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VIBRATIONS: SESSION B: VIBRATION IN TRANSPORT VEHICLES

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ON THE VIBRATIONS OF A BEAM FLOATING IN AN IDEAL FLUID

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Introduction

In analysis of the structural dynamics of a ship as a free free beam, it is common practice to employ "strip theory". This accounts for energy transfer between a vibrating ship and surrounding water by first assuming the flow past hull cross-sections to be two-dimensional, hence obtaining an added mass corresponding to the section. The added mass of every section is then modified by a reduction factor to take account of the three-dimensionality of the flow. This factor J_r is associated with an assumed r^{th} vibration mode, and it is based on the three-dimensional flow around a vibrating spheroid of the same length/breadth ratio as the ship. This paper presents a modal analysis for prolate spheroids, and describes the relation between J_r and the mode shape.

Clearly the modes depend upon the degree of material non-uniformity of the spheroid. Previous investigations [1-4] have given results for mode shapes approximating the behaviour of spheroids only of uniform density and elasticity, and have neglected the influence of added stiffness due to buoyancy effects on floating bodies. The latter might lead to significant distortion in the lowest modes in the vertical plane (conventionally termed heave and pitch) of relatively flexible ships such as Great Lakers; but the phenomenon appears to have been disregarded until recently [5,6].

Uniform and non-uniform spheroids, floating and submerged, are analysed here by the finite element method, and results are compared with those of previous analyses. A consistent added mass matrix for this element is derived from the exact fluid potential theory, and a consistent added stiffness matrix is obtained for buoyancy forces.

Finite element mass and stiffness matrices

The derivation of these matrices is in principle similar to that given by Gallagher and Lee [7]; however the class of uniform elements considered by those authors does not include the spheroidal element.

The beam is divided into N elements by planes perpendicular to its axis. It is assumed that the vertical displacement w within the n^{th} beam element may be expressed in terms of the displacements and rotations at both ends. The assumed displacement function is exact for a uniform beam in static equilibrium under the action of end shears and moments. This leads to element mass and stiffness matrices expressed as integrals over the element length, and it is relatively straightforward to perform these integrations in closed form for each spheroidal element. The overall mass and stiffness matrices, $\underset{\sim}{M}$ and $\underset{\sim}{K}$ respectively, for the assembly of elements that comprise the total spheroid may then be found.

Added mass and added stiffness matrices

In evaluating the added mass matrix it is assumed that the flow is incompressible and inviscid. A velocity potential ϕ may then be used, satisfying a Neumann boundary condition on the surface S of the spheroid. The appropriate solution to Laplace's equation for a deeply submerged spheroid undergoing flexural vibrations at frequency ω is:

$$\phi = \sum_{p=1}^{\infty} c_p T_p{}^1(\mu) U_p{}^1(\nu) \cos\theta \sin\omega t$$

using a curvilinear coordinate system (μ,θ,ν) on S. $T_p{}^1$ and $U_p{}^1$ are respectively associated Legendre functions of the first and second kind. The constants c_p are obtained from the boundary condition on S, using the orthogonality of the functions $T_p{}^1$.

The kinetic energy T_f of the ideal fluid of density ρ_f surrounding the vibrating body is given by

$$T_f = \tfrac{1}{2}\rho_f \int_S \phi \frac{\partial\phi}{\partial n} dS$$

In order to use this expression to obtain a consistent added mass matrix $\underset{\sim}{m}$ for the spheroid, the series form of ϕ must be truncated at some term $p = P$, where P is finite but as large as desired to obtain satisfactory convergence. Then using the assumed displacement of the surface of the n^{th} element, the fluid kinetic energy may be written in the form

$$T_f = \tfrac{1}{2}\omega^2 \underset{\sim}{q}' \underset{\sim}{m} \underset{\sim}{q}$$

where $\underset{\sim}{q}$ is the vector of all nodal generalised displacements.

Evaluation of $\underset{\sim}{m}$ is achieved through use of certain recurrence relations for associated Legendre functions. Each term involves the integral of a polynomial, which may in theory be found in closed form. Because of the large number of terms in each polynomial when P is large, however, it is convenient to compute $\underset{\sim}{m}$ by Gaussian quadrature. The sixteen point quadrature used here is exact for polynomials up to degree 31, hence up to terms given by $P = 29$. The resulting matrix $\underset{\sim}{m}$ is consistent with the mass matrix $\underset{\sim}{M}$, and the two may be added to yield the "virtual" mass matrix for a submerged spheroid. For a floating spheroid the added mass matrix is $\tfrac{1}{2}\underset{\sim}{m}$, for the frequency range of interest here in which wave generation at the free surface may be neglected.

Derivation of the added stiffness matrix for a floating spheroid requires consideration of hydrostatic restoring forces, associated with displacement from a position of equilibrium with the axis of the spheroid lying in the free surface. At a section where the waterline beam is $2y$, the work done in a displacement w is $\tfrac{1}{2}(w^2)(2\rho_f g y)$, and for the n^{th} element the work done is the integral of this quantity over the element length. This leads to the added stiffness matrix for the spheroidal element. The integrals in this matrix may not however be evaluated in closed form, and sixteen point

Gaussian quadrature is again used. Thus the overall added stiffness matrix $\underset{\sim}{k}$ for the assembly of elements may be found, and a "virtual" stiffness matrix obtained for a floating spheroid.

Eigenvalue analysis and results

Free flexural vibrations of the spheroid are governed by the equations

$$(\underset{\sim}{K}+\underset{\sim}{k})\underset{\sim}{q} - \omega^2(\underset{\sim}{M}+\underset{\sim}{m})\underset{\sim}{q} = 0$$

From these the natural frequencies and characteristic modes are found, and reduction factors J_r^* evaluated. (J_r^* is here defined as the ratio of [kinetic energy of fluid in 3D motion for a mode shape calculated assuming 3D motion] to [kinetic energy of fluid in 2D motion, for the same mode shape].) The accuracy of an N element idealisation and a hydrodynamic solution up to P terms is then investigated by convergence studies.

Some results for a deeply submerged spheroid are given in Table I. From these it is concluded that an idealisation of four equal length elements, with eight terms in the series of Legendre functions, gives very accurate values up to the three node flexural mode (i.e. $r = 2$; $r = -1$ and $r = 0$ correspond to the rigid modes of a deeply submerged body, heave and pitch respectively, and $r = 1$ to the two node flexural mode).

Table II and Fig. 1 give some of the results showing the influence of mode shape on J_r^*, and illustrating distortion in the lowest mode of floating non-uniform spheroids. Also shown are the values of J_r obtained by Lockwood Taylor [2] using assumed modes which approximate the characteristic modes of uniform spheroids. The dimensions of the spheroids were selected to provide an analogy with the gross dimensions of a large tanker or Great Laker. The two values of elastic modulus were chosen to give two node natural frequencies of the floating spheroid slightly higher and slightly lower than those of a typical 200,000 dwt tanker. The amount of distortion in the so-called "rigid" mode is surprising. Both for the spheroid and for a real ship this will depend significantly on the distribution of mass. The latter also influences the natural frequencies of the lowest modes of the floating spheroid, as indicated in Table II.

Conclusions

Among the conclusions suggested by the results are the following: (i) Three-dimensional reduction factors depend on characteristic mode shapes which vary with structural properties and buoyancy; accurate calculation of these factors for real ships is probably not worthwhile, since it would be more profitable to perform a modal analysis along the lines indicated here. (ii) The lowest mode of motion of a ship in the vertical plane may theoretically be pitch or heave, depending on waterline and mass distribution. (iii) The lowest two modes of relatively flexible ships, such as tankers and Great Lakers, may entail significant distortion of the neutral axis.

Acknowledgement

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References

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Table I
Convergence study of 10 × 1 free-free spheroid deeply submerged

Number of Elements (N)	Number of Terms (P)	ω_1 (cpm)	ω_2 (cpm)	J_1*	J_2*
2	8	29.0225	68.5939	.82852	.76516
4	5	28.9813	66.6810	.82882	.76382
4	8	28.9813	66.5307	.82882	.77183
4	12	28.9813	66.5304	.82882	.77185
8	5	28.9753	66.5446	.82886	.76372
8	8	28.9753	66.3833	.82886	.77227

Table II
Frequencies and 3D reduction factors for 8 × 1 spheroids

Case		1	2	3	4
Density kg/m^3		1000	500	500	End 1050 Centre 250
Modulus E MN/m^2		1000	1000	200	1000
Configuration		Submerged	Floating	Floating	Floating
Frequencies (cpm)	ω_{-1}	0.0	5.87	5.84	5.86
	ω_0	0.0	6.72	6.71	5.76
	ω_1	23.00	33.32	16.25	30.13
	ω_2	53.00	75.33	34.35	70.34
3D reduction factors†	J_{-1}*(.9447)	.9447	.9452	.9466	.9446
	J_0* (.8525)	.8525	.8528	.8538	.8526
	J_1* (.7647)	.7718	.7715	.7709	.8025
	J_2* (.6842)	.7039	.7038	.7033	.7243

†Lockwood Taylor factors [2] shown in parentheses

Spheroid major axis = 80m
Spheroid minor axis = 10m
ρ_f = 1000 kg/m^3

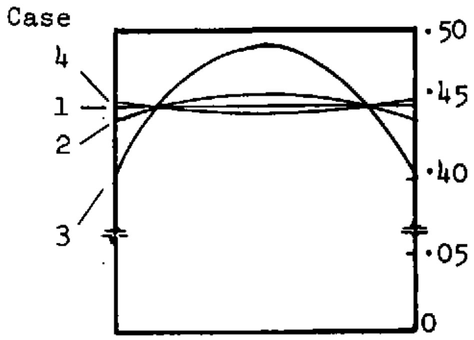

Figure 1 Mode -1 for submerged and floating spheroids