure distribution and intensity vectors on the shell surface and compute the acoustic pressure directivity pattern

Other constraints such as actuator input energy limitations can be taken into account when performing step 3. Typical results that can be obtained according to these computation steps are presented in section 4 for the simple case of an elastic free-free plate.

3- EXPERIMENTAL SET-UP

The experiments to validate the technique of optimisation were carried out on a rectangular plate suspended vertically with free boundary conditions. The filtered X-LMS algorithm was implemented on a Texas Instruments TMS 320C30 processor using the LSI board in a PC 386 computer. The primary input and control inputs were provided using Bruel & Kjaer 4810 shakers and the input forces were measured with force transducers. Acoustic intensity measurements were used to measure total radiated sound power over the surface of the plate and the intensity vectors in a vertical plane normal to the plate were measured to identify specific characteristics of the radiated field in the region of the force inputs. Error signals were obtained using miniature accelerometers and the only information used from the model to implement the control was the position or in the case of two control inputs, the positions of the control shakers. A diagram of the experimental configuration is presented in figure 1.

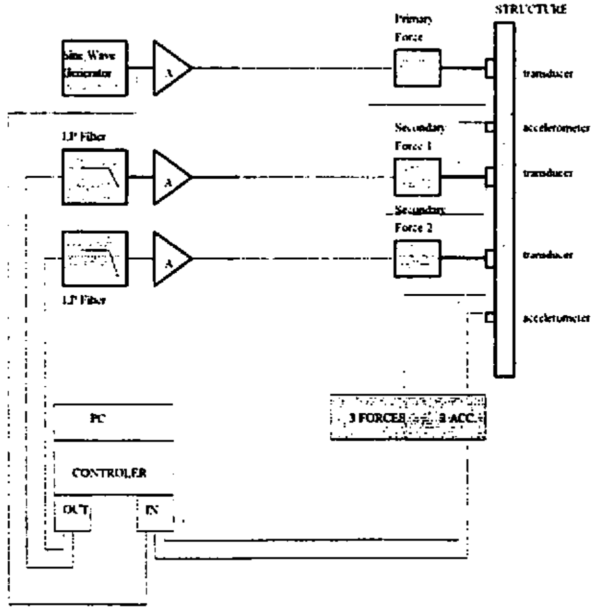

Figure 1. Active control set-up with two control forces and error signals.

Acquisition of the signals from the force transducers, the intensity probe and the accelerometers was acomplished using a DIFA SCADAS data acquisition system linked to an HP 9000 work station using LMS software. The graphics and sound intensity measurement software were developed by ACOVIB in association with the University of Compiègne. Intensity measurements were made using the classical parallel microphone technique and rotating the probe to obtain the two components of the intensity vector in the measurement plane. For lower levels of sound radiation encountered in controlled conditions a referenced measurement of sound intensity could be implemented to obtain more accurate measurements, but this did not prove to be necessary in the present application.

Positions for the accelerometers providing the error signals were chosen initially on the basis of selecting points of maximum displacement with the primary force in action . For a single error transducer the point on the structure with the maximum calculated vibration level reduction between uncontrolled and optimum controlled conditions would seem to be the optimum choice of accelerometer position.

4- NUMERICAL AND EXPERIMENTAL RESULTS

The plate used for this investigation measured 500 x 800 x 2.6 mm and was suspended vertically with free boundary conditions.The finite element mesh is composed of 320 rectangular elements with 6 degrees of freedom per node.The plate is excited by a sinusoïdal input force (amplitude 1N). Thirty six modes are used to calculate the characteristics of the vibration and acoustic response (thirty structural modes and the six rigid body modes).

The vibration caracteristics of the plate used for the experiment were initially identified using pseudo random excitation of the primary input shaker with the control input shakers detached. The resonant frequencies and damping ratios were measured and some of them are compared with the calculated natural frequencies in table 1. The model is in close agreement with the measured values.

The measured values of damping were then integrated into the model to obtain the radiated power and calculate the response of the controlled structure.The numerical solutions for the minimum radiated power were calculated for a single point force input at each node of the structure in a zone defined as the control area .In order to avoid trivial solutions an area was defined around the primary input position where control inputs were excluded.

The optimum position was defined as that giving the maximum reduction in radiated noise.The amplitude and phase of optimum solution could then be compared with the experimental result. Solutions are presented for a frequency of 215 Hz which is the resonant frequency of the fifteenth mode. The uncontrolled displacement (figure 2a) is dominated by the fifteenth mode as expected and this is confirmed by the modal contributions presented figure 2b.

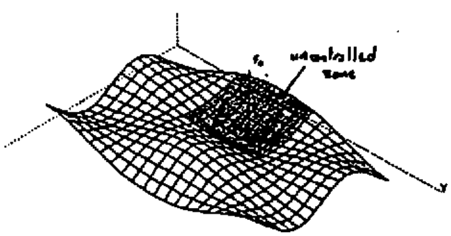

Figure 2 a : Uncontrolled case

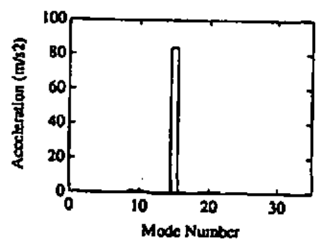

Figure 2 b: Modal amplitudes (m/s^2)

The noise reduction for the controlled structure (1 actuator) is calculated for each of the optimised control input positions over the control surface and this data is presented as a plot of noise reduction versus input position in figure 3.

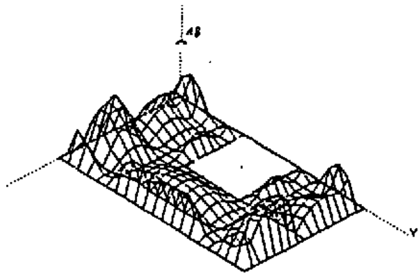

Figure 3 : Noise reduction versus position (dB)

The optimised values of the forces calculated to obtain the maximum noise reduction at each control input position varied between about 0.1N and 4N.The symmetry due to the choice of the position of the primary input can be clearly seen on this plot.As may be expected the points on the nodal lines of the fifteenth mode prove to be poor control input positions and give relatively small noise reductions. Other points which are situated in positions which are antinodes, in phase opposition with the primary input point give bad results, due to the transfer of energy to lower order modes which radiates strongly. The control input positions which result in the maximum noise reduction are situated on the antinodes of vibration of the fifteenth mode in phase with the primary force input position.

The displacement response for the controlled structure using the optimum control input position is presented in figure 4a along with the modal contributions in figure 4b. It can be seen that the energy is redistributed over the most of the modes around the dominant mode of the uncontrolled response, and the control force required to obtain this maximum noise reduction is 1.1N, opposed in phase to the primary force input.

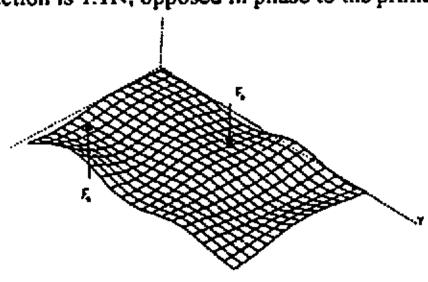

Figure 4 a : One actuator . NR = 17.3 dB
F_1 = - 1.1 N

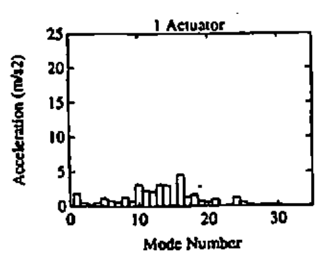

Figure 4 b: Modal amplitudes (m/s^2)

The results obtained experimentally in this configuration were compared in terms of the noise reduction NR. NR= $-10\log_{10}(W_{con}/W_{unc})$ where W_{con} is the total radiated sound power with control and W_{unc} the total radiated sound power without control.

The sound power was measured using scanned sound intensity measurements. The experimental value of the noise reduction, 16.2 dB and the control force amplitude, -1.18 N

obtained using the filtered X LMS controller compares well with numerical results. It may be noted that these experimental results were obtained using an accelerometer position which was not optimised. Better agreement may be expected when this parameter is included in the experimental set-up.
The controlled displacement response using two actuators is shown in figure 5a and b.
The optimum result is given by a moment situated on the line of symmetry of the plate. The noise reduction factor obtained with this configuration was 23 dB. Unfortunately these positions are too close to be implemented with electromagnetic shakers and so another configuration was chosen to compare with experimental results.The configuration chosen resulted in the controlled displacement response depicted in figure 6a and the modal contributions plotted in figure 6b, the noise reduction obtained for these two points was of course inferior to the previous result.

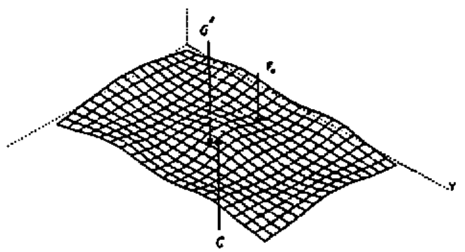

Figure 5 a: Two actuators. NR = 23. dB
G= - 2.94 N G'= 2.71 N

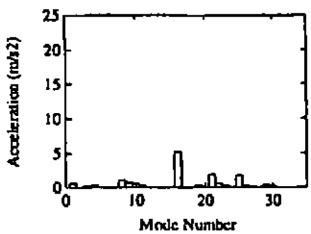

Figure 5 b: Modal amplitudes (m/s^2)

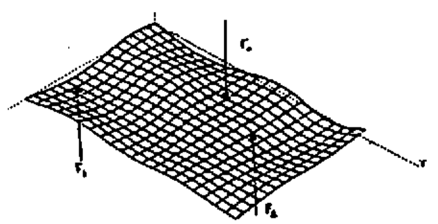

Figure 6 a: Two actuators. NR = 20.5 dB
F_2= -0.46 N F_3= -0.84 N

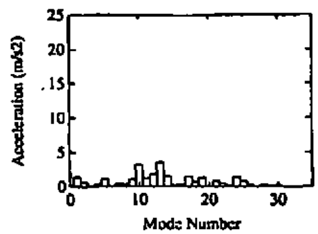

Figure 6 b: Modal amplitudes (m/s^2)

The experimental results of 19 dB for the noise reduction and -0.41 N and -1. N for the two actuator points compare well with the predicted values of the numerical model.
In this configuration the intensity vectors in a vertical plane normal to the plate in the region of the primary excitation were measured with and without control (figure 7). The vectors show a very similar pattern but the amplitudes are reduced from maximum values of around 70 dB to values around 59 dB.A procedure for the calculation of numerical values of intensity is being developed.

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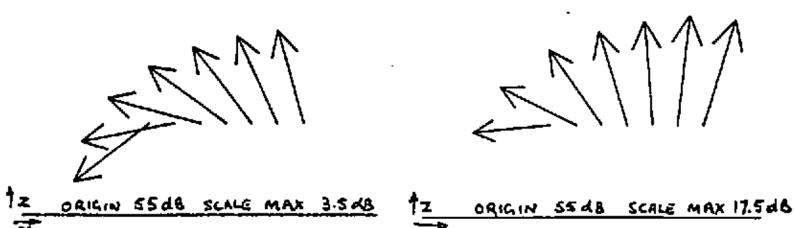

controlled case — uncontrolled case

Figure 7: Intensity vectors in vertical plane normal to plate in controlled and uncontrolled conditions at 215 Hz.

TABLE 1 SOME NATURAL FREQUENCIES

N°		Numerical Hz	Experimental Hz	$100 \times \frac{\lvert F_N - F_E \rvert}{F_E}$ %	Damping D/2f %
1	(0,2)	22.47	21.70	3.54	-
2	(1,1)	23.11	23.40	1.40	2.65
3	(1,2)	52.07	50.07	2.70	0.70
4	(2,0)	58.51	58.30	0.36	0.75
5	(0,3)	61.86	61.20	1.07	0.72
6	(2,1)	75.54	75.20	0.45	0.60
7	(1,3)	93.87	93.65	0.23	0.70
8	(2,2)	112.54	111.77	0.75	0.36
9	(0,4)	124.12	124.07	0.04	0.38
10	(1,4)	152.27	150.73	1.0	-
11	(3,0)	162.86	162.70	0.13	0.30
12	(2,3)	165.14	165.09	0.03	0.26
13	(3,1)	175.33	174.66	0.40	0.32
14	(0,5)	204.38	203.71	0.30	0.25
15	(3,2)	214.97	215.33	0.16	0.23
16	(2,4)	231.03	231.74	0.31	0.19
17	(1,5)	233.42	232.76	0.30	0.21
18	(3,3)	272.47	273.78	0.47	0.15

5- CONCLUSION

This approach can be seen to give useful and applicable results on the simple structure presented here. Extension to more complex structures and different types of source should be technically possible as is the inclusion of the model of other types of actuators. Work will be pursued in these directions when the procedure has been thoroughly validated on simple structures

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