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Vibration Analysis of Frames Consisting of Non-Uniform Beams

by
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Introduction

In the finite element analysis of structures with tapered members, a commonly used approximation is to represent each tapered member by a number of colinear members each of uniform but different properties. Obviously more accuracy is gained by finer divisions, although care should be taken when each element becomes short when compared with the depth. However, such schemes are wasteful in both computer storage and solution time, and also in data preparation and punching. It is this last factor that leads to much of the frustration in computing, since human errors occur infinitely more frequently than those from the machine!

With the vehicle body in mind, beam elements with linear taper in either (or both) plan and elevation (Fig. 1) have been investigated. The work is directed to specific cross-sections of box, channel and top-hat, so that the practical application is more immediate than schemes in which the properties are described by a power series in the lengthwise co-ordinate[1].

Deviation of Stiffness and Mass Matrix

Whilst the exact solution to the differential equation for a beam tapered as described is possible to achieve, such solutions are both numerically cumbersome and difficult to differentiate and integrate. Thus, the well known technique of minimising the error between an assumed (polynomial, in this case) function and the exact solution is used in this paper for the beam-type modes.

For the extensional and torsional modes, exact expressions are used as they are rather more tractable. The torsional case implies that the cross sectional warping is unconstrained, which, although adequate for the box section, is by no means justified for open sections. Further work is proceeding on this latter point (2).

The displacement functions for beam-type behaviour was that which satisfies the differential equation for an untapered beam, namely:-

$$v = A_1 + A_2x + A_3x^2 + A_4x^3$$
$$w = A_5 + A_6x + A_7x^2 + A_8x^3$$

The stiffness matrix for these modes was obtained by the principal of virtual work, giving

$$[k] = [C]^{-1,t}\{ \int [B]^t [D][B]\, dx \} [C]^{-1}$$

The full expressions for the terms of this matrix is given explicitly by Ali[3] for each of the cross sections.

The mass matrix is derived from the same principle[4] giving:

$$[m] = \rho \int [C]^t \ [C] \quad A \ dx$$

Programme Organisation

The generalised structural data format used in a previously developed finite element programme which specifies geometry, nodal connections, element properties, and constrained degrees of freedom was adopted. Since the assembled stiffness and mass matrices are banded, and symmetrical, only one half of each band is stored as "string" matrices. The familiar and simple technique of applying constraints by zeroing the appropriate row and solumn with unity in the diagonal was not applicable in the dynamic case, and thus the row and column was removed and the matrices repacked. Although more efficient from the point of view of smaller storage requirements, the repacking is comparatively time consuming, particularly when many constraints are applied.

Having obtained the reduced mass and stiffness matrices, the former is inverted and although it still retains its diagonal symmetry, its subsequent postmultiplication by the stiffness matrix yields a non-symmetrical full matrix, which of course, takes up considerable storage space. Fig. 2 shows the organisation of the programme.

Extraction of the eigenvalues and eigenvectors from this so called "dynamic matrix" is achieved by the reduction to the Hessenberg form, followed by the QR algorithm, obtaining the complete set of frequencies and mode shapes. No extra storage areas were required for these operations, since the original stiffness and mass matrices were no longer required.

Results

To compare with "exact" solutions, the problem of the flexural vibrations of tapered cantilevers was used as data. Two exact solutions are available [5,6] one for cantilevers which taper to a point (a situation which cannot be exactly catered for by the present element). The other is for the case of the vanishing point being beyond the end by one quarter of the actual length of the beam. Both cater for beams linearly tapered in depth and untapered or tapered in plan, the untapered planform corresponding to the linearly tapered box section considered in this paper.

In terms of the errors between the exact solution and several different element dimensions of the tapered beam element, the accuracy of the latter is excellent, in the fundamental and first overtone, as can be seen in Fig. 3. Also included in this figure are the results for stepped representations involving uniform beam elements from Ref. 1.

References

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3. Ali, R. "Derivation of Stiffness Matrix for a Tapered Beam Element". Dept. of Transport Technology, Loughborough University of Technology, TT7007, 1970.

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5. Cranch & Adler, "Bending Vibrations of Variable Section Beams". J.App. Mech. 23 p.103 (1956).

6. Wang, H.C. "Generalised hypergeometric function solutions on the transverse vibration of a class of non-uniform beams". J.App. Mech., 34 p.702 (1967).

TAPER BEAMS

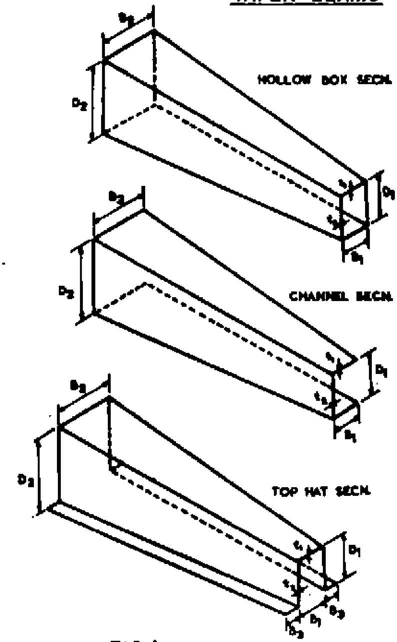
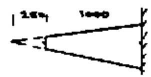
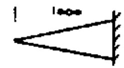
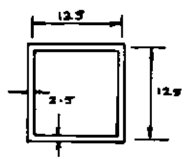


FIG.1.

				
No of ELEMENTS	1st MODE	2nd MODE	1st MODE	2nd MODE
1	5·09	61·5	5·61	211·0
2	1·12 (−16)	1·78 (−18)	1·55	2·89
3	·27 (−7)	·38 (−11)	·51	·52
4	·006 (−6)	·025 (−8)	·005	·181

(Figures in parentheses are for stepped beams[1])



Root Section.

FIG 3. PERCENTAGE ERRORS IN FREQUENCY

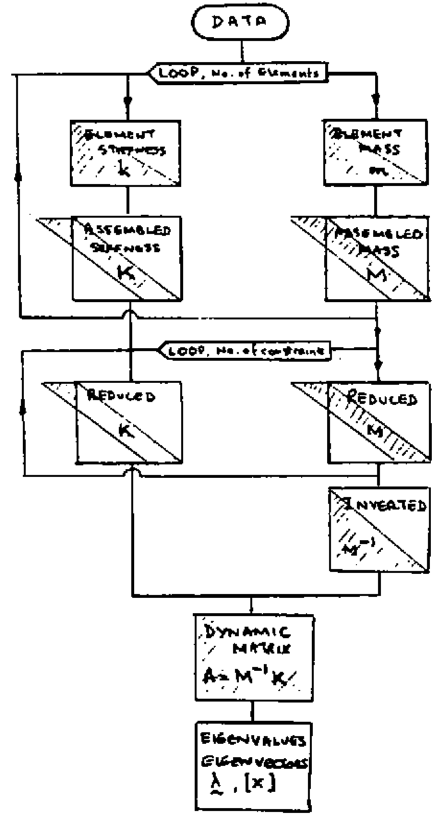


FIG 2 ORGANISATION OF PROGRAMME