NON-DETERMINIST SONAR EQUATION AND ITS USE IN SONAR ASSESSMENT AND RANGE PREDICTION MODELS.

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SUMMARY

The "sonar equation", used in design or assessment of sonar systems, generally gives results with an error or uncertainty which may be important. Its examination permit to propose an improvement of this methodology, leading to a modified results form; needs could be more satisfied and this should lead to an evolution in design of associated noise, propagation,.. models. This improvement can result of a stochastic point of view, or of a method inspired by the fuzzy sets theory. An example is developped, showing the consequences in applying the first of those methodologies.

1. SONAR EQUATION EXTENSION

The design and the assessment of a sonar system lay on the knowing of the signals and noises power levels and of the influence of the processing on those levels. In this aim, the "sonar equation " [1 to 4] is generally the first and main tool. We try here to propose improved methodologies, which would permit to obtain more sophisticated results in the use of this equation. We recall that the sonar equation is expressed in terms of "sonar parameters", whose valuations are in dB ; each term is depending of some "elementary" parameters, and, as a general rule, its value is obtained, for a given set of those elementary parameters, by use of an experimental or theoretic model or possibly by direct measure. For example, for a passive sonar, working in a frequency band W, on a source (spectral line) level SL and a noise spectral level NL, with a transmission loss TL, a directivity index DI and a temporal processing gain TG, the equation gives the "recognition differential" RD :

(1) $$RD = SL - TL - NL - 10 \log W + DI + TG.$$

RD is the signal-to-noise ratio to just perform a certain function, and the equation is valid under the condition making the equality firm.

At the present time, the sonar equation is, as a rule, a determinist equation, linking determinist quantities which correspond to a set of elementary parameters. Knowing the values of this set, we search the system performance RD ; knowing the performance and a part of the parameters, we search the other parameters.

Nevertheless, some sonar parameters values can be known only with imprecision and uncertainty : they are measures with errors, or mean values of varying (or random) quantities, or outputs of models depending of unknown parameters, unverified hypothesis or subjective values. Moreover, we can be interested in a use (perhaps to mask the ignorance generating the preceding imprecision and uncertainty) of the sonar equation conditionned, not by a set of parameters values, but by the belonging of the parameters to a given domain, called "uncertainty domain". Lastly, some values of the parameters may be more likely

than others. We shall call uncertainty state of a parameter, the fact to belong to an uncertainty domain in a given manner.
In that way, the results given by the sonar equation are provided by unknown uncertainty (and imprecision) : they can depend on both a parameters uncertainty state and a non exact (for example random) deduction mecanism between parameters and performance. Then, knowing the parameters uncertainty state, we can search the uncertainty state of the system performance RD ; knowing the uncertainty state of the performance and of a part of the parameters, we can search the other parameters uncertainty state.

We can think of three methods to assume the preceding programme.
- The first method is often use in classical physics : we consider the problem and the result as determinist, we associate a result to a set of values of parameters and we perform an error calculation by interval analysis. This method don't use much information. An uncertainty domain is an interval, with total or null determinist membership.
- The second method is deduced of the subjectivist school in probability theory : we consider the problem and the result as random and we translate the uncertainties and frame the result by means of moments or probability laws. This method asks a good information. An uncertainty state is represented by a random variable state, which implies belonging domain with probability law.
- The third method proceeds from fuzzy sets theory : we consider the problem and the result as provided with uncertainty and we translate the uncertainties and frame the result by means of fuzzy description or by possibility theory. Perhaps this method can compete with the preceding in the problem matching and in the information request. An uncertainty state is a domain with degrees of membership and is represented by a fuzzy quantity.

In this frame of taking into account the uncertainty character, we have to deal with non-determinist sonar relations ; such a relation can be :
- an evaluation of a combination of sonar parameters (A depends of the parameters via B) :

(2) $\Theta \in D(\Theta)$, $A = B(\Theta)$;

- a comparison (equality or inequality) of two combinations of sonar parameters (A and B are each depending on a subset of parameters) :

(3) $\Omega \in D(\Omega)$, $\Theta \in D(\Theta)$, $A(\Omega) \leftrightarrow B(\Theta)$;

In those relations, we have, with $i \in \{1, .., m\}$ and $j \in \{1, .., n\}$,

(4) $A(\Omega) = \sum_i A_i(\Omega_i)$, $B(\Theta) = \sum_j B_j(\Theta_j)$,

where, $\forall\ i \in \{0, .., m\}$ (resp. $j \in \{1, .., n\}$), Ω_i (resp. Θ_j) is the subset of elementary parameters of which A_i (resp. B_j) is depending. We call Ω (resp. Θ) the set of parameters, obtained by reunion of subsets Ω_i (resp. Θ_j). We suppose each parameter provided with an uncertainty state and, then, belonging to an uncertainty domain (a known parameter is translated by a domain reduced to a point, and a total membership) ; we call $D(\Omega)$ (resp. $D(\Theta)$) the uncertainty domain of the whole set of parameters Ω (resp. Θ).

2. STOCHASTIC SONAR EQUATION

We suppose that the parameters of Ω and Θ are random variables, each with a probability law which represents the uncertainty state. Then, A_i and B_j are

also random variables, and so are A and B. The relations (2) and (3) where A and B are random quantities are "sonar stochastic relations".
In the case of the relation (2) (evaluation), we can deduce, conditionally on the fact that Θ belongs to its uncertainty domain, a mean values relation and a variances relation :

(5a,b) $(\forall\ \Theta \in D(\Theta)), \quad E\{A\} = E\{B(\Theta)\}, \quad \sigma^2\{A\} = \sigma^2\{B(\Theta)\}.$

The moments, in those relations, are obtained by :

(6a) $(\forall\ \Theta \in D(\Theta)), \quad E\{B(\Theta)\} = \sum_j E\{B_j(\Theta_j)\},$

and, with statistical independance of the subsets, $\forall\ j$, Θ_j, by :

(6b) $(\forall\ \Theta \in D(\Theta)), \quad \sigma^2\{B(\Theta)\} = \sum_j \sigma^2\{B_j(\Theta_j)\}.$

When some subsets Θ_j are statistically dependant, we can have an over-estimation, by using the Schwarz inequality, in the form :

(7) $\sigma^2\{\sum_i U_i\} \le \sum_i\sum_j\{\sigma^2\{U_i\}\sigma^2\{U_j\}\}^{1/2}.$

In using a normal law, the result can be framed in a confidence interval [5] at level p, $p \in]0, 1[$, such that :

(8) $\Pr[(E\{A\} - k\sigma\{A\}) < A < (E\{A\} + k\sigma\{A\})] = p,$

(or > p when $\sigma\{A\}$ is over estimated). For example : $k = 1.96$ with $p = 0.95$, $k = 1.28$ with $p = 0.8$, $k = 0.67$ with $p = 0.5$.
In the case of the relation (3) (comparison), we can search the uncertainty domain $D(\Omega)$ on which, conditionally to $\Theta \in D(\Theta)$, the probability P_S, that the relation $A(\Omega) \leftrightarrow B(\Theta)$ is satisfied, is not null ; by evaluation of this probability, the result is written :

(9) $\forall\ \Omega \in D(\Omega), \quad P_S(\Omega) = \Pr[A(\Omega) \leftrightarrow B(\Theta)\ /\ \Theta \in D(\Theta)].$

This last method can permit to take into account the sonar temporal aspect.

3. FUZZY SONAR EQUATION

We suppose that the parameters of Ω and Θ are fuzzy quantities, that is to say that each parameter is represented by the giving of a fuzzy set on $\mathbb{R}$. This fuzzy set is, in fact, a mapping μ of $\mathbb{R}$ into [0, 1] ; $\mu(x)$ is the membership degree of x to the fuzzy quantity. The support $\{x : x \in \mathbb{R}, \mu(x) > 0\}$ of the fuzzy quantity is identical to the uncertainty domain of the parameter. Then A_i and B_j are also fuzzy quantities and so are A and B.
More precisely, we use, if possible, representations by [6] "fuzzy intervals" (that is to say "convex fuzzy quantities", i.e. they are such that $\forall\ x \in \mathbb{R}$, $\forall\ y \in \mathbb{R}$, $\forall\ z \in [x, y]$, $\mu(z) \ge \min(\mu(x), \mu(y))$), "closed-generalized fuzzy intervals" (of which μ is upper semicontinuous, i.e. their α-cuts are closed intervals), "compact-generalized fuzzy intervals" (preceding "closed-generalized fuzzy intervals" with compact support), "fuzzy numbers" ("compact-generalized fuzzy intervals" with unique modal value), multimodal fuzzy quantities (finite "union" of "closed-generalized fuzzy intervals").
The relations (2) and (3) where A and B are fuzzy quantities are " sonar fuzzy relations".
In the case of the relation (2) (evaluation), we can deduce the fuzzy representation of A from these of the B_j by means of classic operations on fuzzy quantities [6]. The fuzzy quantities F and G being represented by $\mu_F(x)$ and $\mu_G(x)$, with $x \in \mathbb{R}$, we have the following representations

(10a) $H = F + G : \quad \mu_H(x) = \sup\{\min\{\mu_F(y), \mu_G(x - y)\}, y \in \mathbb{R}\}$;

(10b) $H = F - G$: $\mu_H(x) = \sup\{\min(\mu_F(x + y), \mu_G(y)), y \in \mathbb{R}\}$;

(10c) $H = - F$: $\mu_H(x) = \mu_F(- x)$;

(10d) $H = 1/F$: $\mu_H(x) = \mu_F(1/x), x \neq 0$;

(10e) $H = \alpha F$: $\mu_H(x) = \mu_F(x/\alpha), \alpha \neq 0$.

4. APPLICATION EXAMPLE OF SONAR STOCHASTIC EQUATION

We illustrate those methodologies on a very simple example, in playing essentially on one sonar parameter and one elementary parameter (actually such a use of sonar equation depends on some sonar parameters and on a lot of elementary parameters).
We consider the passive sonar case described in the first paragraph to valuate its performances , on the one hand in term of signal-to-noise ratio, on the other hand in term of range prediction.
The problem is governed, in the determinist point of view, by the sonar equation (1), in which RD has a given value, corresponding to a "standard performance" ; the sonar equation is valid for conditions providing this standard performance. For any conditions, we call [3] "figure of merit" and "signal excess" the combinations :

(11) $FOM = SL - NL - 10 \log W + DI + TG - RD$;

(12) $EX = SL - TL - NL - 10 \log W + DI + TG - RD = FOM - TL$.

For $EX \geq 0$, the conditions generate a performance at least equivalent to the standard one and FOM represents the maximum transmission loss allowing such a performance.

The example is constructed with the following numerical values :

- SL = 136 dB, W = 0.5 Hz, DI = 19 dB, TG = 0 dB, RD = 0 dB.
- We have a set of classical curves [7] which gives the noise level as a function of the sea-state ; we have in dB, for sea-state $j \in \{0, ..., 6\}$, $NL_0 = 43$, $NL_1 = 53$, $NL_2 = 59$, $NL_3 = 63$, $NL_4 = 66$, $NL_5 = 67$, $NL_6 = 68$.
- The noise level of the example is only known in a subjective manner : the various observers agree on a only point, namely the sea-state is about 3.
- We have a transmission loss model [8] giving TL as a function of distance d, on the form : $TL = f(d)$.

The determinist point of view gives two results :

- performances in term of signal-to-noise ratio is represented by EX as a function of d (see figures 1 and 2).
- performances in term of range prediction is represented by the set of distances d such that $EX \geq 0$, that is to say $FOM \geq TL = f(d)$. The figure 2 (in which FOM = 95 dB) shows a range prediction of two intervals [0, 35 km] and [61 km, 68 km], and a punctual possibility at 125 km.

The stochastic point of view pose the problem of obtaining the statistic knowledge about the sonar parameters (or the elementary parameters). We construct this knowledge by :

- keeping the determinist quantities : W = 0.5 Hz, TG = 0 dB, RD = 0 dB.
- deduction of mean values from determinist values : $E\{SL\} = SL_{det} = 136$ dB, $E\{DI\} = DI_{det} = 19$ dB, $E\{TL\} = f(d)$.

– choice of root mean square values : $\sigma_{SL} = 1.5$ dB, $\sigma_{DI} = 1.5$ dB, $\sigma_{TL} = K'd$ (in dB), with $K' = 0.05$ dB/km to have about 3 dB in the first convergence zone.

– choice by association to "sea-state j to k" :

..of mean values : $E\{NL\}_{jk} = (1/2)(NL_j + NL_k)$. We remark that $E\{NL\}_{jk} \neq NL_{(j+k)/2}$

..of root mean square values : $\sigma_{NLjk} = K(NL_{k+1} - NL_{j-1})$, with $K = 0.43$ to have $\sigma_{NL33} = 3$ dB.

We translate the subjectivity about the noise level in looking five cases of sea-state : 3, 2 to 3, 3 to 4, 2 to 4, 1 to 5.

The stochastic point of view gives two results :

– performances in term of signal-to-noise ratio is represented (see figures 3 and 4) by the random variable EX, as a function of d, with :

$E\{EX\} = E\{FOM\} - E\{TL\} = E\{SL\} - E\{TL\} - E\{NL\} - 10 \log W + E\{DI\} + TG - RD$;

$\sigma^2_{EX} = \sigma^2_{FOM} + \sigma^2_{TL} = \sigma^2_{SL} + \sigma^2_{TL} + \sigma^2_{NL} + \sigma^2_{DI}$.

EX, for each value of d, can be framed in a confidence interval at level p (we suppose EX has a normal law) :

$Pr[(E\{FOM\} - E\{PT\} - k\sigma_{EX}) < A < (E\{FOM\} - E\{PT\} + k\sigma_{EX})] = p$.

– performances in term of range prediction is represented (see figures 5 and 6) by the set of distances d such that $P_S(d) = Pr[FOM - TL \geq 0 \; ; \; d]$ is strictly positive, and the value of $P_S(d)$ (we suppose EX has a normal law).

We can note that σ_{EX} is variable with distance d.

We have a synthesis on figures 7 and 8 : they present the distances d, for which, in the random case, the probability $P_S(d)$, of having a performance at least equivalent to the standard one, is upper than 0.7 (fig. 7) and 0.5 (fig. 8), for different sea-states compatible with the subjective noise level knowing (sea-state is about 3) ; the determinist case figures just for comparison.

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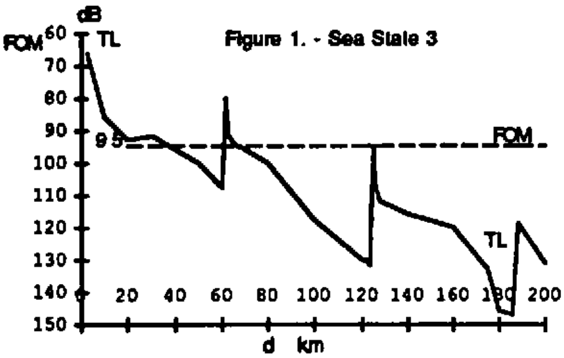

Figure 1. - Sea State 3

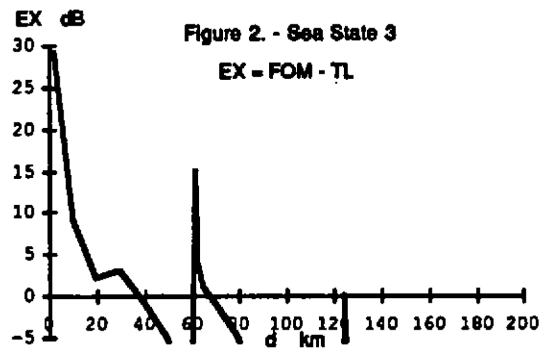

Figure 2. - Sea State 3

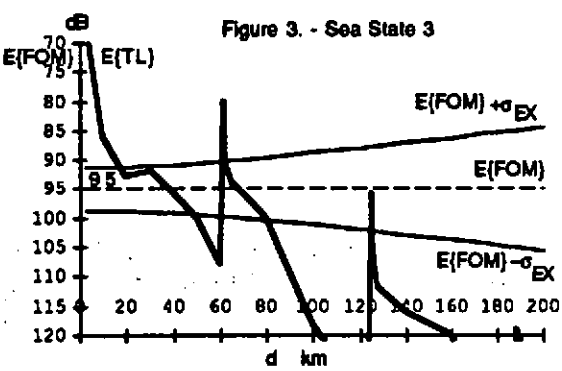

Figure 3. - Sea State 3

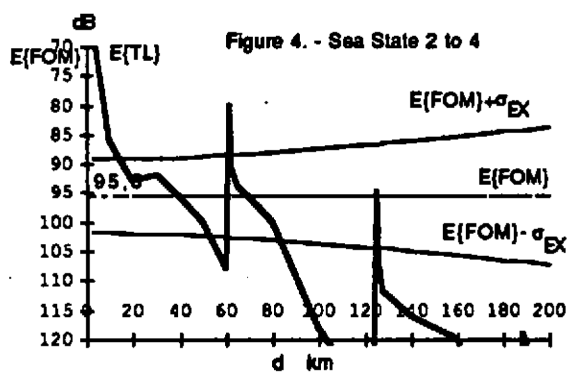

Figure 4. - Sea State 2 to 4

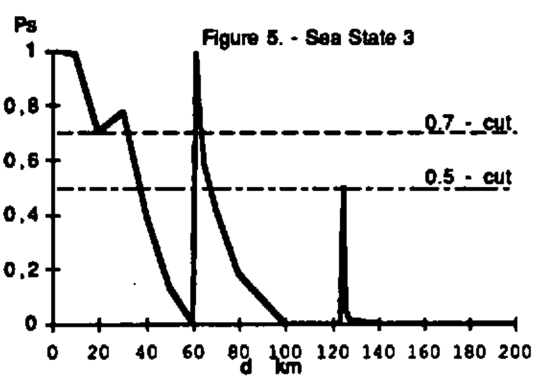

Figure 5. - Sea State 3

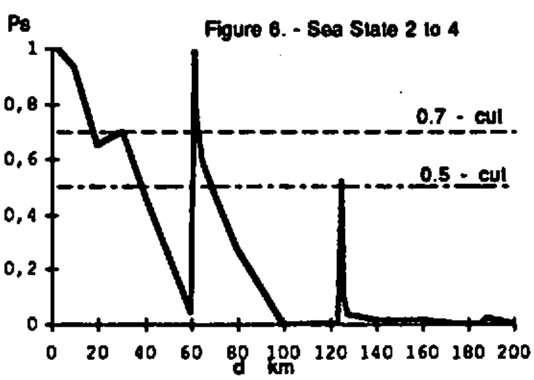

Figure 6. - Sea State 2 to 4

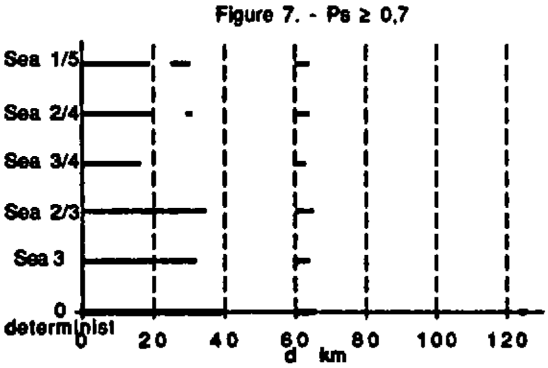

Figure 7. - Ps ≥ 0,7

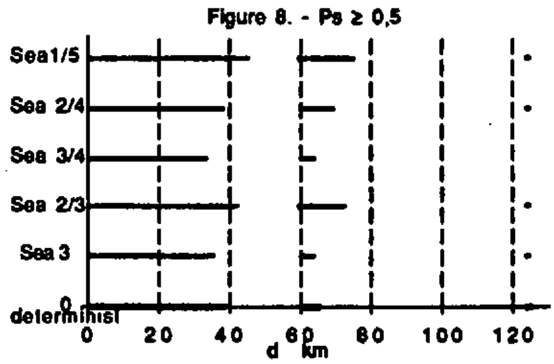

Figure 8. - Ps ≥ 0,5