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# ANALYSIS OF TROMBONE BELL VIBRATIONS

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One of the many controversies surrounding the mechanics of brass instruments is the significance of the deviations from the ideal case of the instrument bell being a rigid boundary for the air column. Yielding walls can interact with the standing wave in the air column, internally dissipate energy and radiate sound from their outer surface: they can do all these whether considered yielding or resonant. If resonant at particular frequencies, e.g. notes of the musical scale, will the effect be constructive, detrimental or insignificant?

The finite element method has been chosen as the best method on which to base a modal analysis of brass instrument bells. This is because a brass instrument bell is of a mathematically arbitrary shape and therefore does not lend itself to analytic techniques, and that the finite element method is relatively simple due to the many computer packages available. A package has been chosen (A.S.A.S. implemented on the IBM 360/195 at the Rutherford Laboratory) and the required data collated to form a set of cases designed to investigate the effect of trombone bell parameters. The parameters which can be represented are:

(a) Material properties: Young's modulus, Poisson's ratio and density.

(b) Wall thickness.

(c) Position of stay.

(d) Geometric shape.

(e) Rim size.

For each data case, a set of modes (both frequencies and shapes) are extracted, a sample of mode shapes being shown in Figure (1). Many of the mode shapes and frequencies come in pairs. This is due to the asymmetry provided by the stay, which implies that the frequency of a particular mode shape will depend upon its geometrical relation to that stay. The frequencies of all the modes extracted lie in the full musical range of frequencies used by the trombone, so there is an initial opportunity for these modes to contribute to the musical quality of the instrument.

In order to calculate the response of brass instrument bells, a generalised method of vibration analysis is used. The method and notation are as in Clarkson (1977).

The method is based on two equations:

$$W(x,y,z,t) = \sum_{r=1}^{\infty} q_r(t).f_r(x,y,z) \qquad (1)$$

$$M_r \ddot{q}_r + C_r \dot{q}_r + K_r q_r = L_r(t) \qquad (2)$$

where W represents the deflected shape of the structure, $q_r$ the amplitude and time dependence of the r-th mode of vibration and is known as the Generalised Co-ordinate, and $f_r(x,y,z)$ is the normalised mode shape of the r-th natural mode of vibration.

Equation (1) implies that any deflected shape of the structure can be represented as a sum of its natural modes of vibration. Equation (2) is of similar form to that of a mass-spring-damper system except $M_r$, $C_r$, $K_r$ and $L_r$ are all generalised parameters.

The generalised parameters are evaluated by a mixture of theoretical and empirical data and equation (2) is solved for the case of acoustic excitation.

When calculating the acoustic excitation force the equations imply that for a "perfectly symmetric" mode shape (viz. for every positive component of displacement there is an equal negative component of displacement) the net coupling to the structure will be zero and thus that particular mode will not be excited. Non-perfect mode shapes are a result of a non-symmetric structure, so it is asymmetries such as the stay which dictate the level of excitation of modes.

The results from the calculation of bell response to acoustic excitation show general agreement with experimental results (Smith 1978, Kitchin 1980), though more information on the damping mechanisms would improve the calculations.

The musical significance of the bell vibration characteristics is not known. Experiments indicate that future work should concentrate on sound radiation from the bell surface and the resultant sound field at the player's ear.

## References

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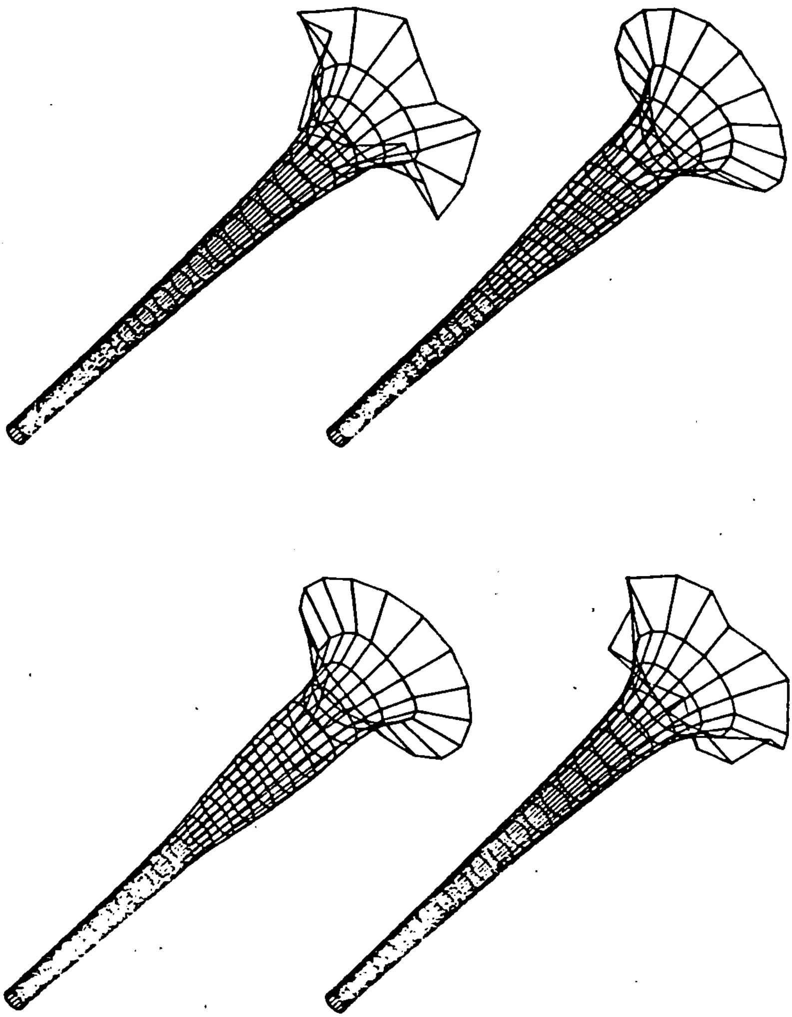

Figure 1. Trombone Bell Mode Shapes