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A NOVEL APPROACH FOR DIRECTION FINDING PROBLEMS

R. FOKA

THOMSON-SINTRA ASM - 1, avenue Artistide Briand
94117 ARCUEIL - CEDEX FRANCE

ABSTRACT

In this paper, a new technique for narrowband, linear equal spaced array processing is derived which is based on what is known as the Toeplitz Approximation Method (TAM) of stochastic system identification. The proposed algorithm provides high resolution signal finding capability and allows accurate separation of directional components of received array data. The method is designed for an arbitrary noise, multipath signal environment. That high resolution direction finding (DF) estimation is important in many sensor systems such as passive sonar, radar, communication, electronic surveillance measures (ESM), etc.

1. INTRODUCTION

Eigenstructure based methods for direction finding have recently been developed. These methods utilize the decomposition of the array data correlation matrix, employing either the signal subspace or the noise subspace basis obtained as eigenvectors of this matrix. This approach is used in the "MUSIC" (Multiple Signal Classification) algorithm of Schmidt (1979) and in the described methods by Bienvenu and Kopp (1980-1981), Owsley (1977), and Johnson and DeGraaf (1982). These methods can be viewed as a generalization of a algorithm which uses only the eigenvector associated with the smallest eigenvalue. This approach was proposed first by Pisarenko (1973) and latter by Cantoni and Godara (1980).

These approaches have been examined in the case where the ambient noise is spatially white. A novel direction finding algorithm, based on a reduced order Toeplitz approximation of an estimated spatial covariance matrix, is proposed in this paper. The estimated covariance matrix, in the case in which sources are uncorrelated and statistically stationary, is Toeplitz. In a multipath environment, however, the source paths are fully correlated, this covariance matrix is no longer Toeplitz. The Toeplitz structure can be guaranteed by employing spatial smoothing, which destroys cross correlation between directional components (1).In the TAM approach, the spatial data may be modeled as the output of a self generating ARMA process with poles, corresponding to arrival directions, on the unit circle. A state representation is estimated from a covariance matrix low order approximate. The algorithm used to obtain this low order matrix, which is based on the Singular Value Decomposition (SVD) of the spatial data matrix, has low sensitivity to data perturbation.

2. MATHEMATICAL FORMUMATION

Consider a linear receiving array with L equally spaced elements and m direct sources with a total of p paths. At the array, source wavefront propagation is assumed planar and source energy is incident on the array a distinct angles $\{\theta_k, k = 1, \ldots p\}$. The propagation channel is homogeneous and the signals are narrowband (i.e. source bandwidth is much smaller than the reciprocal of the propagation time of the signal across the array). The received signal at the ith sensor $y_i(t)$ is given by

$$y_i(t) = \sum_{k=1}^{p} a_k \exp\left(j(\omega_k t - (i-1)2\pi(d/\lambda)\sin\theta_k + \phi_k)\right) + n_i(t) \quad (1)$$

$$= x_i(t) + n_i(t)$$

Rewriting equation (1) in matrix notation, we obtain

$$Y(t) = D\,S(t) + N(t)$$

$$Y(t) = \left(y_1(t), \ldots, y_L(t)\right)^t$$

$$S(t) = \left(s_1(t), s_2(t), \ldots, s_p(t)\right)^t$$

$$D = \left(D_{\theta_1}, D_{\theta_2}, \ldots, D_{\theta_p}\right)$$

$$N(t) = \left(n_1(t), \ldots, n_L(t)\right)^t$$

Therefore, the spatial covariance matrix $R = E\{Y(t)Y'(t)\}$ is

$$R = DR_SD' + \sigma^2 I = R_S + R_N$$

where "'" denotes the complex conjugate transpose ; "t" denotes the simple transpose ; "E" stands for expectation, an ensemble average operator ; I is LXL identity matrix ; $R_S = E\{S(t)S'(t)\}$ is the p x p path signal covariance matrix ; and D is the L x p direction matrix, or a Vandermonde matrix, whose columns are the steering vectors of the impinging planar wavefronts. R is non-negative definite (i.e. eigenvalues of R are non-negative), and

$$||R-R_S||_E = \lambda_{p+1}^2 + \lambda_{p+2}^2 + \ldots + \lambda_L^2$$

In DF problems, all eigenstructure methods of estimating the directions-of-arrival (DOA) $\{\theta_k\}$ are based on exploiting this structure of R. The eigenstructure methods are based on straightforward exploitation of the two following properties :

(1) Since $R_S = \sum_{i=1}^{p} \lambda_i V_i' V_i$ has rank p, the minimal eigenvalue of R is σ^2 with multiplicity L-p, i.e.

$$\lambda_{p+1} = \lambda_{p+2} = \dots = \lambda_L = \sigma^2$$

(2) the eigenvectors corresponding to the minimal eigenvalue are orthogonal to the columns of the Vandermonde matrix D.

In practice the data covariance R is not given, but has to be estimated from y(t). A popular estimate of R is

$$\hat{R} = (1/T) \sum_{t=1}^{T} Y(t)Y'(t).$$

The number of snapshots, T, needed for an adequate estimate of the covariance matrix depends upon the signal-to-noise ratio at the array input and the desired accuracy of the DOA estimates. In the absense of noise, T > p is required in order to completely span the signal subspaces. In the presence of noise, it has been shown by Brennan and Reed (1974) that T must be at least L^2.

Due to errors in $\hat{R}$, its eigenvalues will be perturbed from their true values and the true multiplicity of the minimal eigenvalue may not be evident. A popular approach for determining the underlying eigenvalue multiplicity is an information theoretic method based on the minimum description length (MDL) criterion proposed by Wax and Kailath (1985). The estimate of the number of sources p is given by the value of k for which the following MDL function is minimized :

$$MDL(k) = -\log \left(\prod_{i=k+1}^{L} \lambda_i^{\frac{1}{m-k}} \Big/ \frac{1}{L-k} \sum_{i=k+1}^{L} \hat{\lambda}_i \right)^{(m-k)T} + \frac{k}{2}(2L-k)\log T \; ;$$

where $\hat{\lambda}_i$ are the eigenvalues of $\hat{R}$. The MDL criterion is known to yield asymptotically consistent estimates.

Having obtained an estimate of p, the maximum likelihood estimate of σ^2 conditioned on $\hat{p}$ is given by the average of the smallest L-$\hat{p}$ eigenvalues i.e.,

$$\hat{\sigma}^2 = (1/_{L-p}) \sum_{i=p+1}^{L} \hat{\lambda}_i$$

3. COHERENT PATHS

Conventional eigenstructure methods are applicable only when the path signal covariance matrix R, i non-singular. When some of the path signals are coherent R, will be singular and properties (1) and (2) do not hold.

4. A SOLUTION BY SHAN AND KAILATH

Recently, Kailath et al. (1984) proposed a scheme which is based on the use of spatial smoothing and can guarantee successful performance of the eigenstructure methods regardless of the coherence of the sources. This method has reduced resolution capability (because of the shorter array) and inferior numerical properties.

In this paper, we suggest a new kind of spatial smoothing technique which has better resolution capability. We propose a state-space parameter estimation procedure, instead of polynomial estimation, because it has better numerical property.

5. STATE SPACE FORMULATION

Since the signals received by the array are assumed composed of sinusoids, such a signal can be considered to be the output of a very special ARMA model. Because the spatial covariance of x (defined as noise free measurement) has rank p, x(k) is a pth order Markov process with respect to spatial index k, and x(k) can be predicted from $\{x(k-1), x(k-2), \ldots, x(k-p)\}$. As indicated by Kung et al. (1983) (2) we can formulate a special state-space representation as

$$Z_{k+1} = FZ_k$$

$$x_k = hZ_k$$

One choice of state Z(k) leads to

$$Z_1 = \left(e^{j(\omega_1 t + \phi_1)}, \ldots, e^{j(\omega_m t + \phi_p)}\right)^t$$

$$F = \mathrm{diag}\left(e^{-j\tau_1}, e^{-j\tau_2}, \ldots, e^{-j\tau_p}\right)$$

$$h^{(i)} \cdot X_1^{(i)} = a_i \cdot e^{j\phi_i}$$

Any other choice is related to the representation expressed above as a similarity (coordinate) transformation of F. The state transformation may not be unique but the transfer function is (i.e. poles and zeroes do not change). Therefore, after any coordinate transformation, the eigenvalues of F will always be $\exp(-j\tau_i)$, i = 1,..., p. We can also write $X = (x_1, x_2, \ldots, x_L)^t$ as

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$$X = \left(h,\ hF,\ hF^2,\dots,hF^{L-1}\right)^t Z_1 = \Theta Z_1$$

Therefore, $R_S = \Theta E\{Z_1 Z_1'\}\Theta'$.

In system theory, Θ is called the observability matrix, and if all τ_i's are distinct, i.e. if the p paths do not overlap, the p columns of Θ are independent. Moreover, if all paths are uncorrelated, $E\{Z_1 Z'_1\}$ is diagonal and stricly positive definite, and R_S has rank p. However, when the path signals are coherent, some of the ϕ_k variables are dependent, $E\{Z_1 Z_1'\}$ will be singular and R_S ill have rank < p. In fact, R_S will not be Toeplitz and the spatial process X will no longer be spatially stationary. Thus, the coherence of path signals destroys both the Toeplitz property and the p-rank property of R_S. Clearly, if R_S is non-Toeplitz, the spatial correlation matrix R, pertaining to the observed signal vector, is also non-Toeplitz.

5.1 - Spatial Averaging

If we use spatial averages instead of time averages, then asymptotically we will obtain a Toeplitz matrix C that has the rank p.

If can be easily shown that

$$c(m) = hF^m Ph' \qquad m \geq 0$$

where

$$P = \lim_{L\to\infty}\left(\frac{1}{L}\right)\sum_{i=1}^{L} Z_i Z_i' = FPF'$$

is a p x p state "covariance" matrix. Note that P is not an estimate of $E\{Z_i Z_i'\}$, since $E\{Z_i Z_i'\}$ is a function of i and spatial averaging destroys this dependence.

Now, we form the new Toeplitz covariance matrix as following :

$$C = \begin{pmatrix} c(o) & c(-1) & . & . & . \\ c(1) & c(o) & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \end{pmatrix}$$

Using $c(m) = hF^m Ph'$, we can easily verify the following factorization :

$C = \Theta p \Theta$

Since Θ has only p columns, the rank of C must be $\leq$ p. As stated earlier, Θ and P are always full rank, thus C has rank p irrespective of the coherence of paths.

5.2 - Estimation

Numerous covariance matrix estimators exist in the literature. One very popular choice is the unbiased estimator. When the array length is finite, an unbiased estimate of c(m) is

$$\left(\frac{1}{(L-m)}\right) \sum_{i=o}^{L-m-1} x_i \, x_{i+m}, \qquad m = 0, \ldots, L-1$$

If the time series is composed of $\frac{p}{2}$ sinusoids, the covariance matrix C should ideally be a Toeplitz matrix of rank p. However, because (L) is often relatively small for practical arrays, this estimate may not be a good and the ideal matrix characteristics may not be realized. The estimate may be improved by using a pseudo-ensemble average ; a combined spatial and temporal average. It is therefore suggested that the estimator,

$$c(m) = \left(\frac{1}{T(L-m)}\right) \sum_{t=1}^{T} \sum_{k=o}^{L-m-1} x_k(t) \; x_{k+m}(t)$$

can be used where t is the temporel index.

5.3 - Toeplitz Approximation Method (TAM)

The objective of the Toeplitz approximation method is to retrieve a p-rank property on an estimated covariance matrix C (via (SVD), and then to enforce the structure of D, to obtain estimates F and h from the principal components. In summaray, the two steps of the TAM approach (2,3) are :

Perform an SVD on C and arrange the singular values $\{\sigma_k ; k = 1, \ldots L\}$ in decreasing order. The SVD is

$$\bar{C} = U\Sigma V' = \left(U_1 U_2\right) \begin{pmatrix} \Sigma_1 & o \\ o & \Sigma_2 \end{pmatrix} \begin{pmatrix} V_1' \\ V_2' \end{pmatrix}$$

where the p x p diagonal matrix Σ_1 contains the p largest singular values. A p-rank approximant to $\bar{C}$ is $= U_1 \Sigma_1 V_1'$, and the (minimal) approximation error in the spectral norm is optimal.

In the presence of only white noise, though the singular vectors are unchanged, the singular values are affected. In fact all the singular values are increased by the noise variance, so that the smallest singular value has to be substracted to compensated for the effect. With this, the p rank approximant to $\hat{C}$ is = $U_1\Sigma_1V_1'$, $\hat{\Sigma}_1 = \Sigma_1 - \sigma_M^2 I$, where σ_M^2 is the smallest singular value. This is actually equivalent to the Pisarenko's solution.

The second step involves determining the model parameters. The observability-type matrix is given by $\Theta = U_1\Sigma_1^{1/2}$

Ideally, Θ would have the exact observability matrix structure $\{h\ hF\ hF^2 \ldots\}^t$ and satisfy $\Theta F = \{hF\ hF^2 \ldots\}^t = \Theta\uparrow$. However, because of the approximation, no exact solution exists and we have to resort to a least-squares solution that minimizes $||\Theta F - \Theta\uparrow||_F$, where subscript F denotes the Frobenius norm and $\Theta\uparrow$ is obtained by shifting Θ one row upwards with an added last row of zeros. This leads to least-squares estimates of state-space parameters, as $F = \Theta\dagger\Theta\uparrow$ (state transition matrix).

The eigenvalues of $F = \mathrm{diag}\left(\exp\{-j\tau_1\}, \exp\{-j\tau_2\}, \ldots, \exp\{-j\tau_p\}\right)$, give the directions of the paths, since the eigenvalues are $\exp(-j\tau_i) = \exp(-j2\pi(d/\lambda)\sin\theta_i)$ Also, because $\tilde{F} = QFQ^{-1}$, the amplitude information can be estimated as $a_i = ||h^{(i)}Q||^{1/2}$,

where h is the first row of observability matrix Θ. It is known that the obtained estimate of F is always stable.

Spatial Spectrum

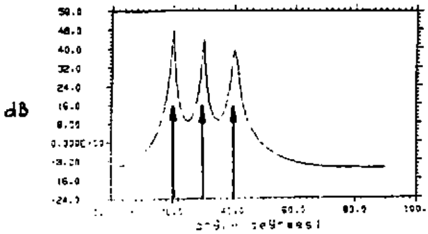
In general, the source energy, distributed as a function of angular direction, can be described as

$$p(\tau) = \sum_{i=1}^{p} \left|\frac{a_i}{1-r_i\exp\left(-j(\tau-\tau_i)\right)}\right|^2 \tag{2}$$

where a_i es the information of source amplitude, r_i is the radius of ith pole.The bearing of the source paths (θ) are determined by finding the spatial frequencies at which (2) achieves a maxima.

6. SIMULATION RESULTS

In order to perform TAM applied to the direction finding, the temporal snapshot data and Gaussian noise data were generated artificially by a computer. The source signals were generated as one direct path and two reflected path which are equal in magnitudes. Figure 1 shows Monte Carlo simulation results for TAM. This figure shows resolution and direction-finding results for three equal-powered emitters at angular locations 20°, 30° and 40°. The SNR was 0 dB.



7. CONCLUDING REMARKS

TAM algorithm for direction finding problems, with application in multipath environment with linear equi-spaced arrays, have been presented. Computational complexity analysis (1) shows that it will be possible to implement these algorithm in real time on high speed parallel processing computer systems within the next five years. Simulations of the algorithms based on a small number of input snapshots (64-128) yield satisfactory results. Preliminary results indicate that one can be optimistic in expecting high resolution and good accuracy when a relatively small number (10-32) of array elements is used.

For future research, it is suggested to extend the TAM algorithms from one dimensional to two dimensional array processing; because in some applications (sonar signal processing) estimation of the elevation angle is important and we need two dimensional array (e.g. planar array). The approach to estimation of the elevation and azimuth angles is the same as we propose in this paper.

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