NONLINEAR ESTIMATION THEORY WITH ACTIVE SONAR OBSERVATIONS*

R. FOKA

THOMSON SINTRA A.S.M 1 avenue A. Briand 94117 ARCUEIL CEDEX FRANCE

1 - INTRODUCTION

The determination of position and velocity of a target using active sonar observations on range and bearing is a problem in nonlinear estimation theory.

Usually dynamics of the target are modeled with respect to a cartesian coordinate frame, while the active sonar's observations consist of measurements on the target's slant range, azimuth. Since pratical estimators, such as maximum likehood and the kalman filter based methods, are derived under assumption of linearity, the transformation relating the target's retangular coordinates to its active sonar coordinates must be linearized prior to estimation.

The nonlinearities of the active sonar measurement model equation are examined and their influence upon the accuracy of filtering and smoothing is determinned. In this paper we derived the linearity errors; it is shown that these errors are proportional to the range measurement. The nature of these linearization errors is examined and it is shown that they can be exploited to reduce the linearization error in active sonar applications to tracking and smoothing.

A simple algorithm by which the effects from these nonlinearities can be significantly reduced is derived. The accuracy of estimates obtained via the algorithm is compared with and shown to be superior to that of the Extended Kalman Filter (EKF) algorithms. It is shown that the use of the algorithm does not increase the computational complexity of estimation to beyond that of a standard EKF.

Finally, application examples on realistic simulated data of the algorithm to the problem of tracking a target is presented.

2 - OBSERVATIONS MODEL AND ERRORS ANALYSIS

Let $(x,y)'$ be the target's position coordinates with respect to asonar-centered cartesian coordinate frame. The transformation to the cartesian coordinate system used for tracking, where β is measured from the y-axis, is

$$x = r \sin\beta \quad (1a)$$
$$y = r \cos\beta \quad (1b)$$

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Let the vector $(x_o,y_o)'$ be an initial unbiased estimated of $(x,y)'$ having covariance matrice P_o. We model the active sonar measurements on range and beasing as :

$$r_m = r + \nu_r \quad (2a)$$
$$\beta_m = \beta + \nu_\beta \quad (2b)$$

Where ν_r and ν_β are assumed to be zero-mean independent gaussian noise with variances $\sigma_r{}^2 = E(\nu^2{}_r)$ and $\sigma_\beta{}^2 = E(\nu_\beta{}^2)$ respectively and

$$r = (x^2 + y^2)^{1/2} \quad (3a)$$
$$= x \sin\beta + y \cos\beta$$

$$\beta = \text{Arctan}\ (x/y) \quad (3b)$$

"E" is the expectation

The EKF algorithm allows us to use the r_m and β_m to update $(x_o,y_o)'$ and obtain a new estimate $(x_e,y_e)'$ of $(x,y)'$ with covariance matrix Pe.

It is well-know that the EKF estimation equation are [1]

$$\begin{pmatrix} x_e \\ y_e \end{pmatrix} = \begin{pmatrix} x_o \\ y_o \end{pmatrix} + Pe\ H_o{}'\ R^{-1} \begin{pmatrix} r - r_o \\ \beta - \beta_o \end{pmatrix} \quad (4)$$

Where

$$P_e{}^{-1} = P_o{}^{-1} + H'oR^{-1}H_o \quad (5)$$

and $Ho = 1/r_o \begin{pmatrix} r_o \sin\beta_o & r_o \cos\beta_o \\ \cos\beta_o & -\sin\beta_o \end{pmatrix}$ (6)

$$R = \text{diag}\ (\sigma_r{}^2,\ \sigma_\beta{}^2) \quad (7)$$

Using the above expression the matrix multiplication in (4) becomes:

$$\begin{pmatrix} x_e \\ y_e \end{pmatrix} = \begin{pmatrix} x_o \\ y_o \end{pmatrix} + Pe \begin{pmatrix} \sin\beta_o \\ \cos\beta_o \end{pmatrix} (r - r_o)/\sigma_r{}^2$$
$$+ Pe \begin{pmatrix} \cos\beta_o \\ -\sin\beta_o \end{pmatrix} (\beta - \beta_o)/r_o \sigma^2{}_\beta \quad (8)$$

we assume there that $P_o{}^{-1}$ is negligible compared with R^{-1} then

$$P_e \simeq H_o{}^{-1}\ R\ H'^{-1} \quad (9)$$

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and we can write

$$\begin{pmatrix} x_e \\ y_e \end{pmatrix} \approx \begin{pmatrix} x_o \\ y_o \end{pmatrix} + \begin{pmatrix} \sin\beta_o \\ \cos\beta_o \end{pmatrix} (r - r_o) + \begin{pmatrix} \cos\beta_o \\ -\sin\beta_o \end{pmatrix} r_o\,(\beta - \beta_o) \qquad (10)$$

we are now going to use equation (10) to calculate the error of the EKF algorithm
let

$$r_e = x_e \sin\beta_e + y_e \cos\beta_e$$
$$\beta_e = \arctan(x_e/y_e)$$

From eq (9) we can note

$$x_e = r\sin\beta_o + r_o(\beta - \beta_o)\cos\beta_o$$
$$y_e = r\cos\beta_o - r_o(\beta - \beta_o)\sin\beta_o$$

The formulation of the EKF algorithme is based on the linearization of eq. (2), then the estimation errors are :

$$re - r = [r^2 + r_o^2(\beta - \beta_o)^2]^{1/2} - r$$

$$\approx r[1 + (\beta - \beta_o)^2]^{1/2} - r \qquad (11)$$

and

$$\beta_e - \beta = \beta_o - \beta + \mathrm{Arctan}\,(r_o/r)\,(\beta - \beta_o)$$

$$\approx -(\beta - \beta_o) + \mathrm{Arctan}\,(\beta - \beta_o) \qquad (12)$$

In writing down equations (11) and (12) we have just assumed that $(r - r_o)/r$ is negligible - We have also used the following trigonometric identity.
$\mathrm{Arctan}\,[(\sin\theta + A\cos\theta)/(\cos\theta - A\sin\theta)] = \theta + \mathrm{Arctan}\,A$ in deriving eq. (12).

It is clear from eq (11) that when the target range is large, the variance of the estimation error may exceed the variance of the range measurement error σ_r^2, and the standard EKF algorithm will tend to diverge.

This above situation is inherent in the EKF when the innovations processus (the difference between the measurements $(r,\beta)'$ and the predicted position $(r_o,\beta_o)'$) are used to update the estimate position of the target.

It is evident now that the standard EKF algorithm does not preserve the accuracy of the range update.
We can improve the estimation accuracy considerably if we include in the calculation of the range innovation the innovation introduced by the bearing update.

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Consider, now the estimate that results from updating (x_o,y_o) with bearing only,

$$\begin{cases} x_1 = xo + r_o\ (\beta - \beta o) \cos \beta o \\ y_1 = yo - r_o\ (\beta - \beta o) \sin \beta o \end{cases}$$

let

$$r_1 = x_1 \sin\beta_1 + y_1 \cos \beta_1$$
$$\beta_1 = \arctan (x_1/y_1)$$

$$r - r_1 = r - r_o\ [1 + (\beta - \beta_o)^2]^{1/2}$$

$$\simeq (r - r_o) - 1/2\ r_o\ (\beta - \beta_o)^2$$

and $\beta - \beta_1 = (\beta - \beta_o) - \text{Arctan}\ (\beta - \beta_o) \simeq 0.$

Now, if instead of using $(r - r_o)$ we use $(r - r_1)$ to update $(x_1,y_1)'$, then,

$$\begin{cases} x_2 = x_1 + (r - r_1) \sin \beta_1 \\ y_2 = y_1 + (r - r_1) \cos \beta_1 \end{cases}$$

Let $r_2 = x_2 \sin\beta_2 + y_2 \cos \beta_2$
$\beta_2 = \arctan (x_2/y_2)$

Then the total estimation error is given by $r - r_2 = 0$

$\beta - \beta_2 = (\beta - \beta o) - \text{Arctan}\ (\beta - \beta_o) \simeq 0.$

In summary, we can say that if we first linearize the azimuth equation at $(x_o,y_o)'$ and use the EKF algorithm to produce an intermediate $\{x_1,y_1\}$ and then use $\{x_1,y_1\}$ to linearize the range equation the estimation errors are considerably reduced and the possibility of filter divergence due to the measurement model linearization is virtually eliminated.

The estimation algorithm described above assumes the sonar observations to be uncorrelated. Unlike the standard EKF algorithm, advantage is taken of the nature of the nonlinearities in the sonar measurement model by processing the observations in this order : azimuth, range. It is evident that this algorithm is not equivalent to the EKF algorithm. In the next section we give specification of this algorithm.

3 - ALGORITHM SPECIFICATION

The source-observer geometry is shown in Fig.1. Assuming constant target velocity the discrete-time propagation of target state $X_T = (r_{Tx}, r_{Ty}, v_{Tx}, v_{Ty})'$ is given by

$$X_T\ (k+1) = \begin{pmatrix} 1 & 0 & t & 0 \\ 0 & 1 & 0 & t \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} X_T\ (k) = \Phi\ (k+1,k)\ X_T\ (k) \quad (13)$$

Where t is the time increment between data samples and $\phi\ (k + 1, k)$ is the state transition matrix. The observer state $X_o = (r_{ox}, r_{oy}, v_{ox}, v_{oy})'$ is given by

$$X_o\ (k+1) = \Phi\ (k+1, k)\ X_o\ (k) + \Gamma_o\ \Delta\ (k), \quad (14)$$

where

$$\Gamma_o = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad (15)$$

is the input matrix, and

$$\Delta\ (k) = \begin{pmatrix} V_{ox}\ (k+1) - V_{ox}\ (k) \\ V_{oy}\ (k+1) - V_{oy}\ (k) \end{pmatrix} \quad (16)$$

represents the incremental change in observer velocity.

The range and the bearing to the target, defined by

$$r\ (k) = (r_{Tx}\ (k) - r_{ox}\ (k)) \sin\beta\ (k) + (r_{Ty}(k) - r_{oy}(k)) \cos\beta(k) \quad (17)$$

$$\beta(k) = \arctan\ (r_{Tx}(k) - r_{ox})/r_{Ty}(k) - r_{oy}(k)) \quad (17b)$$

are noise corrupted when viewed by the observer.

Consequently, what is available are the range and the bearing measurements :

$$r_m\ (k) = r(k) + \nu_r\ (k) \quad (18a)$$
$$\beta_m\ (k) = \beta\ (k) + \nu_\beta\ (k) \quad (18b)$$

Where $\nu_r(k)$ and ν_β are a random sequence.

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If we let the columm vector Y(k) represent the noisy sonar observations at discrete-time k, then

$$Y(k) = h(X(k),k) + \nu(k) \tag{19a}$$

where

$$Y'(k) = (r_m(k), \beta_m(k)) \tag{19b}$$
$$\nu'(k) = (\nu_r(k), \nu_\beta(k)) \tag{19c}$$
$$X(k) = X_T(k) - X_o(k) = (r_x, r_y, v_x, v_y)'$$

we also assume that $\nu(k)$ and $\nu(P)$ are independent for $k \neq p$ and that

$$E(\nu(k)\,\nu'(p)) = \mathrm{diag}(\sigma_r^2, \tau_\beta^2)$$

independent of k.

An EKF algorithm having (13), (14) as its system model and (19) as its measurement model was then applied to the simulated data.
(For details on the derivation and application of the EKF algorithm see for example [2]).

The initial conditions, measurement noise variances, and the state estimate and covariance matrix propagation schemes of the algorithm presented in the last section are identical with those used in the EKF processing.

The update equations used to process the observations Y(k) are the following. First, the propagated state estimate X (k+1/k) at discrete-time k is used to obtain a prediction of the azimuth β (k+1/k) and its system derivative

$$H_\beta(k) = \partial\beta/\partial X|X = X(k + 1k) \tag{20}$$

The azimuth component of Y(k) is then used to form an updated state estimate X_β (k + 1/k + 1) and covariance matrix P_β (k +1/k + 1) where

$$X_\beta(k + 1/k + 1) = X(k + 1/K) + G_\beta(k + 1).$$
$$[\beta_m(k + 1) - \beta(k + 1/k)] \tag{21}$$

$$P_\beta(k + 1/k + 1) = [I - G_\beta(k + 1) H'_\beta(k)] P(k + 1/k) \tag{22}$$

and the gain vector G_β (k + 1) is given by

$$G_\beta(k + 1) = P(k + 1/k) H'_\beta(k).$$
$$[H_\beta(k) P(k + 1/k) H'_\beta(k) + \sigma_\beta^2]^{-1} \tag{23}$$

Next, using X_β (k + 1/k + 1), a prediction of the range r(k + 1/k + 1) and its system derivative

$$H_r(k + 1) = \partial r/\partial X|X = X_\beta(k + 1/k + 1) \tag{24}$$

are evaluated.

The range component of Y(k) is used to form an updated state $X_{\beta r}$ (k + 1/k + 1) and covariance matrix $P_{\beta r}$ (k + 1/k + 1) where

$$X_{\beta r} (k + 1/k + 1) = X_{\beta} (k + 1/k + 1) + G_{\beta r} (k + 1). [r_m(k + 1) - r(k + 1/k + 1] \quad (25)$$

$$P_{\beta r} (k + 1/ k + 1) = [I - G_{\beta r} (k + 1) H_r (k + 1)] P_{\beta} (k+1/K+1) \quad (26)$$

and the gain vector $G_{\beta r}$ (k + 1) is given by

$$G_{\beta r} (k + 1) = P_{\beta} (k + 1/k + 1) H'_r (k + 1). [H_r (k + 1)P_{\beta} (k + 1/k + 1) H'_r (k + 1) + \sigma_r^2]^{-1} \quad (27)$$

The quantities $x_{\beta r}$ (k + 1/k + 1) and $P_{\beta r}$ (k + 1/k + 1) are then used as the initial conditions for the propagation equations.
The estimator defined by our algorithm equations (20) - (27) together with the EKF propagation equations, is formally an unbiased minimum variance estimator to first order in the estimator error.

4 - SIMULATIONS AND RESULTS

As an application of the algorithm presented in this paper, we consider the problem of determining, from active sonar observations, the position and velocity of target moving in straight line.

A set of simulated was generated by applying (13) - (19) to the generated trajectory and azimuth wiht σ_r = 100 m, σ_β = 1° for a modern active sonar system. The initial conditions are approximately, to a target (r_o = 85 km, β_o = 45°) with a velocity of 5 m/s.

The results are presented in Figs 2.6. It is obvious from these figures that EKF estimates are biased and tend to diverge [3] and the estimates generated by the filter of our algorithm are unbiased and convergent.

5 - CONCLUSION

An estimation algorithm by which the effects of nonlinearities in the active sonar measurements model may be reduced, have been presented. The simulations of algorithm yield satisfactory results. Preliminary results indicate that one can be optimistic in expecting good accuracy.
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6 - REFERENCE

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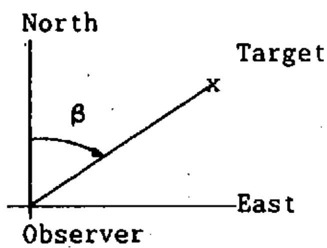

Fig. 1

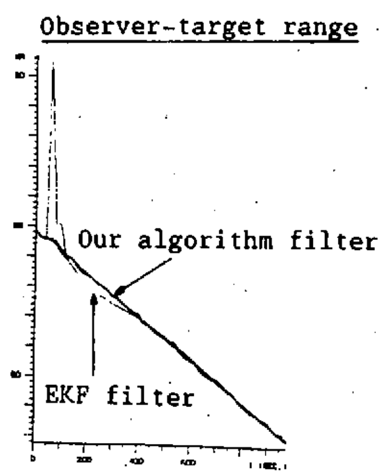

Fig. 2

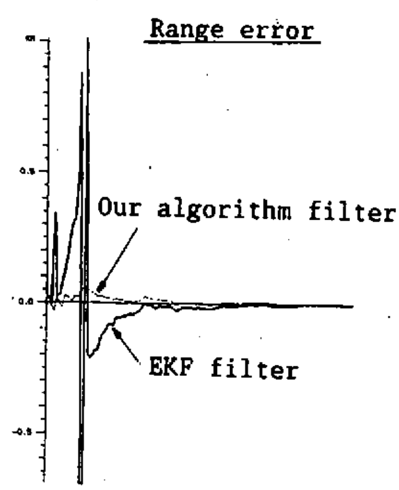

Fig. 3

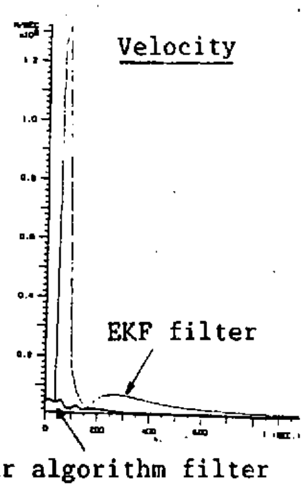

Fig. 4

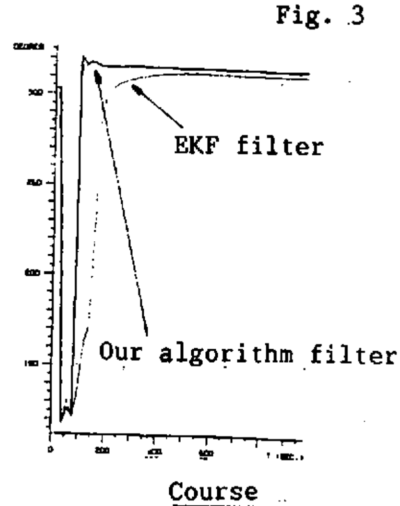

Fig. 5