A NUMERICAL METHOD FOR ACOUSTIC INTERACTION WITH UNDERWATER STRUCTURES BASED ON NEAR-FIELD ARTIFICIAL BOUNDARY

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Recent years, large amount of simulations on acoustic radiation from large complex underwater structures was practiced to find out the mechanisms of structure noise controlling. It has been learnt that for large-scale complex structures, the structural vibration is usually a summation of vibration wave components of different structural wavelengths even if the excited force is harmonic. The far field noise mainly comes from the structural vibration components with relatively long wavelength while short waves only exist in the near field around the structure to satisfy the kinetic continue condition at the structure-fluid interface. Based on this observation, a numerical method, called FE-FE-BE method, is presented in this paper for evaluating acoustic radiation from a fluid-loaded vibrating structure at low frequency. In this method, the structure is modeled with Finite Element method. But the unbounded acoustic domain outside the structure is divided into an interior region and an exterior region by an artificial boundary surface which satisfies the pressure and velocity continuum condition. In the interior region the Finite Element method is adopted for the fluid acoustic simulation, while in the exterior region the Boundary Element method is employed for the unbounded fluid acoustic computation. Numerical results are in good agreement with experiment data. Traditionally structural Finite Element method coupled with fluid Boundary Element method (FE-BE method) is employed for prediction of underwater noise from large-scale structures. However the computation time becomes so long that the application of the FE-BE method is confined. Compared with the traditional FE-BE method, the computation time greatly decreases when the proposed method is adopted.

Keywords: artificial boundary, finite element method, boundary element method, acoustic prediction, ship noise and vibration

1. Introduction

Traditionally, structural Finite Element method coupled with fluid Boundary Element method (called FE-BE method) is usually employed to predict the noise radiated from underwater vibrating structures due to exciting forces acting on the structures [1,2]. However, for large complex structures, such as submarines or ships, even at low frequency the computation time can become so long that the application of the FE-BE method is confined. The reason for this is that the number of boundary elements at the fluid-structure interface becomes extremely large since the boundary element size at the interface has to be conformed [3] to the finite element size on the wet structural surface due to the requirement of satisfying the kinematic continuum condition on the interface.

This is also the reason why the acoustic infinite element method [4, 5] was becoming prosperous in 1990s.

Recent years, large amount of simulations on acoustic radiation from large complex underwater structures was practiced to find out the mechanisms of structure noise controlling. It has been learnt that for large-scale complex structures, the structural vibration is usually a summation of vibration wave components of different structural wavelengths even if the excited force is harmonic. The far field noise mainly comes from the structural vibration components with relatively long wavelength while short waves only exist in the near field around the structure to satisfy the kinematic continuum condition at the structure-fluid interface and to form the evanescent near-field waves. When the structural bending wavelength λ_b on the wet structural surface is smaller than the acoustic wavelength λ in the fluid domain, the evanescent waves [6], decayed exponentially with distance r_s normal to the fluid-structure interface, are formed in the near field around the wet structural surface. Especially, when $\lambda >> \lambda_b$, the evanescent wave decays as $\exp(-2\pi r_s/\lambda_b)$. This means that the short waves only exist in the near field around the wet structural surface. It is preferable to employ fluid Finite Element method to simulate the near field acoustic waves in the interior region Ω_1 as shown in Fig.1, since the Boundary Element method will lead to a huge fully-populated matrix that makes the computation time unacceptable.

Based on the idea that the acoustic short waves only exist in the near field around the structure for the low-frequency structural vibration, and on the idea that the exterior outgoing acoustic wavelength is long at the low frequency, the traditional numerical method, of structural Finite Element coupled with fluid Boundary Element on the structure-fluid interface, is extended to a new numerical method. This method may be called FE-FE-BE method that divides acoustic fluid domain into an interior domain Ω_1, and an exterior domain Ω by the artificial boundary surface Γ_A as shown in Fig. 1. For the dynamic response of the structure (dry and wet), the structural Finite Element is employed to be coupled with the fluid Finite Element in the interior fluid domain Ω_1, and outside the interior domain Ω_1 the fluid Boundary Element is adopted for the simulation of the acoustic radiation in the exterior domain Ω. The coupling effect between the fluid FE in the domain Ω_1 and the fluid BE in the domain Ω is included by satisfying the pressure continuum condition on the artificial boundary surface Γ_A at a proper distance away from the wet structural surface Γ_S. On the artificial boundary surface Γ_A, large element scale is used for Boundary Element (BE) method due to the long acoustic wavelength. Both the number of elements and the computation time are largely reduced. Therefore the acoustical problems of large complex structures are expected to be solved fast comparing with the FE-BE method.

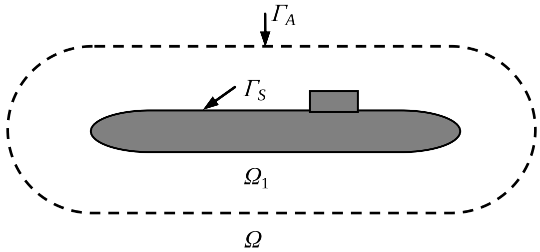

Figure 1: Geometry of the structural acoustic problem of the FE-FE-BE method.

The FE-FE-BE method is different from the traditional FE-BE method in that for the FE-BE method, the BE mesh is partitioned on the wet structural surface Γ_S while for the FE-FE-BE method, the BE mesh is discretized on the artificial boundary surface Γ_A at a certain distance from the wet structural surface.

Structural Finite Element coupled with Infinite Element method (called FE-IE method) [7] is another numerical technique for solving the noise radiation problem due to the vibrations of large underwater structures. The Infinite Element method is a kind of specialized Finite Element method in the sense that it satisfies the Sommerfeld radiation condition on a fairly far boundary surface Γ_X which truncates the infinite exterior domain into a finite exterior domain Ω_t, as shown in Fig. 2.

This truncation domain Ω_t is further divided into an inner subregion Ω_1 and an outer subregion Ω_0 separated by an artificial boundary surface Γ_A.

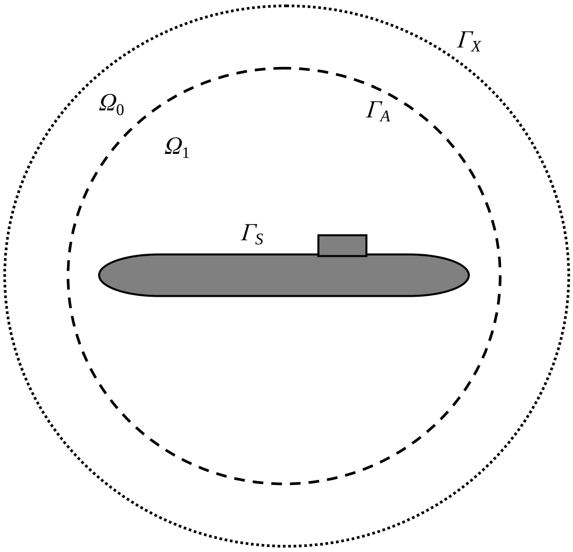

Figure 2: Geometry of the structural acoustic problem of the FE-IE method.

Both FE-IE method and FE-FE-BE method model the interior domain with the Finite Element method. But the FE-IE method models the acoustic field of the finite exterior domain Ω_0 with Infinite Element (IE) and the FE-FE-BE method models the infinite exterior domain Ω by the Boundary Element (BE) on the artificial boundary surface Γ_A. Another difference between the FE-IE method and the FE-FE-BE method is that, for the FE-IE method the artificial boundary surface Γ_A has to be a sphere, or spheroidal, or ellipsoidal surface [4, 5] while for the FE-FE-BE method, the artificial boundary surface Γ_A can be an arbitrary surface surrounding the vibrating structure.

Although some researchers[4, 8] conclude that the traditional FE-BE method is not superior to the FE-IE method in computation time, the new FE-FE-BE method may be comparable with the FE-IE method in time cost and provide a alternative method with exact solution for far-field acoustic prediction of large structures. Furthermore, the FE-FE-BE method has advantage of easily satisfying the boundary condition of the free water surface, or the boundary condition of sea floor reflection through changing the form of Green functions.

Before the new FE-FE-BE method can be used practically, following questions should be answered: (1) the validation of this method by comparing with the FE-BE method and experimental data, (2) determining the position of the artificial boundary surface, (3) comparison of the computation time of the FE-FE-BE with FE-IE method. The validation of the proposed method is discussed in this paper by a comparison between computational results and experimental data. The investigations of questions (2) and (3) will be discussed in further paper.

Due to the limitation of the paper pages, only the main formulations of the FE-FE-BE method are summarized in the following sections. For the derivation of these formulations, readers may refer to the reference 9.

2. BE method for exterior acoustic domain

The motion of a fluid particle is assumed to exist long enough that all transient effects have decayed, the time dependence $\exp(\mathbf{i}\omega t)$ has been suppressed, where $\mathbf{i}=(-1)^{1/2}$, and ω is the circular frequency of excitation. In order to compute the sound field of the exterior fluid region Ω, the artificial boundary surface is discretized into n_A triangular elements. Let $\overline{P}_j$ denote the average sound pressure amplitude of element j on the artificial boundary surface Γ_A. The pressure amplitude at any field point $\mathbf{x}$ in the exterior region Ω and on the artificial boundary surface Γ_A can be expressed as [9]

$$\bar{p}(\mathbf{x}) = -\rho\omega^2 \sum_{j=1}^{n_A} \varphi_j \bar{P}_j \ , \tag{1}$$

where ρ is the fluid density. The normalized pressure function φ_j represents the exterior acoustic field produced by the piston vibration of element j with unit pressure amplitude and with all other elements rigid on the artificial boundary surface. This pressure function can be calculated from [9]

$$\varphi_j(\mathbf{x}_k) = \sum_{t=1}^{n_A} A_{kt}\sigma_{tj}, \qquad A_{kt}(\mathbf{x}_k) = \iint_{S_t} G(\mathbf{x}_k, \mathbf{y}_t)\mathrm{d}S(\mathbf{y}_t) = A_{kt}^c + \mathrm{i}A_{kt}^s \ , \tag{2}$$

where $G(\mathbf{x}_k,\mathbf{y}_t)$ is free space Green function [1] with satisfying the boundary condition of free water surface, $\mathbf{x}_k$ is a viewer point on the element k and $\mathbf{y}_t$ is a source point on the element t, S_t is the surface of the element t. The unknown source strength $\sigma_{tj}(\mathbf{y}_t) = \sigma_{tj}^c + \mathrm{i}\sigma_{tj}^s$ at source point $\mathbf{y}$ on the artificial boundary surface can be obtained by solving the BEM equations [9]

$$\sum_{t=1}^{n_A} \begin{bmatrix} \mathbf{A}_{kt}^c & -\mathbf{A}_{kt}^s \\ \mathbf{A}_{kt}^s & \mathbf{A}_{kt}^c \end{bmatrix} \begin{Bmatrix} \boldsymbol{\sigma}_{tj}^c \\ \boldsymbol{\sigma}_{tj}^s \end{Bmatrix} = \frac{1}{\rho\omega^2} \begin{Bmatrix} \boldsymbol{\delta}_{kj} \\ 0 \end{Bmatrix}, \tag{3}$$

where $\boldsymbol{\delta}_{kj}$ is Kronecker delta function，the element number $k, t, j = 1, 2, 3 \text{L} \ n_A$. It can be seen that the Eq. (3) can be solved independently before the structural response and interior acoustic field are obtained. This solution will provide a radiation condition on the artificial boundary for the acoustic propagation in the interior domain Ω_1. And the unknown pressure amplitude $\bar{P}_j$ can be obtained by solving interior acoustic FE equations coupled with structural FE equations in the following sections.

3. FE method for interior acoustic region

This part states the FE method for the acoustic propagation in the interior domain Ω_1, which relates with the FE method for the structure vibration and relates with the BE method for the acoustic propagation in the exterior domain Ω. The sound pressure amplitude $\bar{p}_1$ at any point in the interior acoustic domain Ω_1 must satisfy Helmholtz equation, and is subjects to the kinematic condition on the wet structural surface Γ_S and the sound pressure continuum condition on the artificial boundary surface Γ_A. The interior fluid region Ω_1 is modeled by n_Ω numbers of tetrahedral elements. The matrix FE equation for the unknown acoustic pressure in the interior acoustic field can be obtained from [9]

$$[\mathbf{K}_1 - \omega^2(\mathbf{M}_1 - \rho\mathbf{A}_a)]\{\bar{\mathbf{p}}_1\} = -\rho\omega^2\mathbf{A}_w\{\bar{\mathbf{u}}\}, \tag{4}$$

where $\mathbf{A}_a = \mathbf{A}'_a \cdot \mathbf{A}_0$ is influence coefficient matrix due to satisfying radiation condition on the artificial boundary Γ_A, and the matrix $\mathbf{A}_0$ is a transformation matrix to transform the nodal sound pressure amplitude vector $\{\bar{\mathbf{p}}_1\}$ in the whole interior fluid region to the nodal sound pressure amplitude vector $\{\bar{\mathbf{p}}_1\}^A$ on the artificial boundary surface, that is $\{\bar{\mathbf{p}}_1\}^A = \mathbf{A}_0\{\bar{\mathbf{p}}_1\}$. The matrix $\mathbf{A}'_a$ can be calculated from the solution σ_{tj} obtained in the previous section.

$$\mathbf{A}'_a = [\iint_{S_k} \mathbf{N}^T \cdot \mathbf{E}\mathrm{d}S], \qquad \mathbf{E} = [-\sum_{t=1}^{n_A} I_{kt}\sigma_{tj}], \qquad I_{kt} = \mathbf{n}_k \cdot \iint_{S_t} \nabla G(\mathbf{x}_k, \mathbf{y}_t)\mathrm{d}S \ , \tag{5}$$

where $\mathbf{N}$ is the element shape function matrix and $\mathbf{N}^T$ is the transposed matrix of $\mathbf{N}$, $\mathbf{n}_k$ is the outward normal (point to the exterior region) of element k on the artificial boundary surface, S_t denotes that the integral is taken on the surface of element t, ∇ is the Laplacian operator, the bracket [ ] indicates the matrix is assembled from sub-matrices in the bracket or from matrix elements in the bracket.

The mass matrix $\mathbf{M}_1$ and the stiffness matrix $\mathbf{K}_1$ in the Eq. (4) are

$$\mathbf{M}_1 = [\frac{1}{c^2}\iiint_{\Omega_m} \mathbf{N}^T \cdot \mathbf{N} \mathrm{d}\Omega], \quad \mathbf{K}_1 = [\iiint_{\Omega_m} \nabla\mathbf{N}^T \cdot \nabla\mathbf{N} \mathrm{d}\Omega], \tag{6}$$

where Ω_m is the volume of the tetrahedral element m in the interior acoustic region.

The coefficient matrix $\mathbf{A}_w = \mathbf{A}'_w \cdot \mathbf{A}_1$ due to the wet structural vibration in the Eq.(4) is related with the wet-structure geometry by the matrix

$$\mathbf{A}'_w = [\iint_{S_j} \mathbf{N}^T \cdot \bar{\mathbf{n}}_j \cdot \mathbf{N} \mathrm{d}S], \tag{7}$$

where S_j denotes that the integral is taken on the surface of the element j on the wet structural surface Γ_S, the $\bar{\mathbf{n}}_j$ is the inward normal (point to dry structures) of element j. And the matrix $\mathbf{A}_1$ is a transformation matrix to transform the nodal displacement amplitude vector of the whole structure to the nodal displacement amplitude vector $\{\bar{\mathbf{u}}\}^w$ of the wet structural surface, that is $\{\bar{\mathbf{u}}\}^w = \mathbf{A}_1\{\bar{\mathbf{u}}\}$. There are two unknowns of vectors, $\{\bar{\mathbf{p}}_1\}$ and $\{\bar{\mathbf{u}}\}$, in the Eq. (4). The solution can only be solved by combining the FE equation in the near-field acoustic domain Ω_1 with the structural FE equation explained in the following section.

4. FE method for structures

The structure is modeled with finite elements, and the dynamic equation of the structure is given by [9]

$$[-\omega^2\mathbf{M} + \mathrm{i}\omega\mathbf{B} + \mathbf{K}]\{\bar{\mathbf{u}}\} = \{\mathbf{F}\} + \mathbf{A}_p\{\bar{\mathbf{p}}_1\}, \tag{8}$$

where **M**, **B** and **K** are the structural mass, damping, stiffness matrix, respectively, $\{\mathbf{F}\}$ is the vector of mechanical forces applied on the structure. $\{\mathbf{F}_1\} = \mathbf{A}_p\{\bar{\mathbf{p}}_1\}$ is the acoustic loading vector on fluid-structure interface exerted by the fluid in the interior region Ω_1, the matrix $\mathbf{A}_p$ is a transform matrix to transform the nodal sound pressure amplitude vector of the whole interior fluid region to the nodal sound pressure amplitude vector of the fluid-structure interface, and to transform the element pressure vector of the fluid-structure interface into the nodal force vector imposed on the wet structural surface.

There are two groups of unknowns in the equation. One is structure vibration $\{\bar{\mathbf{u}}\}$, another is acoustic pressure $\{\bar{\mathbf{p}}_1\}$ in the interior region Ω_1. To solve it, the interior fluid equation (4) has to be included.

5. Computation of structural response and acoustic pressure

Combining the structure dynamic equation (8) with the interior acoustic equation (4) yields fluid-structure coupled equations of the form

$$-\omega^2\begin{bmatrix}\mathbf{M}_1 - \rho\mathbf{A}_a & -\rho\mathbf{A}_w \\ 0 & \mathbf{M}\end{bmatrix}\begin{Bmatrix}\bar{\mathbf{p}}_1 \\ \bar{\mathbf{u}}\end{Bmatrix} + \mathrm{i}\omega\begin{bmatrix}0 & 0 \\ 0 & \mathbf{B}\end{bmatrix}\begin{Bmatrix}\bar{\mathbf{p}}_1 \\ \bar{\mathbf{u}}\end{Bmatrix} + \begin{bmatrix}\mathbf{K}_1 & 0 \\ -\mathbf{A}_p & \mathbf{K}\end{bmatrix}\begin{Bmatrix}\bar{\mathbf{p}}_1 \\ \bar{\mathbf{u}}\end{Bmatrix} = \begin{Bmatrix}0 \\ \mathbf{F}\end{Bmatrix}. \tag{9}$$

The solution of the fluid-coupled FE equation (9) gives the structural dynamic response $\{\bar{\mathbf{u}}\}$, the nodal sound pressure amplitude vector $\{\bar{\mathbf{p}}_1\}$ in the interior domain and the nodal sound pressure amplitude vector $\{\bar{\mathbf{p}}_1\}^A = \mathbf{A}_0\{\bar{\mathbf{p}}_1\}$ on the artificial boundary. Once the solution $\{\bar{\mathbf{p}}_1\}^A$ is obtained, the average sound pressure amplitude $\bar{P}_j$ of element j can be calculated by $\bar{P}_j = \mathbf{D}\{\bar{\mathbf{p}}_1\}_j^A$, where

D=[1/3,1/3,1/3]. The acoustic pressure at any field point **x** in the exterior domain can be computed by equation (1).

6. Numerical results and validation of the method

To validate the FE-FE-BE method, the dynamic response and acoustic radiation from a cylindrical shell with ring-stiffeners is computed, and the sound pressure level (SPL) is compared with the experiment data due to Chen and Schweikert [10]. Fig. 3 shows the dimensions of the cylindrical shell and the experimental installation. The excitation force, $F=\cos(2\pi f\tau)$ pounds, with frequency f=258 Hz is applied to a point on the middle stiffener, as shown in Fig. 3. The material properties of the cylinder are as follows: Young's modulus $E=2.06\times1011\ N\cdot m^{-2}$, Poisson's ratio v=0.3, structure damping ratio 2ξ=0.06 and density $\rho_s=7850\ kg\cdot m^{-3}$. The density of the fluid is $\rho=1000\ kg\cdot m^{-3}$ and the speed of sound in water is $c=1461\ m\cdot s^{-1}$.

The artificial boundary surface is a cylinder surface with length L_1=2.158 m and radius R=1.0795 m, as shown in Fig. 4. The element size on the wet structural surface for FE-FE-BE method is same as FE-BE method, and the size of triangular elements on the artificial boundary surface is about $\lambda/12$, where λ is the acoustic wavelength. The scales of tetrahedral elements in the interior fluid region increase gradually from the wet structural surface to the artificial boundary. Near the artificial boundary surface, the element scales are much larger than the scales near the wet-structure surface. Fig. 5 shows the comparison of numerical SPL with the experiment data and the FE-BE results. It can be seen that both the numerical results of the FE-FE-BE method and that of the FE-BE method agree well with the experiment data. However, the computation time is quite different. Table 1 shows the computation time of the FE-FE-BE method and the computation time of FE-BE method for f=258 Hz. The results indicate that the computation time of the FE-FE-BE method decreases by 94.1% comparing with the conventional FE-BE method.

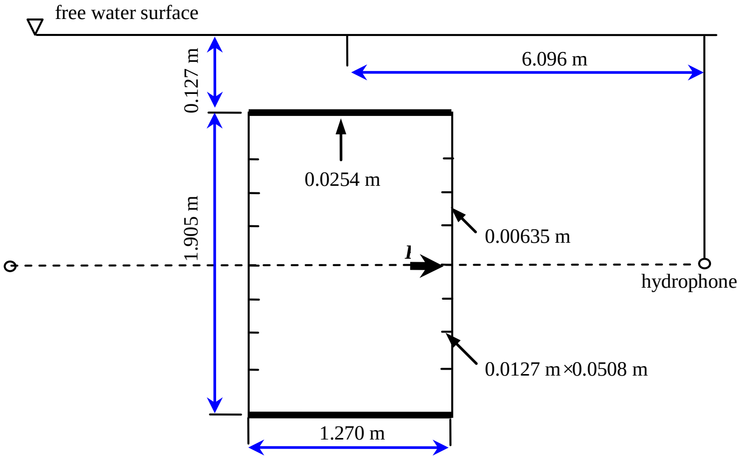

Figure 3: Dimension of the cylindrical shell and experimental installation.

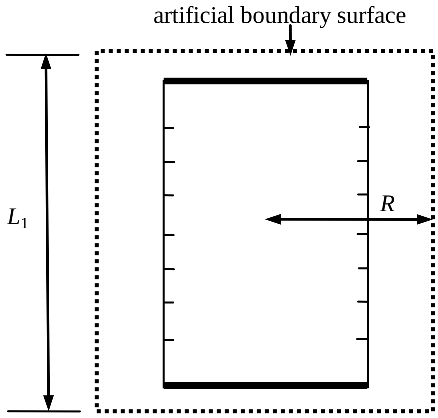

Figure 4: The artificial boundary surface.

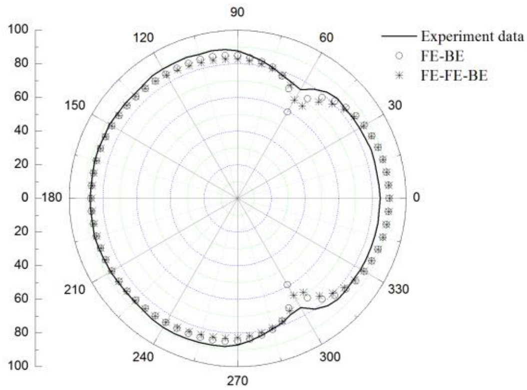

Figure 5: Comparison of numerical results with experiment data.

Table 1: Computation time of the FE-FE-BE method and FE-BE method for f=258 Hz.

Method	Case	Number of finite elements	Number of BE elements	Computation time (s)
FE-BE	Case 2	2788	2452	490
FE-FE-BE	Case 2	8195	212	29

7. Conclusions

A numerical approach called FE-FE-BE method is presented to compute the dynamic response and acoustic radiation from an underwater large complex structure at low frequency. In this method, an unbounded domain is separated into an interior region and an exterior region by an artificial boundary surface. The conventional FE method is employed in the interior region, and the BE method is adopted in the exterior region. The method can be used not only for the computation of the near field acoustic radiation, but also for the computation of the far field acoustic radiation. The technique is validated through the dynamic response and acoustic radiation from an underwater ring-stiffened cylindrical shell. Numerical results are in good agreement with experiment data. Comparing with the conventional FE-BE method, the computation time decreases by 94.1% when FE-FE-BE method is adopted.

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