DESIGN CURVES FOR ELBOW SILENCERS

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Passive elbow silencers with acoustic fill such as glass fiber, rock wool or foam are commonly used in conventional heating, ventilation and air conditioning systems. Acoustic performance has been estimated for a few basic silencers through the use of simple regression curves available in the literature. Recently, a commercially available finite element program was used to generate design curves to cover a wide range of manufactured elbow silencers. Insertion loss of the silencers is estimated from attenuation rates calculated from the finite element method. The sound-absorbing material is considered to be bulk reacting. Wave propagation in the material is thus included. Results from the model are compared to actual test data. The results showed good comparison between the current model and the test data. Design curves are grouped by using a set of non-dimensional parameters, thereby covering a wide range of conventional elbow duct silencers. Details of the design curves and the mathematical model are presented. Development of the design curves for easy estimation of the acoustic performance of elbow silencers is also presented.

Keywords: Elbow Silencers, Bulk Materials, Insertion Loss, Design Curves

1. Introduction

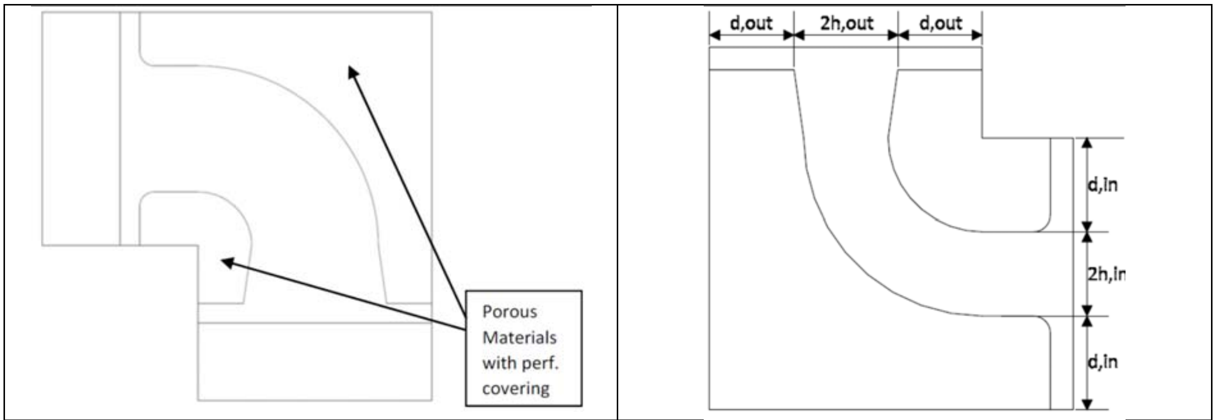
Acoustical performances of simple elbow (round and rectangular) silencers, used in building HVAC systems, have been conventionally evaluated using empirical relations based on laboratory and/or field measurements [1, 2]. However, propagation of sound in duct bends have been studied extensively using modal analysis [3, 4, 5, 6, 7]. Modal analysis was seen to provide insight into the physics of the propagation mechanisms, but the calculation of the insertion loss depends on accurate representation of the modal coefficients. Applying a finite element scheme was shown to be a better estimate of the overall insertion loss of the passive silencers [8, 9, 10, 11].

Simple experiments were conducted, using a standing wave set-up, to evaluate the performance of elbow ducts with duct liners. The experimental results from the liner set-up are applied to calibrate and validate simulation results of commercially available software. Further results from a conventional HVAC system silencer, tested in a certified laboratory, as well as results from a Japanese study are also used to fine-tune the simulation results. The preliminary results of the validation efforts were presented in Ramakrishnan and Shinbin [12].

The details of the generation of the design curves for elbow silencers used in HVAC system ducts are presented in this paper. Details of the mathematical model along with the assumption used in the model derivation are also presented. The results also include the formulation of the design curves in terms of octave band frequencies.

2. Theoretical model

Details of a typical elbow silencer are shown in Figure 1. The main focus is to evaluate the transmission loss of the sound as the flows traverses the corner, treated with porous materials.



(a) (b)

Figure 1: Schematic details of an elbow silencer.

The transmission loss can be evaluated by solving the governing wave equation to determine the inlet and outlet sound power in the duct system. In its most basic form, the acoustic wave equation describes acoustic pressure change in a free field, lossless medium (i.e.,) assuming that the wave is not impeded by heat dissipation, viscosity of the medium or any boundary effects.

In order to apply the acoustic wave equation to an enclosed duct system, boundary conditions that define the effect of the various system components on the acoustic pressure must be applied to the wave equation. Examples include accounting for the effect of the duct walls on the free field sound propagation as predicted by the wave equation, and the change in sound propagation characteristics when the wave travelling through air encounters a different medium (i.e., the liner), having different properties than air.

The behavior of the acoustic pressure in the porous liner material was calculated in accordance with the results of Delany and Bazley [13] and Miki [14]. According to Delany and Bazley, propagation of sound in an isotropic porous medium can be described in terms of the complex characteristic impedance (Z_o) and complex propagation coefficient (γ). The porous materials varied in bulk density and fibre size, described in terms of the flow resistivity (σ) of the material. The flow resistivity was used as the basis for normalizing the results for the purpose of defining the expressions for the Z_o, and γ. Miki updated the results of the Delany and Bazley tests using newer regression models to address an issue found with the performance of the Delany Bazley model at lower frequencies [14]. Z_o, and γ can be evaluated from:

$$Z_o = \rho_o c\left[1 + C_1\left(\frac{\rho_o f}{\sigma}\right)^{-C_2} - iC_3\left(\frac{\rho_o f}{\sigma}\right)^{-C_4}\right] \tag{1}$$

$$\gamma = \frac{\omega}{c}\left[1 + C_5\left(\frac{\rho_o f}{\sigma}\right)^{C_6} - iC_7\left(\frac{\rho_o f}{\sigma}\right)^{C_8}\right] \tag{2}$$

where C_i are constants defined either by Delany and Bazley or Miki, depending on the user selection. As noted above, ρ_o is the density of the medium, c is the speed of sound in the medium, f is the frequency in Hz, and σ is the flow resistivity of the liner material.

As noted above, the wave equation describes the acoustic pressure change in a free field, whereas the system being modelled is restricted by the duct walls. The effect of the duct wall has been accounted for by applying a boundary condition that sets the normal acoustic velocity to zero at the system boundary of the elbow configuration shown in Figure 1. The liner casing also includes perforated metal sheets that form a boundary between the air flow and the liner material. These were accounted for by applying a boundary condition describing the acoustic impedance per Bauer [15],

$$\frac{Z}{\rho_c c_c} = \left(\frac{1}{\sigma} \sqrt{\frac{8\mu k_{eq}}{\rho_c c_c}} \left(1 + \frac{t_p}{d_h} \right) + \theta_f \right) + i \frac{k_{eq}}{\sigma} \left(t_p + \delta_h \right) \tag{3}$$

where μ is the dynamic viscosity, σ is the area porosity (the percentage of the plate area that is represented by holes), t_p is the plate thickness, d_h is the hole diameter, δ_h is the end correction to reactance, and θ_f is the flow resistance. These were set either in accordance with the manufacturer specifications for the actual perforated sheets used (σ, t_p, d_h) or typical values from literature.

The solution of the wave equation with the appropriate boundary condition is used to evaluate the inlet sound power and outlet sound power of the elbow silencer. The transmission loss is then given by,

$$\mathrm{TL} = 10\log\left(\frac{L_{w,in}}{L_{w,out}} \right) \tag{4}$$

where, $L_{w,in}$ is the sound power at the inlet plane and $L_{w,out}$ is the sound power at the exit plane. The solution procedure to evaluate TL is described in the next section.

3. Numerical simulation

FEM (Finite Element Methods) methods were applied to evaluate the TL. The powerful software COMSOL Multiphysiscs was used as the FEM solver [16]. A typical numerical model of a double elbow silencer is shown in Figure 2 below.

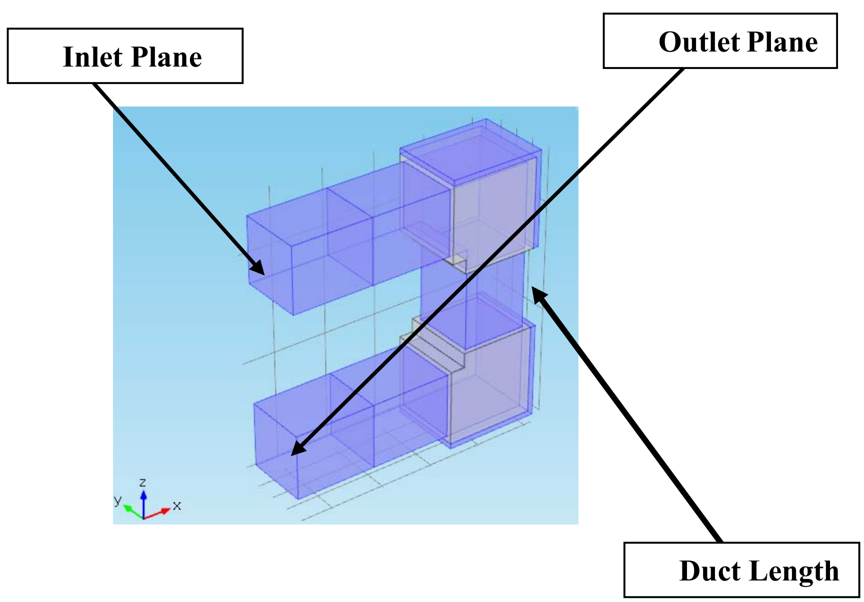


Figure 2: COMSOL Model of a typical 180º Elbow Silencer.

FEM techniques result in very accurate results, provided the data for the input results are accurate. The only major disadvantage is the computing time and storage capacity of the machine used to solve the fundamental wave equations governing both the free air and porous material regions.

The propagation through the material requires the complex wave speed and acoustic density and they can be obtained from either of the two References 13 and 14. The main input parameter for the determination of the wave speed and density is the flow resistivity of the porous material. Two different set of experimental results were used to validate the simulation results of the FEM models and are described below.

3.1 COMSOL model validation

The acoustic simulation of COMSOL was validated with experimental results from two sources. The application of COMSOL to elbow silencers is presented in the following sections.

3.1.1 Laboratory results from Reference 17

Several test models from Itamoto et al. were used to validate the FEM simulation results [17]. The test units in the paper included two square lined elbows, of size 600 mm x 600 mm, identified as 'E' and 'Es'. The unit 'Es' contained a split vane, while 'E' did not. In addition to the insertion loss provided by the two elbows alone, a number of combinations were also tested in the paper. These included two elbows separated by varying lengths of duct, and with the second elbow rotated either 0, 90 or 180 degrees relative to the first elbow. The length of duct, between the two elbows, as shown in Figure 2), varied for each test, between 0 mm and 2400 mm.

Two elbow silencers 'E' and 'Es', as well as three of the 180 degree combinations were modelled in FEM simulations. The combinations included test models with 0 mm and 900 mm of duct length between the silencers: E-0-Es, E-900-Es and E-900-E. The units were tested in 2D, and a comparison to the test data indicated that the model was not replicating the actual results with much accuracy. The large differences could be due to the fact that Reference 17 did not include any description with regard to whether only two or four sides of the duct were lined. Based on the figures provided, it was assumed that the ducts were lined on four sides, meaning that the 2D modelling approach would not be suitable for testing these units if this were so. Furthermore, the flow resistivity of the liner material was not indicated in the paper, so typical values provided by the manufacturer of the liner material (18,000 Rayls/m) was used. One of the elbow silencer (Type E) was modelled in 3D to check if it would result in improved accuracy. It should be noted that modelling in 3D requires much more computer resources than modelling in 2D due to the number of mesh points required to populate the entire duct system at a density of 5 mesh points per wavelength. As such, some of the larger systems with two silencers and extended duct lengths between silencers could not be modelled in 3D due to limited memory and excessive run times. Furthermore, the actual test units in Reference 17 included inlet and outlet duct extensions of over 5 m in length. These too could not be included in the modelling as they represent large volumes that would need to be populated by mesh points at the same density as the system under evaluation. The results of the modelling are compared to the Reference 17 data in Table 1 below.

Table 1: Comparison of COMSOL predictions to Reference 17 tests

Unit	Octave Band Centre Frequency (Hz)					
	63	125	250	500	1000	2000
Elbow E (Itamoto)	2	5	7	9	12	12
Elbow E (COMSOL, 2D)	0	1	4	7	12	14
Elbow E (COMSOL, 3D)	0	4	6	11	14	15

The above results show that the 3D model assuming an elbow lined on four sides performed slightly better, with a maximum difference of 3 dB in the 2,000 Hz octave band.

3.1.2 Conventional HVAC system elbow silencers

One of the silencer manufacturers, located in Ontario, provided a number of existing elbow silencer designs for which insertion losses had been measured in an external accredited laboratory. The silencers were tested under both forward and reverse flow conditions at various velocities, as well as no-flow conditions. The no-flow test results were applied in COMSOL simulations.

It should also be noted that the tested silencers encase the liner within perforated metal sheets and non-perforated metal sheets in order to hold the liners in place. The COMSOL modelling approach was updated to account for the presence of the metal sheet surfaces in the test elbow silencers, using the *Interior Perforated Metal Plate* and *Interior Sound Hard Boundary* conditions in COMSOL (see equations 1, 2, and 3). Results for four typical silencers are presented in Table 2 below.

Table 2: Comparison of elbow predictions to test results

		Octave Band Centre Frequency (Hz)						
		63	**125**	**250**	**500**	**1000**	**2000**	**4000**
SL-1	*a*	10	14	22	36	38	36	30
	b	9	15	23	33	35	42	27
SL-2	*a*	5	7	14	26	24	24	21
	b	4	7	15	27	17	29	23
SL-3	*a*	6	10	20	26	36	35	30
	b	6	8	17	32	34	41	32
SL-4	*a*	10	14	26	42	43	45	38
	b	9	15	23	34	34	42	31

a – average no flow test results (Intertek)
b – average COMSOL prediction for flow resistivity ranging from 8,000 to 18,000

It should be noted, however, that any compression of the liner material to fit the liner casing may result in changes to the air flow properties though the liner material, and in turn add uncertainty to the manufacturer stated flow resistivity value of the liner material. The bulk density of a material factors into the associated flow resistivity value, and compression increases the bulk density which in turn affects the flow resistivity value. It was expected that the insertion losses were generally predicted to agree within 3 dB of the actual test data; however, often a single octave band was out by greater than 3 dB. This is likely due to a change in flow resistivity resulting from the liner configuration. Additional simulations were conducted with varying flow resistivity values, and were able to obtain results that had an improved agreement with the measured data. It is seen therefore, that the flow resistivity provided by the manufacturer may change when the liner material is encased, which strongly influences the insertion loss distribution across the modelled frequency range. Differences in the high frequency range would be subject to the same type of error as suggested by Ramakrishnan and Watson [10, 11], attributable to entrance/exit losses and/or experimental error.

4. Normalizing curves

The main objective of this study was to generate design curves that could be applied to estimate the insertion loss of an elbow silencer based on normalized parameters describing the dimensions of the unit and characteristics of the lining material. Historically such design curves have been established through physical testing or empirically by solving complex equations. Each of these methods can be costly in terms of labour, equipment and computer resources. As such, design curves may be created by running iterations of the model for a variety of elbow silencer dimensions and liner characteristics without having to construct the physical unit. By normalizing these elements of the specimen under evaluation, the design curves may be applied to any similar lined elbow with known silencer information. The design curves provide the attenuation rate (decibels per metre of lining) across a frequency range of 63 Hz to 4,000 Hz octave bands.

The dimension of the duct was normalized using the parameter N. The physical properties of the liner are described in the normalized term, R. The final normalizing parameter applied in the analysis was the normalized frequency, μ. N, R and μ are defined in Eqn. 5 below, (see also Figure 1).

$$N = \frac{d}{h}; \quad R = \frac{\sigma d}{z_{\circ}}; \quad \mu = \frac{2fh}{c_{\circ}} \qquad [5]$$

where σ is the flow resistivity of the liner material in MKS Rayls/m, and $z_o = \rho_o c_o$ is the characteristic impedance of air, f is the frequency in Hz, and c_o is the speed of sound. Both *N* and *R* are dimensionless parameters that describe the properties pertinent to sound propagation in a lined elbow. The remaining property is the centreline length through the lined section of the duct. This was accounted for by calculating the attenuation through the duct, and dividing the results by the length of the lined section such that the results were presented in terms of decibels per unit length. The design curves were then prepared on the basis of the attenuation rate.

In order to create the normalized curves that would be applicable to a wide range of physical elbow properties and liner materials, a number of test units were designed in AutoCAD using one of the typical silencer design provided by a manufacturer. The design that was the basis of the exercise is included in Figure 1. It should be noted that these liner configurations include a taper at the outlet, meaning that the liner width, *d*, and open airspace width, *2h*, are slightly different at the inlet and outlet for each unit. The normalized parameters from the reference papers were derived on the basis of a constant liner thickness. As such, the average values of *d* and *2h* at the inlet and outlet were applied in this study. The values for *2h* in the test cases ranged between 305 mm and 610 mm. The centerline length of the open airway, representing the lined section length of the silencer, varied between 750 mm and 1250 mm.

Varying the unit size and liner thickness caused the parameter *N* to change, while varying the flow resistivity of the liner material, σ, caused the parameter *R* to change. Three test elbow designs were established, each with a different *N* value, and each were modelled in COMSOL with flow resistivity values that resulted in consistent *R* for each unit (as *R* is also a function of liner thickness, which varied between each unit, the flow resistivity required to result in a consistent *R* value varied among the test units). Above approach yielded three values of *N*, and three common values of *R* for each N. Varying these parameters allowed for the design curves to describe the insertion loss for a range of elbow designs. The final set carried forward for modelling included lined elbows that ranged in N (dimensional) values between 0.7 and 1.9, and for *R* values (descriptive of the liner properties) of 5, 10 and 20. Sample design curves for two cases are presented in Figures 3 and 4.

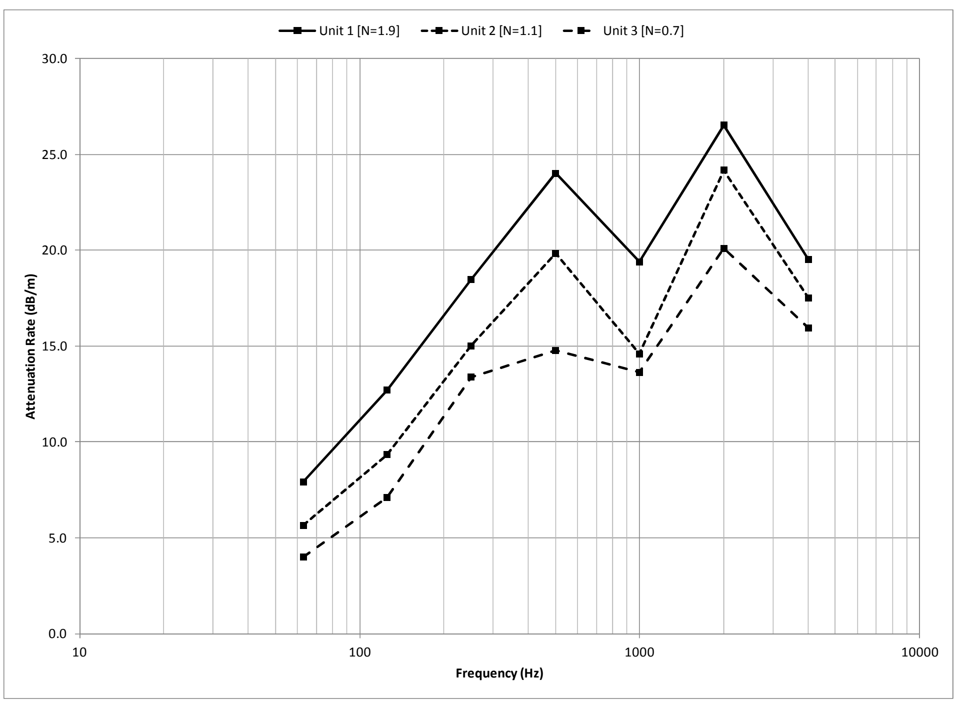


Figure 3: Single Octave Band Attenuation Rates for R = 5; (N =1.9, 1.1, and 0.7)

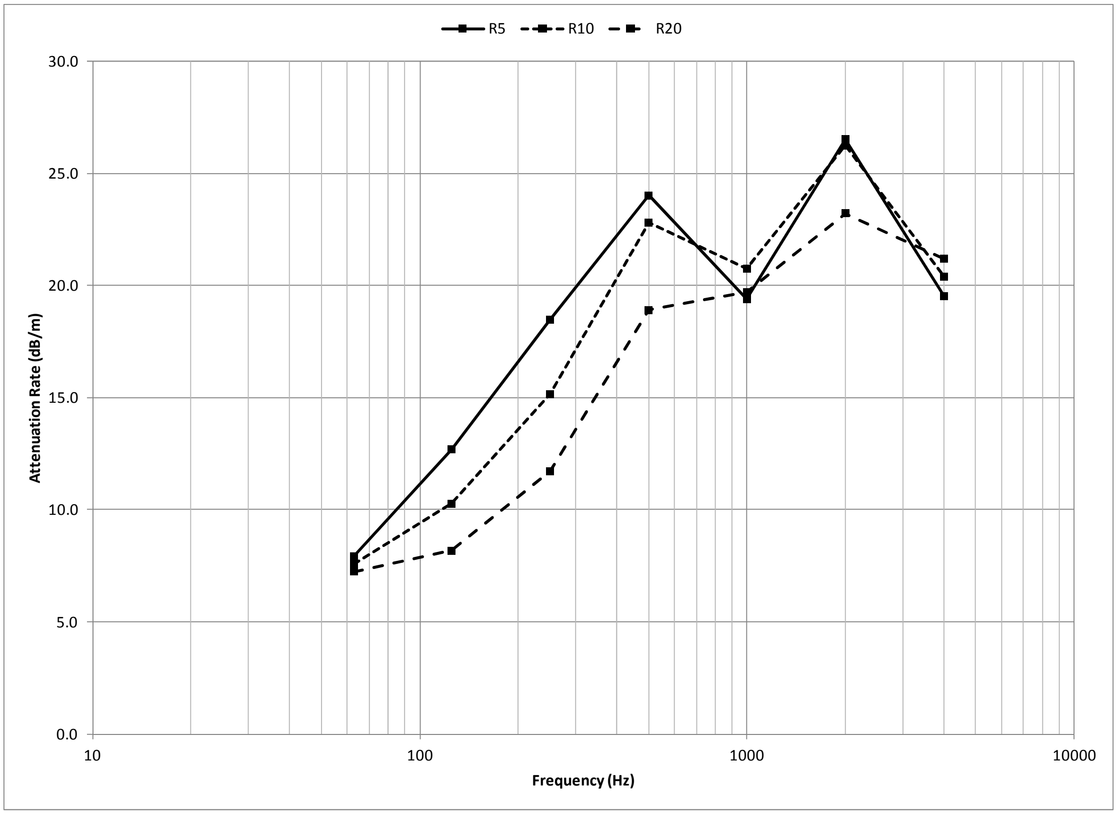


Figure 4: Single Octave Band Attenuation Rates for N = 1.9; (R = 5, 10, and 20)

The above two result sets are quite similar to the behaviour of the performance of elbow silencers presented by Ver [1].

5. Conclusions

Earlier results of elbow silencer performances evaluated in the 1980s were updated by using 3-D FEM simulation techniques. With the aid of three normalizing parameters, the transmission loss of a wide range of conventional elbow silencers used in HVAC system ducts can be easily computed. Only a small set of design curves in octave bands were presented to highlight the usefulness of the design curves.

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