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OPTIMUM ARRAY PROCESSING FAMILY

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ABSTRACT

We present a family of optimum array processings. It is shown that various types of processing, such as "classical beamforming" or "maximum likelihood (or optimum) beamforming" can be represented by structures generated with a unique optimization model by playing upon the criterion and the constraints or by considering peculiar input processes. Also, are permitted the qualifying and the comparison of the obtained structures.

INTRODUCTION

This study is placed in the frame of spatial array processing, for frequency-direction analysis, with the following characteristics :

- Passive mode : the signals and the eventual jammers are random and carried by plane waves. When necessary,the input functions are stationary in time and space.

- Basic geometry : the antenna is a linear array of equispaced sensors.

- Narrowband output : the system performs a frequency-direction analysis.

- Processing limited to a static point of view.

Such an analysis can be built on various resolution methods, each generating one or several structures. To their performances are connected several general problems such as improvement, qualifying (in the matters of measure and optimality), or comparison.

Some methods are well known : Conventionnal Beamforming Method (C.B.M.), Maximum Likelihood (or Capon Optimum) Method (M.L.M.), Maximum Entropy (or Burg) Method (M.E.M.), Eigenvectors Decomposition (or Pisarenko) Method (E.D.M.). The second one (M.L.M.) results of an optimization procedure, applied to the spatial part of the processing, with a criterion of Capon's type ; the processing can be imagined with a spatial part following or preceding the temporal part. In the same way it can be thought of the similar systems in which the optimization is applied to the temporal part. So, at the reference structure of the M.L.M, correspond other structures by 3 different types of duality. [1,2]

OPTIMUM ARRAY PROCESSING FAMILY

The basic idea of MLM can be generalized by extending the optimization procedure to the temporal part of the structure : it gives a "Capon Bioptimum Method"(B.O.M.) or a "Capon Global Optimum Method"(G.O.M.) according to a separate or to a global mode of temporal optimization extension. [3]

The Capon's conditions of optimization, used in MLM, BOM, GOM, corresponds to a linear structure with a unit gain for the considered output variable and a minimum output mean power.

We show in the following that the various types of processing derived by the four methods CBM, MLM, BOM, GOM, can be represented by structures generated with a unique optimization model : the latter is modulated by the parameters of the criterion, submitted to constraints or considered for peculiar input processes ; its optimization criterion generalizes the Capon's conditions. Thus are permitted the qualifying and the comparison of the derived structures. For example, in table 1, we can found some characteristics of structures generated by the four methods, with reference to points of the model of the next section and to figures (1.a,b,c,). There, we denote : Temporal Fourier Transform : FT;Spatial Fourier Transform : sFT; Classical Beamforming (time delays) : CB; Capon's Optimum Beamforming : COB; Capon's Optimum Spectral Analysis : OSA; Spatial Capon's Optimum Spectral Analysis : sOSA; followed by : ⟶ ; replaced by : ⟷ .

Method	Type of beamformer -analyser	Type of linear structure	Reference structures *	Dual structures ***	Derived structures
CBM	classical	separable	FT ⟶ CB	CB ⟶ FT	CB ⟷ sFT
MLM * *	classical -optimum	separable	FT ⟶ COB	COB ⟶ FT CB ⟶ OSA OSA ⟶ CB	COB ⟷ sOSA CB ⟷ sFT
BOM * *	bi-optimum	separable	OSA ⟶ COB	COB ⟶ OSA	COB ⟷ sOSA
GOM	global -optimum	general	..... unique structure		

Remarks :

* the reference structures have an imposed "narrowband input".

** in MLM and BOM the optimization is built on the input of the optimized sub-structure ; in BOM the second optimization is conditionned by the first.

*** - CBM, MLM (1), BOM : spatial-temporal order duality;
- CBM, MLM (2), BOM : spatial-temporal exchange duality;
- MLM (3) : optimization support exchange duality.

OPTIMIZATION MODEL

Problem

We consider a linear array of n_e equispaced sensors. The sensor spacing is L. The observation is constituted by N_e samples of the complex input n_e-vector $V(t_k)$, with $t_k = t_o + kT$. The sampling period is T. The system is a discrete linear structure which extracts, at time t_k, from the time period $[t_k - (N+1)T, t_k]$ the component at frequency ν in the direction θ ; the variable θ can be replaced by the spatial frequency μ, with $\mu = \nu \cos\theta / c_o$, where c_o is the constant wave speed.

Structure of the processing family

The processing is performed by an imposed linear structure varying with ν and θ ; this structure can be (figures 1-a,b,c) :
- general (without constraint) : it is a $n_e N_e$-filter H varying with ν and θ.
- separable (by constraint) : it is a structure built on a N_e-filter T ("temporal") varying with ν and a n_e-filter S ("spatial") varying with μ (or θ and ν) ; it can be of two types, temporal-spatial and spatial-temporal, leading to an equivalent $n_e N_e$-filter which can be put in the form $H = S \otimes T$ or $H' = T \otimes S$ where H,S,T are row-matrices and $\otimes$ represents the Kronecker product of 2 matrices. X^T and X^+ are the transposed and conjugate-transposed of the matrix X.

Optimization of the processing family

The optimization is global when using a general structure and separate when imposing a separable structure. The criterion dictates to the structure :
- to have a unit gain for the considered variable (ν , θ , (ν , θ)),
- to minimize the output mean power bound to a pseudo-input process represented by a given covariance matrix G called "optimization matrix".

When the optimization matrix equals the covariance matrix of the (true) input process, this criterion reduces to Capon's conditions ; then we say that the optimization matrix is matched (to the input process).

Cancelling constraint

By constraint the optimized structure can cancel the effect, on the selected steering, of a given "set to cancel" of frequency-direction couples (for a general structure), of frequencies (for a temporal sub-structure), of directions (for a spatial sub-structure).

Such a set to cancel can correspond, for example, to intermittent jammers, when the input statistical second-order is difficult to know or to estimate.

This constraint is obtained by adding to the criterion the cancelling condition : the structure must have a zero gain for the given values of the variable to cancel.

Separability (cf[4])
The structure, solution of the optimization model, can be separable

- by constraint ; in this case both the optimization and the cancellation are separated for the two sub-structures.

- by given separability of the optimization matrix : $G = A \otimes B$; the eventual cancellation condition must permit it.

- by consequence of the separability of the input covariance matrix Γ with Capon's conditions ($G = \Gamma = a\, \Gamma' \otimes \Gamma''$) ; the eventual cancellation condition must permit it.

Optimization procedure [4]
We call :

X a complex random n-vector and $\Gamma = E\left\{X \; X^+\right\}$ its covariance matrix ;

δ, $c_1, \ldots, c_m$ different, complex, deterministic n-vectors ;

G a complex, hermitian, strictly positive definite, optimization matrix ;

$c = \left[c_1 \vdots \ldots \vdots c_m\right]$, $P_{cG} = I - c\,(c^+ G^{-1} c)^- c^+ G^{-1}$;

$P_{cG} = I$, if there is no c_j ($m = 0$).

The complex row-matrix h, with order n,

satisfying $\begin{cases} h\,\delta = 1 \\ hc = 0 \\ h : \inf\limits_h h\,G\,h^+ \end{cases}$ is $h(\delta, c, G) = \dfrac{\delta^+ G^{-1} P_{cG}}{\delta^+ G^{-1} P_{cG}\, \delta}$,

with the associated minimum : $p(\delta, c, G) = \inf\limits_h h\,G\,h^+ = \dfrac{1}{\delta^+ G^{-1} P_{cG}\, \delta}$.

The output mean power of h, for X at the input, is

$$p_\Gamma(\delta, c, G) = \frac{\delta^+ G^{-1}\, P_{cG}\, \Gamma\, P^+_{cG}\; G^{-1}\, \delta}{(\delta^+ G^{-1} P_{cG}\, \delta)^2}$$

and $G = \Gamma \;\Rightarrow\; p_\Gamma(\delta, c, \Gamma) = p(\delta, c, \Gamma)$.

Here I = unit matrix,
A^{-1} = inverse matrix of A,
A^- = generalized inverse matrix of A.

RESULTS

Some results are given here for processing without cancellation ; others are presented in [3, 4, 5] .

First, characteristics and performances of some members of the family can be found in tables 2 and 3. They correspond to the following notations :

$V_j(t_k)$: complex input sample in t_k on sensor j ;

$V_k = [V_1(t_k), \ldots, V_{n_e}(t_k)]^T$

$Y_j = [V_j(t_{k-N_e+1}), \ldots, V_j(t_k)]^T$

$U^+ = [Y_1^+ \vdots \ldots \vdots Y_{n_e}^+]$

$U'^+ = [V_{k-N_e+1}^+ \vdots \ldots \vdots V_k^+]$

$\Gamma_V = E\{V_k V_k^+\}$

$\Gamma_Y = E\{Y_j Y_j^+\}$

$\Gamma_U = E\{U\ U^+\}$

$\Gamma'_U = E\{U'\ U'^+\}$

$f = [1, e^{-2\pi i \nu T}, \ldots, e^{-2\pi i \nu (N_e-1)T}]^+$

$d = [1, e^{2\pi i \mu L}, \ldots, e^{2\pi i \mu (n_e-1)L}]^+$

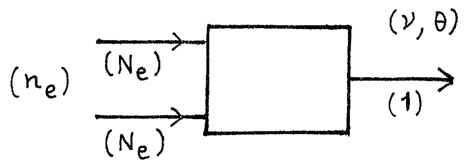

Figure 1 - a

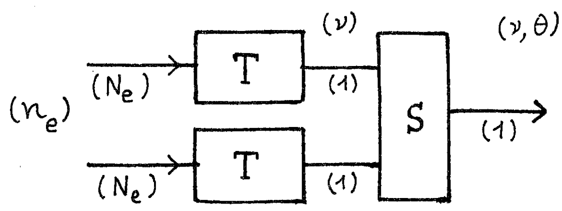

Figure 1 - b

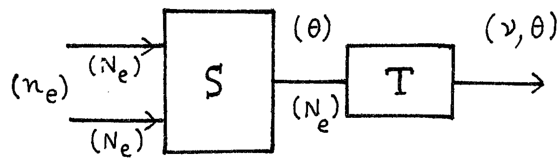

Figure 1 - c

Afterwards, discrimination diagrams of a set of four sources in white noise by four beamformer-analyser (classical, classical-optimum, bi-optimum, global-optimum). They correspond to the following problem :

- number of sensors : $n_e = 5$;
- number of samples : $N_e = 10$;
- 4 sources of signal to noise ratio 8, located in the (ν, μ) - plan according to the chart figure 2 ;
- the level lines of the 4 diagrams are spaced by 0.5 dB. (figure 3).

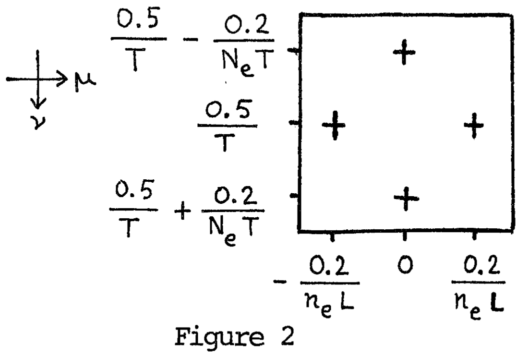

Figure 2

Optimization caracteristics about structure St and optimization matrix OM	Resulting structure for any input process
I St : general OM : matched	1 general structure "global optimum beamformer-analyser"
II St : separable OM : both matched	2 separable structures "bi-optimum beamformer-analyser"
III St : separable OM : one unit matrix and one matched	4 separable structures "classical-optimum beamformer-analyser"
IV St : general OM : imposed and separable ($G_S \otimes G_T$) or St : separable OM : both imposed (G_S, G_T)	1 commutative separable structure
V St : general OM : unit matrix or St : separable OM : both unit matrix	1 commutative separable structure "classical beamformer-analyser"

Table 2

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(4) P.Y. ARQUÈS, "Eléments pour l'étude d'une famille de traitements d'antenne optimaux", à paraître.

(5) P.Y. ARQUÈS et B. LUCAS, "Etude d'une famille de traitements d'antenne pour l'analyse en fréquence-direction", à paraître.

I - $$p = \frac{1}{(d^+ \otimes f^+)\, \Gamma_U^{-1} (d \otimes f)}$$

II - $$p = \frac{1}{d^+ \left[(I \otimes (f^+ \Gamma_Y^{-1}))\ \Gamma_U (I \otimes (\Gamma_Y^{-1} f))\right]^{-1} d} \quad \frac{1}{(f^+ \ \Gamma_Y^{-1} \ f)^2}$$

temporal - spatial

$$p = \frac{1}{(d^+ \Gamma_V^{-1} \ d)^2} \quad \frac{1}{f^+ \left[(I \otimes (d^+ \ \Gamma_V^{-1}))\ \Gamma_U' \ (I \otimes (\Gamma_V^{-1} \ d))\right]^{-1} f}$$

spatial - temporal

III - $$p = \frac{1}{N_e^2} \quad \frac{1}{d^+ \left[(I \otimes f^+)\ \Gamma_U \ (I \otimes f)\right]^{-1} d}$$

temporal - spatial
$G_T = I$
G_S matched

IV - $$p = \frac{((d^+ \ G_S^{-1}) \otimes (f^+ G_T^{-1}))\ \Gamma_U \ ((G_S^{-1} \ d) \otimes (G_T^{-1} \ f))}{(d^+ G_S^{-1} \ d)^2 \ \ (f^+ G_T^{-1} \ f)^2}$$

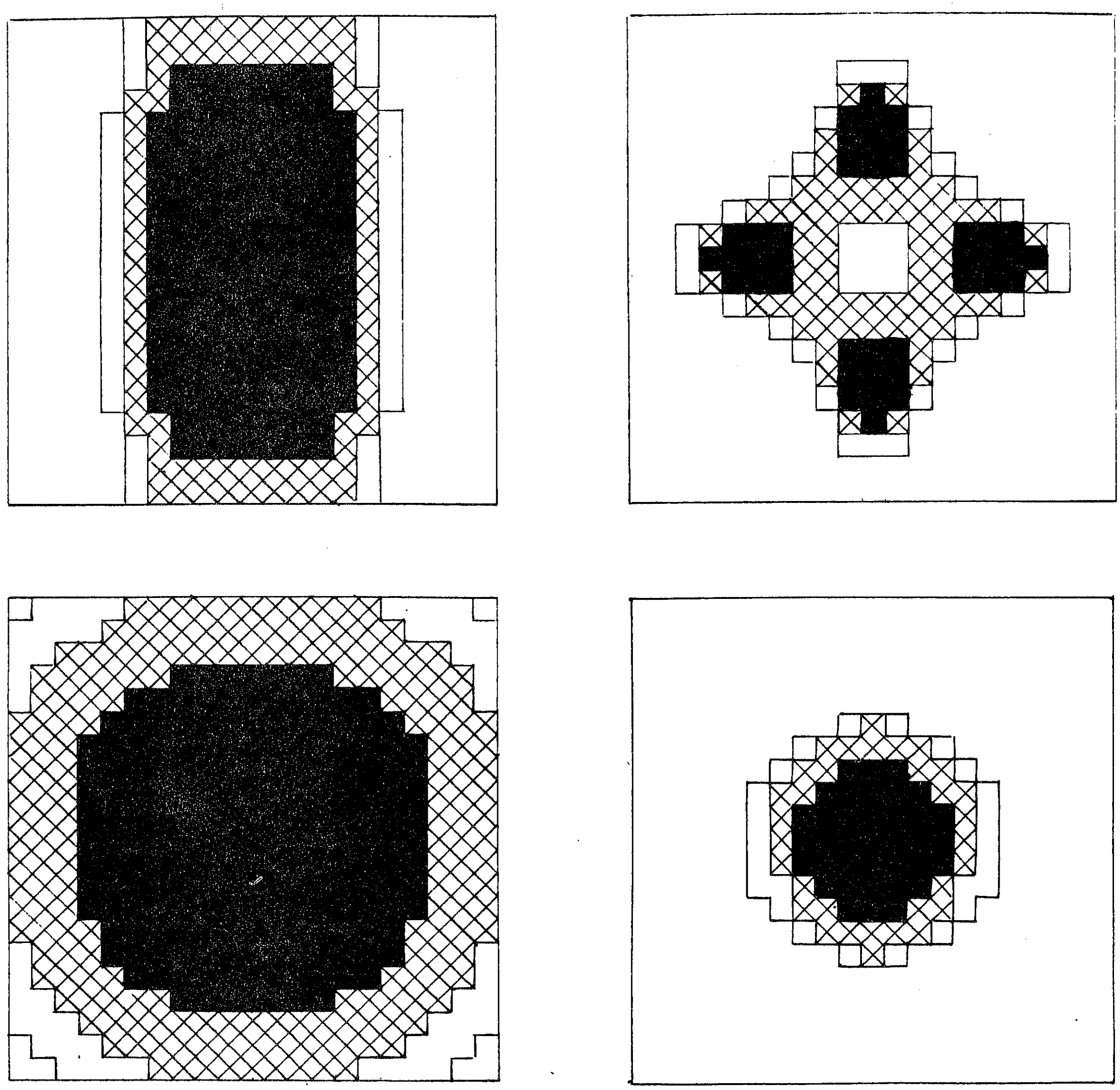
V - $$p = \frac{(d^+ \otimes f^+)\ \Gamma_U (d \otimes f)}{n_e^2 \ N_e^2}$$

Table 3

Remark : In case III there are 3 other analogous formulas.

Classical-optimum

Global-optimum



Classical

Bi-optimum

Figure 3