24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

ELASTIC WAVE PROPAGATION IN LIQUID FILLED BURIED PIPES

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The non-destructive testing of pipelines encounters many different environmental conditions, which includes pipelines that are buried in materials such as soil, sand and even concrete. In addition, these pipelines may carry gases or liquids, such as oil or water. The location of cracks or areas of corrosion in pipelines often encounters varied environmental conditions and this requires knowledge of the influence these conditions have on the propagation of elastic waves travelling down the pipe wall. This includes knowledge of modal group velocity, which is used to estimate the time-of-flight of a pulse scattered by a defect, as well as the reduction in inspection range caused by the attenuation of energy in the pulse. Accordingly, it is desirable to develop theoretical models to help in understanding the influence of different environmental conditions, and so in this article the semi-analytic finite element (SAFE) method is used to obtain the dispersion curves for buried liquid filled pipes. It is shown that through a Galerkin based formulation a governing eigenequation may be formulated in a way that delivers a fast and efficient solution. Crucially, this involves the utilization of a semi-orthogonality relation so that modes can be sorted quickly and easily in order to focus on those modes relevant in commercial non-destructive testing.
Keywords: elastic waves, buried pipes

1. Introduction

Pipelines are found in many different engineering applications and it is common for pipes to be filled with a liquid and to be buried underground. This presents many operational risks associated with the possibility of degradation and ultimately failure of a pipeline, and so it is important to develop techniques to monitor and predict structural integrity. For example, one may use passive sensors to detect a pipe rupture, and this normally involves measuring the presence of acoustic waves that travel away from a pipe rupture along pipe walls [1]. Alternatively, one may attempt to actively monitor the structural integrity of a pipe using an appropriate inspection regime. This falls under the category of non-destructive testing, and normally involves an acoustic wave being launched down a pipe wall from an array of transducers, and then echoes from defects such as cracks or areas of corrosion being picked up as they return to the transducers. This method is commonly known as Long range Ultrasonic Testing (LRUT) [2]. The two methods rely on similar phenomena, namely the ability of pipe walls to transmit sound energy over distance, however passive techniques tend to operate in the low frequency range, whereas active techniques focus on much higher frequencies, normally in the ultrasonic range. Thus, if one is interested in developing an understanding of how these waves propagate then it is necessary to develop techniques that are capable of covering a wide frequency range.

In order to better understand the propagation of sound waves along pipelines, it is desirable to develop mathematical models that enable detection techniques to be improved and optimised. Moreover, it is common for pipelines to be buried and also to be filled with a liquid and this further complicates the mathematical analysis. Accordingly, this article presents a numerical model based on the Semi-Analytic Finite Element (SAFE) method, and shows that a model of this type is capable of accommodating internal liquids as well as a surrounding material.

A number of computational methods are available for obtaining the eigenmodes for sound propagation along the walls of fluid filled buried structures. For example, one may adopt analytic techniques based on the use of a transfer matrix approach [3], or utilise low frequency approximations to generate analytic expressions for low order modes [4]. However, an analytic approach is not readily applied over a wide frequency range, this is because at higher frequencies many modes propagate and it can become difficult to locate and track all of the roots of the dispersion relation. Accordingly, it is attractive to develop numerical techniques capable of being applied over a wider frequency range, and here the SAFE method provides the most robust methodology for doing this.

The SAFE method is now a popular technique for analysing guided waves. See for example, Hayashi et al [5] who apply the SAFE method to a pipe, and the series of papers by the first author for coated pipes [6, 7, 8]. Nguyen et al. [9] later showed how the SAFE method could be extended to the study of buried pipes through the use of a perfectly matched layer (PML), see also the article by Treyssède [10]. Duan and Kirby [11] later showed that by separating out the radial and circumferential components in the PML region, it was possible to develop a one-dimensional approach to obtaining the eigenvalues for buried pipes. This was seen to provide a fast and efficient technique so that eigenmodes for buried pipes could be obtained over a wide frequency range and for relatively large pipes.

This article extends the recent work of Duan and Kirby [11] to include a fluid inside the pipe, so that the pipe is now buried and fluid filled. A one dimensional approach is maintained throughout and the SAFE-PML development is described in section 2. In section 3 dispersion curves are then presented and the effects of the fluid in the pipe are analysed.

2. Theory

The SAFE method is applied here to a fluid-filled pipe buried in soil, see Fig. 1.

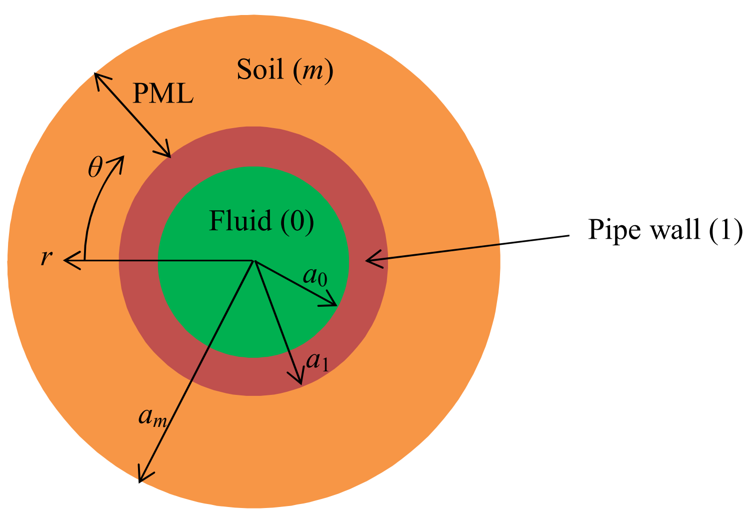

Figure 1: Geometry of fluid-filled buried pipe.

The pipe wall (region 1) and the surrounding soil (region *m*) are assumed to support elastic wave propagation, and so the governing equation for each region is Navier's equation:

$$(\lambda+\mu)\nabla(\nabla\cdot\boldsymbol{u}')+\mu\nabla^2\boldsymbol{u}'=\rho\frac{\partial\boldsymbol{u}'}{\partial t^2} \tag{1}$$

where λ and μ are the Lamé constants, $\boldsymbol{u}'$ is the displacement vector, ρ is density and t is time. A time dependence of $e^{\mathrm{i}\omega t}$ is assumed, where ω is the radian frequency and $\mathrm{i}=\sqrt{-1}$. The orthogonal coordinate system (r,θ,z) is used here, where r and θ are shown in Fig. 1, and z is the axial co-ordinate, so that $\boldsymbol{u}'=[u_r' \quad u_\theta' \quad u_z']$. The displacement in each direction is then expanded over the guide eigenmodes to give

$$u_r'=u_r e^{i(\omega t-n\theta-k_{T_1}\gamma z)} \tag{2}$$

$$u_\theta'=u_\theta e^{i(\omega t-n\theta-k_{T_1}\gamma z)} \tag{3}$$

$$u_z'=u_z e^{i(\omega t-n\theta-k_{T_1}\gamma z)} \tag{4}$$

This expansion assumes symmetry in the circumferential direction so that the theta dependence may be written as $e^{in\theta}$, where n denotes circumferential mode order. In addition, γ is the coupled [dimensionless] wavenumber for the problem, and $k_{T_1}=\omega/c_{T_1}$ where c_{T_1} is the shear wave velocity for the pipe wall. The assumed ansatz for displacement is then substituted back into the governing equation, and here it is convenient to separate Navier's equation into its three components. Once this has been done, a finite element based solution is sought, so that each equation is weighted by the function w and integrated over regions 1 and *m*. Further, a weak formulation yields

$$\int_\Omega\left\{\sigma_{rr}\frac{\partial w_r}{\partial r}+\frac{in}{r}w_r\sigma_{r\theta}+ik_{T_1}\gamma w_r\sigma_{rz}-\frac{w_r}{r}[\sigma_{rr}-\sigma_{\theta\theta}]-\rho\omega^2 w_r u_r\right\}d\Omega=\int_\Gamma w_r\sigma_{rr}n_r d\Gamma \tag{5}$$

$$\int_\Omega\left\{\sigma_{\theta r}\frac{\partial w_\theta}{\partial r}+\frac{in}{r}w_\theta\sigma_{\theta\theta}+ik_{T_1}\gamma w_\theta\sigma_{\theta z}-\frac{2w_\theta}{r}\sigma_{r\theta}-\rho\omega^2 w_\theta u_\theta\right\}d\Omega=\int_\Gamma w_\theta\sigma_{\theta r}n_r d\Gamma \tag{6}$$

$$\int_\Omega\left\{\sigma_{zr}\frac{\partial w_z}{\partial r}+\frac{in}{r}w_z\sigma_{z\theta}+ik_{T_1}\gamma w_z\sigma_{zz}-\frac{w_z}{r}\sigma_{rz}-\rho\omega^2 w_z u_z\right\}d\Omega=\int_\Gamma w_z\sigma_{zr}n_r d\Gamma \tag{7}$$

The FE method proceeds by approximating the displacement using the shape function N, so that in direction q,

$$u_q(r)=\sum_{i=1}^{p_q}N_{qi}(r)u_{qi}=\mathbf{N}_q\boldsymbol{u}_q \tag{8}$$

where p_q is the number of nodes in the mesh for direction q. If we assume isoparametric elements, then $\mathrm{N}_q=w_q$, and it is convenient also to set $w_r=w_\theta=w_z=N_r=N_\theta=N_z$. The shape and weighting functions are then substituted back into Eqs (5)-(7). Note that in the PML region it is necessary to perform the integral over a stretched co-ordinate, which is defined as

$$\tilde{r}=\int_0^r\xi_r(s)ds \tag{9}$$

In the fluid region, the governing equation is Helmholtz's equation, which gives

$$\nabla^2 p'-\frac{1}{c_0^2}\frac{\partial p'}{\partial t^2}=0. \tag{10}$$

The pressure in the fluid region is then expanded in a similar way to that used for the solid region, and so

$$p'=pe^{i(\omega t-n\theta-k_{T_1}\gamma z)} \tag{11}$$

Finite element discretisation of this equation, then yields

$$\int_\Omega\left\{\frac{\partial p}{\partial r}\frac{\partial w_0}{\partial r}-\frac{1}{r}w_0\frac{\partial p}{\partial r}+\frac{n^2}{r^2}w_0 p+k_{T_1}^2\gamma^2 w_0 p-k_0^2 w_0 p\right\}d\Omega=\int_\Gamma w_0\frac{\partial p}{\partial r}n_r d\Gamma \tag{12}$$

Equations (5)-(7) and (12) represent the governing equations of the problem and they are coupled together through the boundary conditions at the interface between the fluid and the solid. These are given as

$$\text{at } r = r_0: \ \frac{\partial p}{\partial r} = \rho_0 \omega^2 u_r, \text{ and } p = -\sigma_{rr}; \tag{13}$$

$$\text{and at } r = r_0: \ \sigma_{\theta r} = \sigma_{zr} = 0. \tag{14}$$

It is necessary also to close the problem by applying boundary conditions at the outer edge of the PML. These boundary conditions are arbitrary, and the following are chosen for convenience

$$\text{at } r = r_m: \ \sigma_{rr} = \sigma_{\theta r} = \sigma_{zr} = 0. \tag{15}$$

This then delivers four simultaneous and coupled equations, which can be solved to recover the wavenumber γ for the problem. The phase speed for each mode is then given as $c = \mathrm{Re}(c_{\mathrm{T}_1}/\gamma)$ and the attenuation as $\Delta = -8.686 \, \mathrm{Im}(k_{T_1}\gamma)$.

3. Results

In this section, dispersion curves are presented for a steel pipe filled with water and buried in soil. The properties of the water are $\rho_0 = 1000 \text{ kg/m}^3$ and $c_0 = 1480 \text{ m/s}$. An 8inch Schedule 40 steel pipe is examined here, so that $a_0 = 101.36 \text{ mm}$, $a_1 = 109.54 \text{ mm}$, $\rho_1 = 7932 \text{ kg/m}^3$, and the shear velocity is $c_{T_1} = 3260 \text{ m/s}$ and the longitudinal velocity $c_{L_1} = 5960 \text{ m/s}$. For the surrounding soil, $\rho_m = 1900 \text{ kg/m}^3$, and $c_{T_m} = 80 \text{ m/s}$; the outer radius of the PML is $a_m = 273.12 \text{ mm}$. For the longitudinal velocity in the soil, two values are investigated: $c_{L_m} = 1000 \text{ m/s}$ and $c_{L_m} = 1600 \text{ m/s}$.

To solve the problem, 50 elements are used in the water, 50 elements are used in the pipe wall, and 800 elements are used for the PML layer, with the properties of the PML layer being identical to that used by Duan and Kirby [11]. Note that the PML settings here are mainly used to capture so-called leaky modes [9, 10], as these are most relevant to non-destructive testing applications. For trapped modes [9, 10], it might be possible to use different PML settings, which will be investigated in a different article. This delivers a total of 5204 degrees of freedom for the coupled system and 150 modes are computed at each frequency.

In Figs. 2 and 4, the phase speed of leaky modes is presented for the two different longitudinal velocities in the soil, and in Figs. 3 and 5 the corresponding values for attenuation are presented. A lower frequency range is used in these plots to ensure that they remain reasonably clear and it is easy to follow the different propagating modes. The model can readily compute eigenmodes at higher frequencies, and into the ultrasonic range, however this delivers much more complicated dispersion curves and this will be discussed below. It can be seen in Figs. 2-5 that the presence of the liquid begins to influence the propagation of the structural modes as the frequency is increased. For example, for L(0,1) (the first longitudinal structural mode) the phase velocity and attenuation begins to change as the frequency is increased and here it is interesting to note that the degree of influence imparted by the fluid is also dependent on the properties of the soil, as the difference between the two sets of figures illustrates. Obviously, the presence of the water does not affect the propagation of T(0,1) (the first shear, or torsional, structural mode) as this is a shear mode, however it is useful to include this as a check that the model is delivering physically plausible results. Note that the soil has a significant effect on T(0,1) as the frequency is reduced, this is because the energy from the structure begins to transfer into the soil at low frequency and this causes the attenuation to increase rapidly.

The addition of water now supports additional modes. At low frequencies, the energy of mode α_1 is concentrated in the fluid and pipe wall; however, as the frequency is increased, some of this energy now begins to transfer into the soil, which causes a rapid rise in the attenuation of α_1 for a value of $c_{L_m} = 1000 \text{ m/s}$. Thus, as one would expect, if the properties of the soil are changed, then it is seen that the attenuation of α_1 is also changed, and in this case it is lowered across the frequency range when c_{L_m} is increased. However, it is clear that energy contained within fluid borne modes may leak out of the fluid at higher frequencies and encounter high levels of attenuation, which means that one must be careful when looking for information carried by this mode and at the very least some prior estimation of the behaviour for this mode will be necessary before using it for detection purposes.

The α_2 mode is a more complex mode that is discontinuous in the curves presented here. This is because the mode changes from radiation to so-called leaky. That is, at lower frequencies the vast majority of the energy in this mode is located in the soil, and so this is considered to be a radiation type mode (and hence is not shown on the figure). As the frequency increases, the energy in the pipe and fluid increases so that it is now propagating with the majority of its energy in the pipe/fluid – this is known as a leaky type mode. It is the leaky modes that are of interest in non-destructive testing, and it is seen that this mode is not strongly affected by the presence of the fluid, at least when the longitudinal speed in the soil is increased. This mode serves to illustrate the complexity of this type of problem, whereby the energy in the modes moves from the solid into the pipe and/or the fluid across the frequency spectrum. This makes it difficult to track particular modes, especially as a large number of radiation modes are often detected. Accordingly, a major issue with the SAFE method, in which hundreds of unsorted eigenmodes are computed, is the identification and sorting of eigenmodes relevant to non-destructive testing.

The results presented here show that as the frequency is increased, the attenuation of particular modes is dependent on the conditions in the soil, as well as the presence of the fluid. However, it is seen that even in such a complex situation, some modes have attenuations that are acceptable for the detection of defects and/or ruptures in pipes. For example, L(0,1) consistently delivers relatively low values of attenuation over the frequency range, and provided one avoids very low frequencies, T(0,1) will also be suitable for detecting defects and leaks. However, at higher frequencies, and into the ultrasonic regime, the problem becomes considerably more complex, and further work is necessary in the identification and sorting of modes. Moreover, the plots presented here are for a large steel pipe, and a reduction in the size of the pipe will facilitate the simplification of the dispersion curves at higher frequencies.

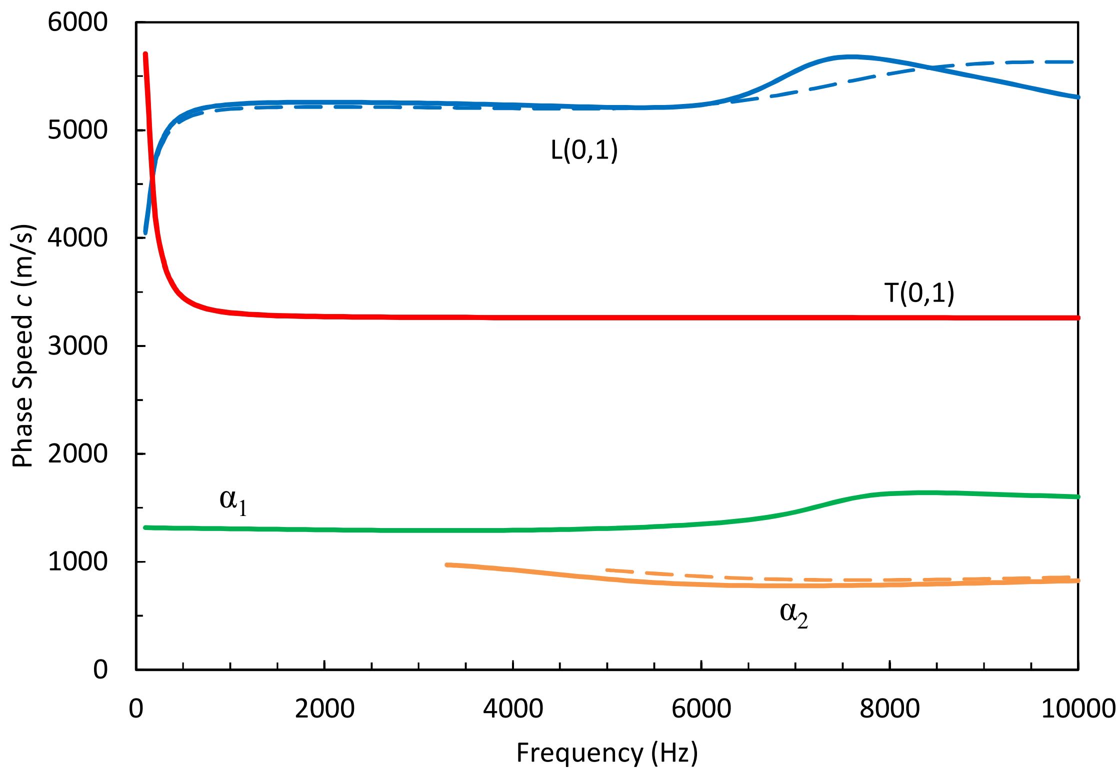

Figure 2: Phase velocity for c_{Lm}=1000 m/s. ———— , water-pipe-soil; — — — · , pipe-soil.

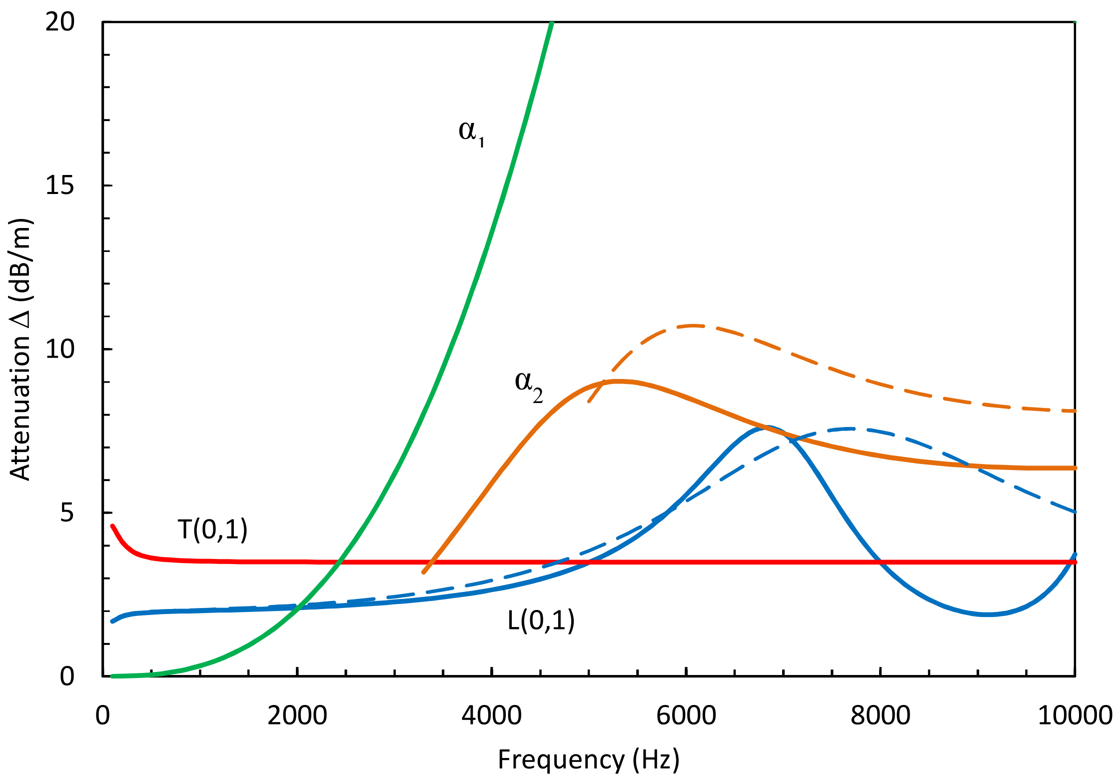

Figure 3: Attenuation for c_{Lm}=1000 m/s. ———— , water-pipe-soil; — — — · , pipe-soil.

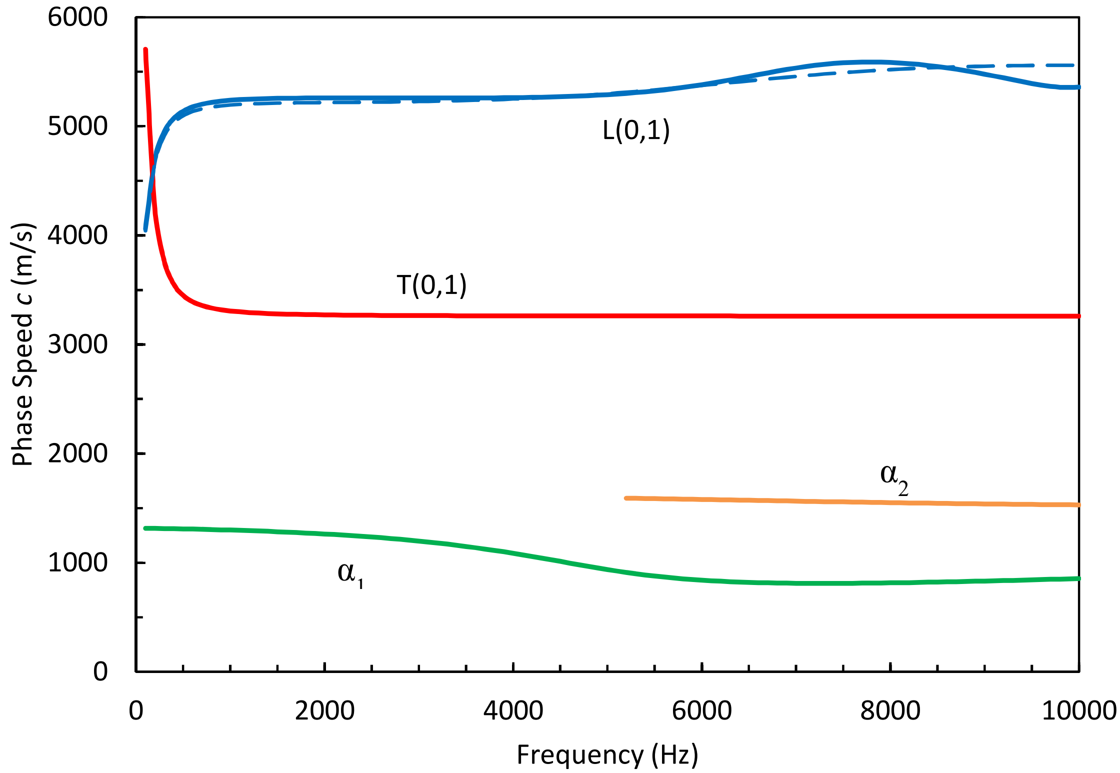

Figure 4: Phase velocity for c_{Lm}=1600 m/s. ——— , water-pipe-soil; – – – · , pipe-soil.

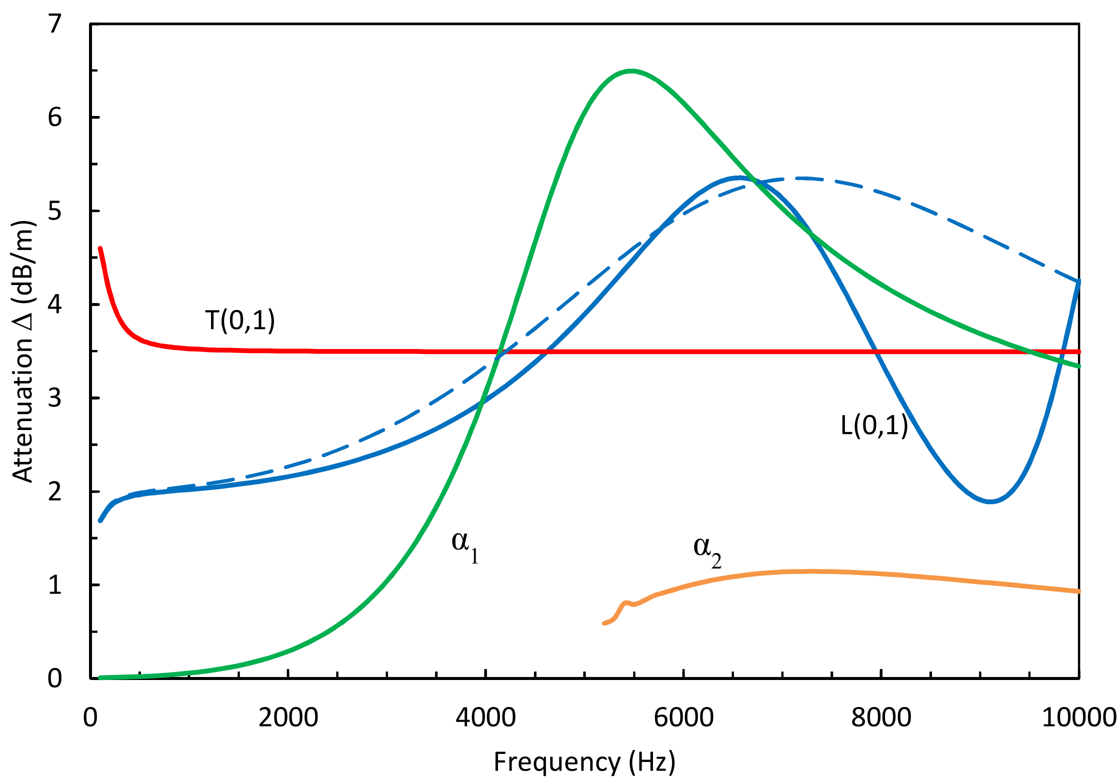

Figure 5: Attenuation for c_{Lm}=1600. m/s. ——— , water-pipe-soil; – – – · , pipe-soil

4. Conclusions

This article presents a summary of a SAFE method suitable for obtaining the leaky eigenmodes for buried pipes filled with a fluid. Dispersion curves are presented over a frequency range of 100 Hz-10 kHz, and it is shown that the fluid has a significant effect on the propagation of the first longitudinal structural mode. Moreover, this effect is also influenced by the properties of the surrounding medium, which in this case is soil. This illustrates the strong coupling between each region, and in order to solve this problem over an extended frequency range it is necessary to include this coupling in any theoretical model. Of course, in reality the boundary conditions are not always as simple as those ones applied here, especially in the coupling between the pipe and the soil, although the results presented here suggest that any changes in this coupling are likely to have a significant effect on wave propagation.

The SAFE method provides a flexible and robust approach to find the properties of guided waves, however the solution of the governing eigenequation delivers an unordered list of eigenvalues. When one is faced with a complex multi-layered problem such as the one presented here, it is a significant challenge to sort and order eigenmodes that are relevant to engineering applications such as non-destructive testing and leak detection. Moreover, the presence of a PML further complicates the issue, and the choice of the properties of this PML can influence the ability of the method to find the desired eigenmodes. Accordingly, there is further work to be done to deliver a SAFE algorithm suitable for efficiently extracting the relevant eigenmodes, and doing this over an extended frequency range that includes the ultrasonic region. This is the subject of ongoing work.

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