COMPUTER DRIVEN METHODS FOR ACOUSTIC TRANSDUCER DESIGN

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Abstract

This paper discusses two aspects of the problem of developing design tools for axi-symmetric structures within the TransCAD acoustic transducer design environment. A brief review of modelling techniques and software is presented and the difficulties of obtaining a purely prescriptive design basis for broadband structures are outlined. The possible use of Transmission Line Matrix modelling, as an alternative to the Finite Element method is discussed. The use of computer optimisation techniques is then considered and the organisation of a powerful, affordable TransCAD workstation architecture is reviewed.

1. Introduction

For operation between 1 kHz and 100 kHz, piston-type axi-symmetric transducers provide the design basis for the majority of practical ultrasound projectors. Transducers such as these are employed in a wide variety of sonar and industrial ultrasound systems. Often, narrowband operation is all that is required and the engineering design of the transducer is then relatively straightforward.

Underwater acoustic system designers naturally seek to make use of signal processing electronics which offers increasing computational power at yet lower cost. The transducer then becomes a bandwidth bottleneck, starving the processor of information. Consequently, a demand is placed upon the designer to provide transducers of greater proportional transmission bandwidth and more uniform transmission response. Unfortunately, design rules then begin to exhibit serious inadequacies. This paper discusses some limitations in the modelling techniques currently used and suggests two possible ways of circumventing this dilemma.

2. Transducer Models

Although a lumped-mass model [1] provides the simplest representation of an axi-symmetric transducer structure it predicts only a single mode of resonance. Experiments with long, passive bar resonators reveal a multitude of resonances. The model may be improved by recognising, in formulating the differential equations which describe behaviour, that the bar resonator has mass distributed along its length. This approach leads ultimately, to the transmission line representation [2], figure 1, which predicts not only the fundamental, or "grave" longitudinal resonance, but also the existence of harmonically related overtone resonances. Elementary theory also predicts that sound-speed in such a resonator is given in terms of ρ, the material density and E, the modulus of elasticity as $c = c_0 = (\rho/E)^{1/2}$. For this model

$$Z_1 = j\sigma A \tan(kL/2) \qquad Z_2 = -j\sigma A/\sin(kL) \qquad Z_L = \sigma_w A\{R + jX\}$$

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where $R = 1 - 2J_1(2ka)/2ka$ and $X = 2S_1(ka)/4k^2a^2$. Here, L is the transmission line section length, A its cross sectional area and $\sigma = \rho c$ its specific acoustic impedance. $k = 2\pi f/c$ is wavenumber. J_1 and S_1 are first-order Bessel and Struve functions, respectively. Finally $\sigma_w = 1.5 \times 10^6$ is the specific acoustic impedance of the water load.

Experiments with long bar resonators [3] reveal that, at high values of the ratio of radius, r, to wavelength, λ, (which is to say, at high overtone frequencies) the overtones are in fact not perfect harmonics of the grave resonance frequency but exhibit a decreasing frequency separation equivalent to a pronounced drop in sound speed in the resonator material. Over a century ago, Pochhammer [4] and Chree [5] predicted that such an effect would govern propagation in bars of infinite length.

An exact theoretical treatment for finite-length resonators and, most particularly, for resonators of complex mechanical structure (such as sonar transducers) is confounded by the difficulty of establishing meaningful boundary conditions for the solution of the relevant differential equations which are, of themselves, exceedingly complex. However, application of the Pochhammer-Chree sound speed correction to transmission line models at least permits a significant improvement in modelling accuracy for structures wherein sound propagation occurs at relatively large values of r/λ. In this context, the empirical curve fit

$$c = c_0 (1 + 7.46 (r/\lambda)^3)/(1 + 13 (r/\lambda)^3)$$

shown in figure 2 is good to within 7% for values of $r/\lambda \sim 1$ and is asymptotically exact for $r/\lambda \to 0$ or ∞.

It is a straightforward matter to extend the theory of transmission line models of bar structures to include hollow bars, as well as solid and hollow cones [6]. Furthermore, piezoelectric driving elements may also be modelled as transmission lines [7]. Using such techniques allows quite detailed models of acoustic transducers to be assembled.

3. Transmission Line Modelling Software

Although transmission line methods are well-known and widely used in transducer design, well-crafted software specifically intended for ease of use is not widely available. The author is aware of three examples of such software: SEADUCER [8], Astre [9] and TransCAD [10]. All three approaches allow modelling of sonar transducers constructed of axi-symmetric solid and hollow bar elements, with disc, ring or tube piezoelectric ceramic drive elements.

In structures which are in some sense "squat", such as the radiating head of a Tonpiltz sonar transducer, yet other non-longitudinal resonances will be observed which often correspond to flexure-mode or "flapping" behaviour. SEADUCER uses a numerical approximation (not a transmission line model) which purports to include head-flexure.

Of the three approaches, only TransCAD builds in the Pochhammer sound-speed correction. TransCAD also offers a sophisticated Graphics User Interface (GUI) which allows complex on-screen modifications to the engineering design of a transducer to be made with great ease.

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4. Spatially Discrete Models

Although transmission-line modelling software can provide valuable insight into transducer design possibilities, it has inevitable shortcomings when the design of squat or broadband transducers is contemplated. This is because other, non-longitudinal, resonances will intrude upon the design process. In order to circumvent this problem, it is customary to invoke the use of the Finite Element method [11]. Two major, partially customised FE modelling suites which include the ability to include piezoelectric elements and which are in some sense commercially available, are PAFEC [12] and ATILA [13].

FE modelling is one of three classes of "spatially discrete" simulation techniques which could be brought to bear upon this problem and is, without doubt, the most widely used. The other two are the Finite Difference (FD) [14] and Transmission Line Matrix (TLM) [15], [16] methods. Of these latter techniques, the FD approach has been used infrequently and, so far as the author is aware, never within the context of a generalised design package.

TLM, it would appear, has never been used for modelling acoustic transducers. However, it may offer some interesting possibilities in providing an alternative to the FE method. Recent studies conducted in the author's laboratory lead to the conclusion that a correct formulation for a two-dimensional TLM model describing acoustic propagation in the solid may at last have been achieved.

5. Transmission Line Matrix Modelling Applied to Transducer Design

Transmission Line Matrix (TLM) modelling is a relatively new numerical method for obtaining the solution to partial differential equations [15]. TLM has several potential advantages over competing techniques. For example, it is particularly easy to include nonlinear effects. TLM is also inherently stable, whereas Finite Difference and Finite Element methods may not be. Finally TLM often offers attractive physical insights into the processes being modelled.

Although no use has been made of TLM in transducer design, there have been numerous other applications in the recent past, including: electromagnetic compatibility [17], radar cross-section analysis [18], nonlinear modelling of mechanical structures [19] and heat diffusion in semiconductor charge transport models [20]. This last example highlights the use of TLM in "coupled models", wherein one physical attribute, such as heat build-up, may affect another, such as electron diffusion. Similar parameter cross-coupling occurs between the thermal and acoustical behaviour of sonar transducers.

Studies in progress in the authors' laboratory seek to establish the viability of a linked plane-wave and TLM model, with the TLM mesh providing, for example, a representation of the head of a Tonpiltz-type transducer. For axi-symmetric structures, which account for the greater proportion of acoustic transducers of this type, a two-dimensional mesh model must be interfaced to the one-dimensional plane-wave transmission-line model which constitutes the remainder of the structure, as shown in figure 3. The presumption is, of course, that it is only the non-longitudinal head-flap modes which will be excited at frequencies comparable with the overall longitudinal resonance of the entire structure. Proceeding in this manner we seek to develop models which will offer accurate solutions to the problem of coupled longitudinal and head-flap resonances, whilst retaining the high computational speeds of the plane-wave and TLM models, by comparison with the FE approach.

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One consequence of the modelling of a Tonpiltz head using the TLM method is that, given a rectangular mesh, both mass and stiffness increase with increasing radial distance from the longitudinal axis (the axis of rotational symmetry) of the structure. This means that the effective line parameters for the two-dimensional transmission line elements must increase with outwards radial displacement. An alternative approach involves employing orthogonal but non-rectangular meshes [20].

6. Theoretical Basis of Transmission Line Matrix Modelling

TLM modelling purports to provide a solution to the Wave Equation in one of its several forms, some of which are illustrated in figure 4, together with examples of application. The objective in TLM modelling is to produce an electrical network which contains, amongst other (possibly nonlinear) components, 1, 2 or 3-dimensional nets of transmission lines as space-discretising elements. The line electrical terminations determine boundary conditions. The transmission line parameters L, C and R may, themselves, relate nonlinearly both to operating frequency and to the spatial dimensions of the object being modelled. An impulsive excitation is impressed on the TLM mesh at some appropriate source location. Analysis is then performed upon the time-discretised impulse response at any other location of interest within the mesh.

In two dimensions, the region of interest is quantised into a square (Cartesian coordinate) mesh overlain in both the x-direction and the y-direction by parallel transmission lines. Node-point inter-connections linking the x-lines and y-lines consist of capacitive stubs which model the effect of Poisson's ratio. The distance between adjacent nodes is defined as Δl and the time taken for a simulated impulse to travel between nodes on any one line, Δt, is a function of the propagation velocity $c = (LC)^{-1/2}$ in the medium and $\Delta\lambda$, so that

$$\Delta t = \Delta\lambda / c$$

The model operates by allowing ideal impulses to propagate along the lines and, due to the simulated discontinuity at the nodes, results in transmitted and reflected impulses being scattered back into the lines. These scattered impulses then become incident on adjoining nodes at the next time instant. The weighted sum of impulses incident on a node provides a solution for the acoustic pressure at that point in space and time. The computation at each node, within each iteration, is defined by the following equations

$$_{k+1}V_n^r = \frac{1}{2}\left[\sum_{m=1}^{4} {}_kV_m^i\right] - {}_kV_n^i$$

where

$$_{k+1}V_1^i\,(z, x) = {}_{k+1}V_3^r(z, x-1)$$

$$_{k+1}V_2^i\,(z, x) = {}_{k+1}V_4^r(z-1, x)$$

$$_{k+1}V_3^i\,(z, x) = {}_{k+1}V_1^r(z, x+1)$$

$$_{k+1}V_4^i\,(z, x) = {}_{k+1}V_2^r(z+1, x)$$

and ${}_kV_n{}^i$ is the incident impulse at timestep k on line n.

The state of the mesh at time $(k+1)\Delta t$ can be calculated from a knowledge of the magnitudes and positions of all impulses at time $k\Delta t$ by employing these equations. It should be noted that the computations required at each node, during each iteration may be performed in parallel. Mutual passing of data between nodes is only necessary at the end of each iteration. This means that TLM computations would appear to be ideally suited to a parallel architecture. The author and a colleague have investigated the use of a MEIKO surface for propagation modelling using the TLM method [21].

The accuracy of a solution obtained from a TLM model is subject to a number of approximations and limitations, (the majority are of no interest in the present context, see [22] for further details), but, when considering memory and execution time requirements, it is important to be aware of the number of nodes and iterations necessary for a given physical situation. It has been empirically discovered that the node spacing, $\Delta\lambda$, must be equal to or less than 0.1λ, where λ is the wavelength of the highest acoustic frequency of interest. The number of iterations necessary for a solution of given accuracy depends on the particular problem in hand. In general it is the case that the more iterations, the greater will be the accuracy of the final solution. Ultimately, the criterion of performance, in comparing TLM and FE, is the time required to attain given accuracy of response. As yet, no clear perspective emerges to illuminate this particular issue.

7. Prescriptive Design

One of the curious mythologies associated with sonar transducer design is that it is first and foremost, a "black art". It is interesting to reflect that there are no truly prescriptive texts on transducer design. That is, there are no identifiable rules which state "do this and such-and-such will happen". Why, we may ask, should this be so? And what may we do to rectify the matter?

We have already considered the problem of developing differential equations which provide an exact solution for vibration in axi-symmetric structures. Even the evolution of such equations is a matter of immense difficulty. When it comes to the problem of specifying conditions at interfaces between adjoining sections of a complex radiating structure, the exact solution becomes intractable.

Consider instead the possibility of developing empirical design rules. The first difficulty is that if we do succeed in making a reasonably good transducer, and then seek dimensional scalings which might allow us to translate a good response at one frequency into an equally good, scaled response at another frequency, we find that the complex loading of one part of the structure on another, and the intrinsically covert and non-linear behaviour of the translation results, often, in an unusably degraded transmission characteristic.

Put in another way, if we observe a particular resonance, one of several in a broadband transducer characteristic, it is not possible to find a single dimensional or material attribute of the actual device which controls only that resonance. It is this feature which has made the development of prescriptive design rules impossible for all but the most extreme of transducer structures. In this context "extreme" means long and thin, as in the case of, for example, of ultrasonic machining and welding heads, or short and wide, as in the case of "thin-disc" designs, after the manner of Goll [23] and DeSillets [24].

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8. Computer Optimisation

One solution to the problem of a lack of prescriptive design rules is to employ computer optimisation methods. This strategy has been proposed by McCammon and Thomson [25] and also by Thjissen [26]. Both have suggested that this be done using transmission line models. As we have seen, little success is likely to attend such a venture, if we consider success to equate to being able to pass to the machinist, an engineering drawing which will result, without further modification, in a working prototype which meets its initial specification.

That is not to suggest, however, that the computer optimisation method may not be of value. Nor indeed, the use of transmission line models. Perhaps the best approach to utilising computer optimisation would be to invoke the use of, for example, transmission and polar response templates, much as the communication engineer might employ filter templates.

The procedure might then be for the acoustical engineer to provide the optimisation programme with a "good starting structure" based, perhaps on past experience. The engineer would also provide "fit within" template responses, much as is shown in figure 5. The optimisation might then use a (fast) transmission line model to coarse-tune the design, handing over to a (slow) FE model to complete the design.

Within the current TransCAD environment this presents some problems. For certain strategic reasons pertaining to project resources, TransCAD was developed in interpreted Basic on a RISC Acorn Archimedes computer. At the time development commenced, in 1987, the Archimedes provided an excellent compromise between speed, RAM-size and cost. By way of example, using a transmission line model as the basis of the simulation, TransCAD computes a 100 point transmission frequency response for an 8 section transducer model in approximately 12 seconds. This is acceptably fast for normal human intervention with the screen. Interesting work can be done, without the feeling that the machine is overly slow in response, thereby inhibiting creativity.

If the optimisation route were implemented, then if (as seems probable) an optimisation would require at least 100 relaxations, the computation time to obtain a solution would be at least 20 minutes. This also might well be acceptable. However, handing over to the FE model to home in on a design capable of immediate fabrication could well result in this time increasing by one, or more probably two orders of magnitude. That is, the optimisation could well be expected to take between one and ten days. Clearly, a more powerful processor would be desirable.

Towards the end of the development of the first version of TransCAD, a European Community (EC) MAST (Marine Science and Technology) Directorate research grant, held jointly between the University of Birmingham and Thomson Sintra Activites Sous Marin, Valbonne, France, was obtained. This grant was to enable the further development of TransCAD including the introduction of FE code. Certain EC caveats were placed on the software development. The software was required to run under either the DOS or the UNIX operating system being written, for preference, in the C language.

Among the objectives is carrying out the new research were: the relocation of TransCAD on a PC and, later, the enhancing of the PC by means of some more powerful second processor, capable of taking advantage of the open-architecture of this type of machine. At first it was thought that the Transputer would provide such enhancement. A significantly more economical but (per motherboard) equally powerful solution to this problem was found in the Intel i860 processor. Table 1 clearly indicates the relative power of these various options.

Figure 6 illustrates the gross architectural disposition of tasks between the Graphics User Interface (GUI) which is being rewritten using Windows 3.0 and the scientific core-code, written in C or (in the case of the FE code) FORTRAN. In an unenhanced 486 PC, the core code would simply run sequentially with the GUI code. The objective in installing the i860 processor is essentially to have it appear as a very fast, hardware "subroutine". One of the major initial problems in implementing the accelerated version of TransCAD has to do with ensuring that the i860 (and this is a problem it shares with a "transputer" solution) does not force the 486 to become a "dumb terminal". The 486 is, of itself, a machine of more than adequate power to support the GUI. Furthermore, Windows 3.0 is supplied in object code and cannot be run under the non-DOS i860 operating system. The sole purpose of using the i860 is thus as a "number cruncher" for the scientific core code.

9. Conclusions

This paper has described current developments of the hardware and software involved in extending the TransCAD design environment for acoustic transducers. Of particular interest have been the possibilites inherent in the use of the TLM approach as a possible alternative to FE modelling of transducer behaviour. It is believed that the first valid TLM model for acoustic propagation in the solid structure has been identified and it is intended that this topic will be pursued in the near future in order to ascertain the value of TLM, by comparison with the FE method.

The paper has also discussed the use of optimisation in developing transducer structures and recognises this as a two-part process. A first, crude optimisation using the Transmission Line modelling approach would hope to obtain, rapidly, a good starting point for a slower but more accurate optimisation based upon the FE method. The paper discusses possible, affordable computer architectures which might enable such optimisations to be conducted within a realistic time-frame. It is suggested that one particularly interesting approach involves utilising an i860 second processor to provide a substantial improvement in data throughput, when installed in a 386/486 PC.

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Table 1

Version	Machine	Language	Benchmark	Time to Optimise
TransCAD 1.0	Acorn Archimedes	Interpreted Basic	100 s	1 - 10 days
TransCAD 2.0	Elonex 486 PC	Compiled C	1.5 s	20 min - 3 hr
TransCAD 2.1	486 + i860	Compiled C	0.28 s	4 - 40 min

Notes: Benchmark consisted of 10^5 complex multiply-adds; Time to Optimise is a best guess based on between 100 and 1000 relaxations to attain the required result assuming the FE method to be two orders of magnitude slower than the TL method.

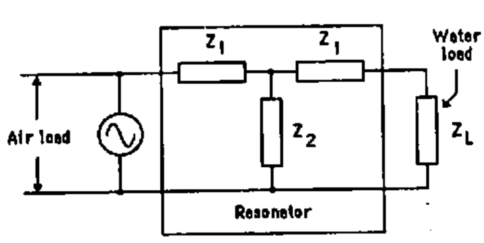

Figure 1. The bar resonator represented as an elementary transmission line segment

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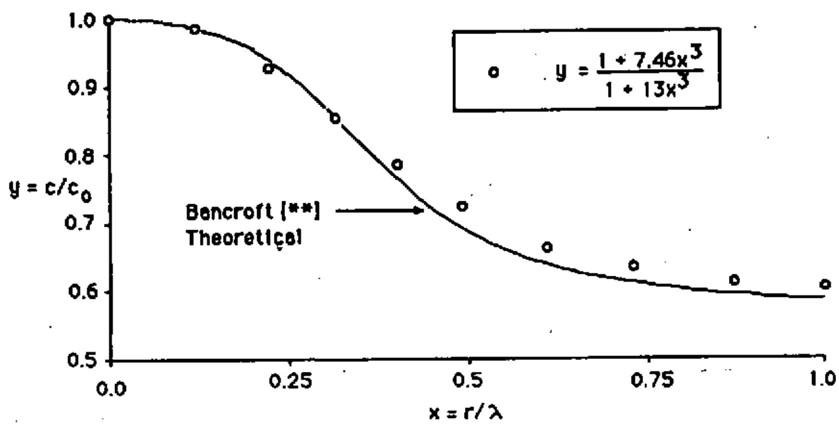

Figure 2. The Pochhammer-Chree sound speed variability curve fit

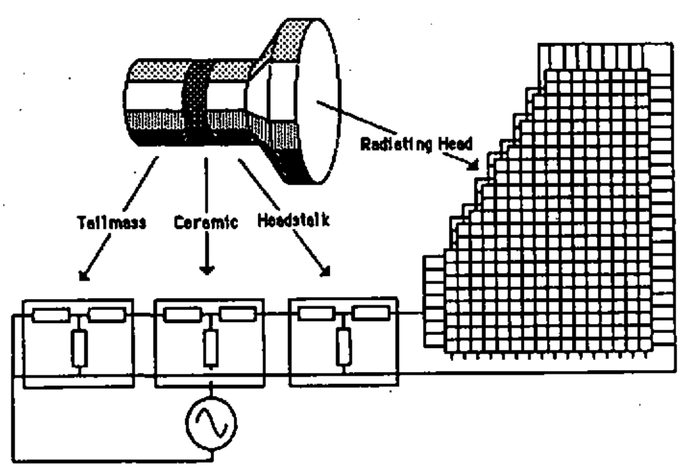

Figure 3. Use of the TLM method in modelling head resonances

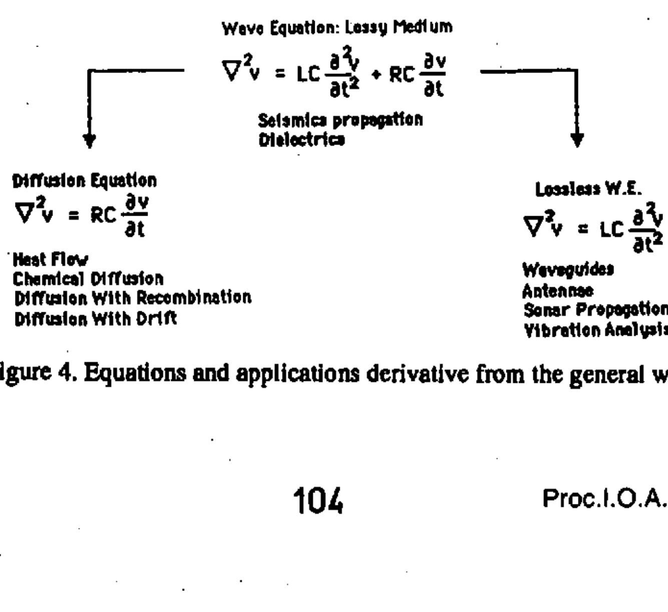

Figure 4. Equations and applications derivative from the general wave equation

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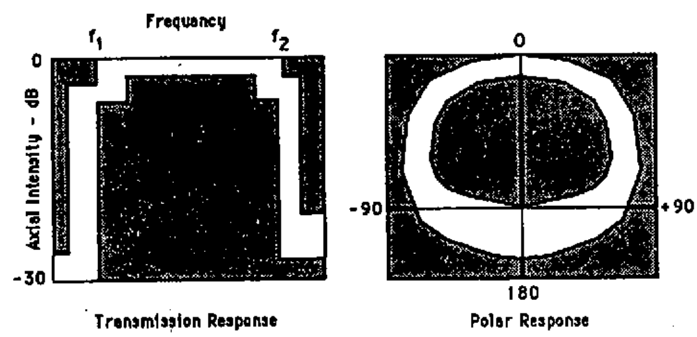

Figure 5. Transmission and polar resonse templates for transducer optimisation

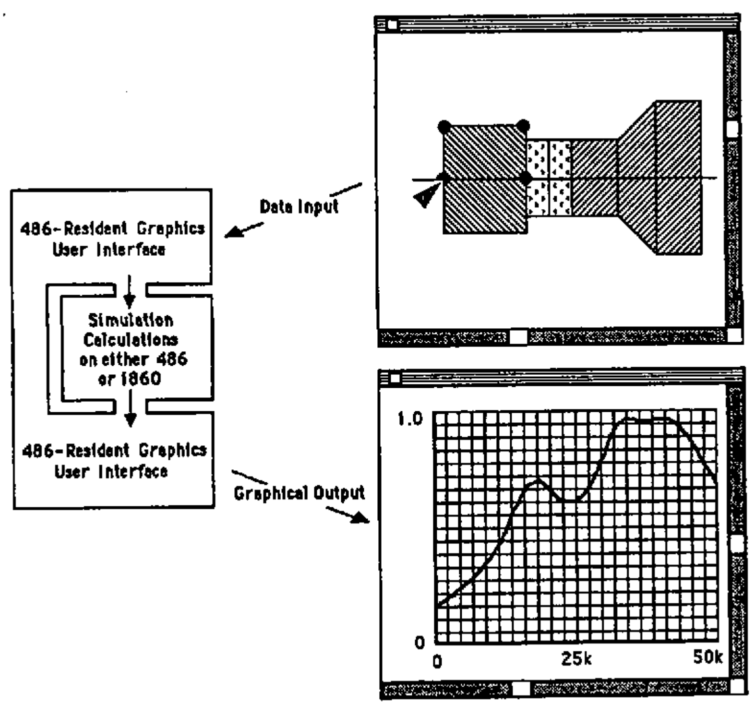

Figure 6. Architecture of the proposed TransCAD 2.0 design environment