EVALUATING UNCERTAINTY IN THE PREDICTION OF THE PERFORMANCE OF BASE-ISOLATION SYSTEMS DESIGNED TO REDUCE GROUNDBORNE NOISE IN BUILDINGS

Rupert Thornely-Taylor

Rupert Taylor Ltd, Saxtead Hall, Saxtead, Woodbridge, Suffolk, IP13 9QT, UK
email: rmtt@rupertaylor.com

With increasing shortage of land for development in major cities it is becoming increasingly common for high quality residential development to be planned on sites above or near to urban rail systems, many of them constructed without resilient track support. In order to achieve high quality internal acoustical conditions, it is necessary to construct the buildings on base-isolation systems consisting of polymer bearings or springs. In such circumstances it is necessary to predict the future internal noise levels due to groundborne noise from the passage of trains, and practitioners currently use a range of methods, often involving classical vibration isolation theory. This paper will review the underlying physics involved in placing a dynamic system such as a building on resilient bearings, and will identify the principal causes and magnitudes of uncertainty or error in methods used for the calculation of the insert gain of the bearings. A case study is described involving the numerical prediction of the performance of bearings installed to support a tower block directly above a railway tunnel.
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1. Introduction

Predicting the performance of base-isolation systems for buildings for the control of groundborne noise requires the use of a model of the dynamic behaviour of the building. Frequently the model used is that of a single-degree-of-freedom (SDOF) system consisting of a mass on a spring, and the classical equation for the transmissibility of the spring foundation is used as a means of predicting the behaviour of the base isolation system. Such a model necessitates an assumption that the building is a lumped mass m, and the input excitation for all the bearings of stiffness k is equal in magnitude and phase so that only the vertical motion need be considered. The classical equation leads to the conclusion that above a frequency of $\sqrt{2}f_n$ where f_n is the bearing natural frequency $\sqrt{(k/m)}/2\pi$, there is a reduction in the vibration amplitude of the isolated mass, which decreases at the rate of 6 dB per octave for a viscously damped system with critical damping and up to 12 dB per octave for an undamped system, with no upper frequency limit.

In practice no real building satisfies these assumptions. It is common practice to derive the values for the mass supported by each bearing from the structural engineer's load calculations and divide the load in Newtons by the gravitational acceleration g. This does not necessarily give the correct value for the mass above the bearing, as the load force can be due in part to strain in structural members and not simply to the weight of the supported mass. Even a building which approximates a lumped mass may rock, pitch or roll at one of the coupled natural frequencies associated with those degrees of freedom. Few, if any, buildings approximate a lumped mass, and almost all have internal natural frequencies of columns, floorplates and other structures. Tall buildings, which are increasingly the subject of base-isolation designs, may behave as a stack of mass-spring systems with the floors as masses and the columns as springs.

2. The dynamic behaviour of buildings

2.1 Theory

The equation commonly used for calculating the performance of a base-isolation system using a SDOF model is [1]

$$T = \sqrt{\frac{1+\left[2\left(\frac{f}{f_n}\right)\left(\frac{b}{b_c}\right)\right]^2}{\left[1-\left(\frac{f}{f_n}\right)^2\right]^2+\left[2\left(\frac{f}{f_n}\right)\left(\frac{b}{b_c}\right)\right]^2}} \tag{1}$$

where f is the forcing frequency, $f_n = \sqrt{(k/m)}/2\pi$ is the natural frequency of the mass-spring system and b/b_c is the fraction of critical damping. This equation is identical [1] to the equation for the force transmissibility of a vibrating mass mounted on a spring, except that for a case of base-isolation the transmissibility, T, is of displacement, velocity or acceleration and not force. Since groundborne noise is broadly a function of the vibration velocity of radiating surfaces, equation 1 will be regarded as a velocity transmissibility equation in this article.

It follows that even in a case where a building behaves like a lumped mass and is excited only vertically by an input in-phase at all mountings, the velocity transmissibility is not necessarily the insertion loss of the bearings. The velocity of the bearing foundation will not be the same as it is with rigid blocks in place of bearings. For a given input force, the effect of inserting a low impedance element in the transmission path may be to increase the velocity, reducing the isolation calculated using equation 1. However, this assumes that the source has constant force characteristics. The driving point impedance of the base of a mass-on-a-spring system, at resonance, is infinite for zero damping. Consequently, the velocity of the base is zero at resonance, because it cannot move against an infinite impedance. The steady-state transmissibility is infinite, however, because there will be some excitation of the mass at the start of the excitation and its velocity divided by that of its foundation involves division by zero to give infinite transmissibility.

In almost all real cases, in addition to vertical movement, a supported mass will also be excited asymmetrically causing it to rock or pitch. These modes will have their own natural frequencies, each coupled to the vertical SDOF natural frequency. The effect of the coupling is to cause two new coupled frequencies in each case, one lower than the SDOF frequency and one higher. Lateral components in the foundation vibration will excite either or both the two lateral natural frequencies of the mass, and may also excite the yawing mode (rotation about a vertical axis). Thus there are potentially six natural frequencies associated with a lumped mass on bearings.

Most buildings are constructed as assemblies of plates and beams, including beams forming columns to support floor plates. The bearings are likely to be placed beneath columns. The transmissibility of the bearing system will be dependent on the driving point impedance of the structure "seen" by the top of the bearing. An infinitely long column standing on end on a bearing has a driving point impedance of $\rho c_L S$ where ρ is the mass density of the column, c_L is the longitudinal compression wave speed and S is the cross-sectional area of the column.

Vibration is not only transmitted through columns by means of longitudinal compression waves, but also by means of bending waves if the motion of the foundation is not in-phase across the base of the foundation. Plates supported by columns will be excited in one or more of their plate modes as a result of both vertical and bending wave inputs from columns. An example of the driving point impedance of a simply supported, undamped, plate excited near its corner [2] is shown in Figure 1. This shows excursions of the order of 60dB between resonances and anti-resonance at frequencies quite closely spaced.

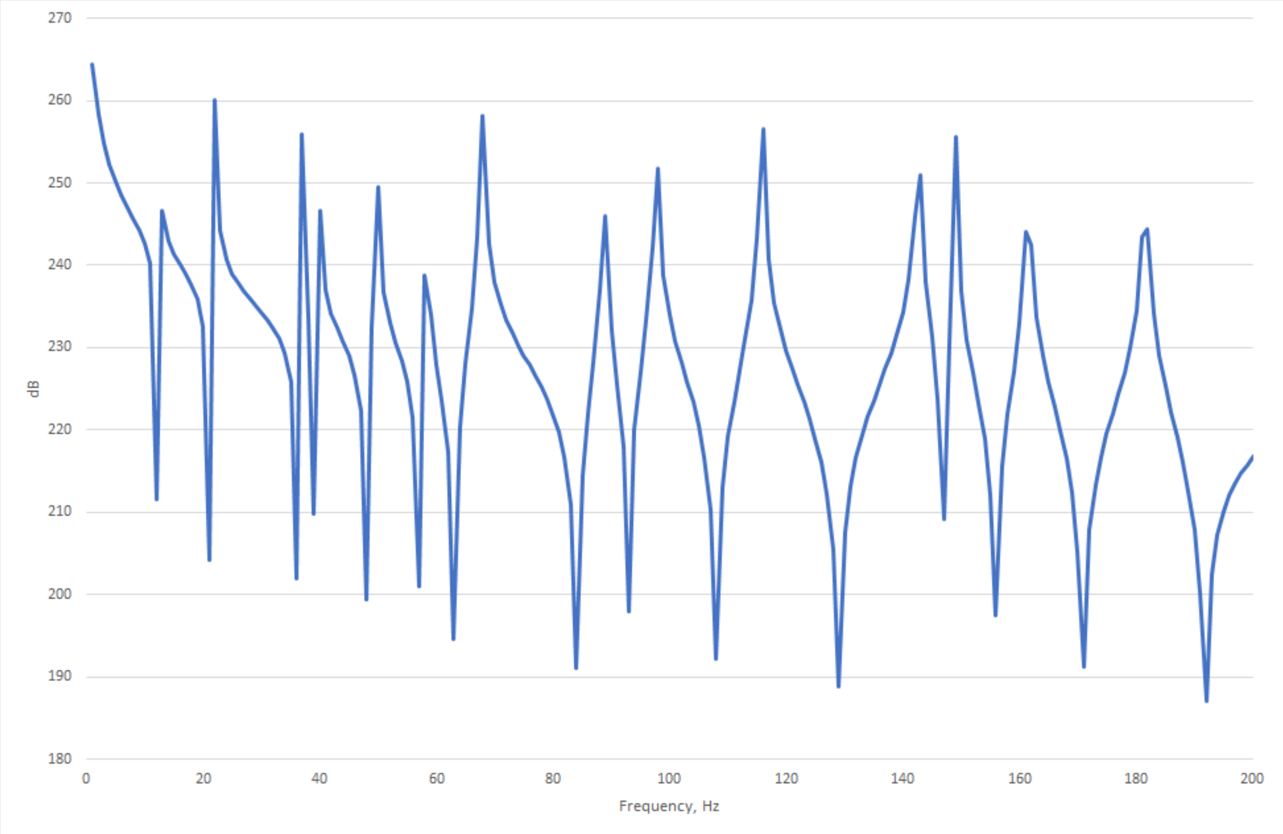

Figure 1. Example of the driving point impedance of a simply supported plate excited near a corner..

The consequence of the existence of the significant number of mechanisms that cause a building to depart from the lumped mass assumption is that whereas the impedance of a mass is simply $i\omega m$ (where ω is the angular frequency, m is the mass in kilograms and i is √-1), the impedance of a building contains many resonances at which the transmissibility of the bearings will be significantly above the SDOF value, and anti-resonances at which it is greater.

The precise behaviour of a base-isolated building is dependent on the source impedance. This is a function not only of the nature of the foundations below the bearings, but also on the coupling between the building and the soil or rock supporting it. A finite plate resting on the top of an elastic halfspace behaves itself like a mass on a spring [3][4], with the mass being not only the building foundation structure, but also a volume of soil near it and the spring constant being a function of the dimensions and the soil dynamic properties. In these circumstances, the SDOF mass-spring system becomes a mass-spring-mass-spring system, possessing two coupled natural frequencies, one below the foundation natural frequency and the other above the supported mass-spring combination.

In contrast to the simple case of a rigid body resting on a half-space is a tall building with many floors supported on columns. Such a case has interesting dynamic properties [5]. For example, if the load on the ground floor columns is summed and considered as a single degree-of-freedom system with the ground floor columns treated as springs based on the compressive stiffness of the columns, it can be the case that the natural frequency of system is in single figures. Should it be necessary to consider base-isolation of such a building, the effect of inserting bearings below the columns would seem intuitively to be somewhat nugatory. However, the actual situation is much more complex than that of a single-degree-of-freedom system, as illustrated in figure 2. The figure is a plot of the velocity transmissibility between the foundation and the first floor of a nine-storey building, showing the peaks caused by the SDOF resonances of the all the columns and masses of the slabs above. The upper curve is for an unisolated building, and the lower curve shows the effect of inserting resilient bearings below the lower floor level, having a natural frequency of 7Hz calculated using the total mass of all the floors as if they were a lumped mass. Although the improvement in transmissibility is much less than the SDOF equation predicts, there is nonetheless an improvement of 10-15 dB. It is notable, however, that there is already a significant reduction in transmissibility in the uni-solated case at frequencies above the natural frequency of the top floor on its columns, in this case above 80 Hz.

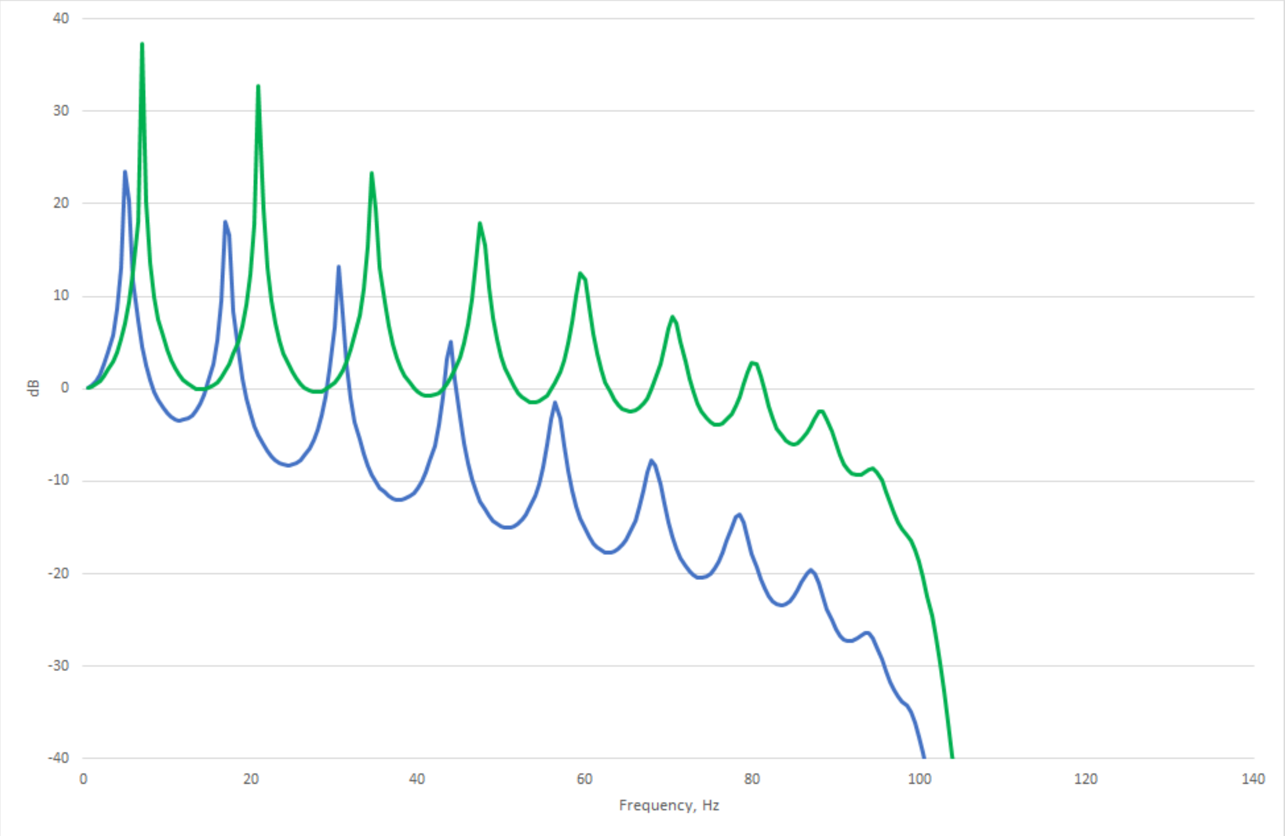

Figure 2. Coupled natural frequencies in a nine-storey building with and without 7Hz base-isolation bearings inserted.

2.2 Numerical modelling

The stack of springs and masses considered in Figure 2 is a major oversimplification, particularly in an asymmetric building structure. In order to study the effect of introducing resilient bearings in a tall building, a numerical model has been created using a finite-difference-time-domain (FDTD) software package [6]. The building in question is situated over an underground railway and has a number of deep basements and podium structures, above which is an 18-storey residential tower with irregularly shaped floors. In order to control groundborne noise from the underground railway in the residential apartments the building is designed with resilient bearings at the heads of the columns and in the cores just below first floor level, being the first of the eighteen floors of the tower. The bearings were assigned stiffnesses based on a SDOF calculation taking the entire mass of the building above the bearings such that they would have a natural frequency of 10Hz.

Many of the dynamic properties of a building are very difficult if not impossible to measure in-situ, and the advantage of a numerical model is that information can be output that would be unavailable for the actual building.

The driving point impedance was output from the model at the top of a bearing underneath a column line, defined as $\mathcal{F}\{F(f)\}^*/\mathcal{F}\{v(f)\}^*$ where $\mathcal{F}$ signifies the Fourier transform of the force, F(f) and velocity, v(f) and * signifies the complex conjugate. It is plotted in dB in figure 3 along with the two idealised lumped-parameter impedances of $i\omega m$ and $\rho c_L S$. It can be seen that while peaks in the spectrum reach the upper $i\omega m$ curve, at most frequencies the curve is closer to the lower $\rho c_L S$.

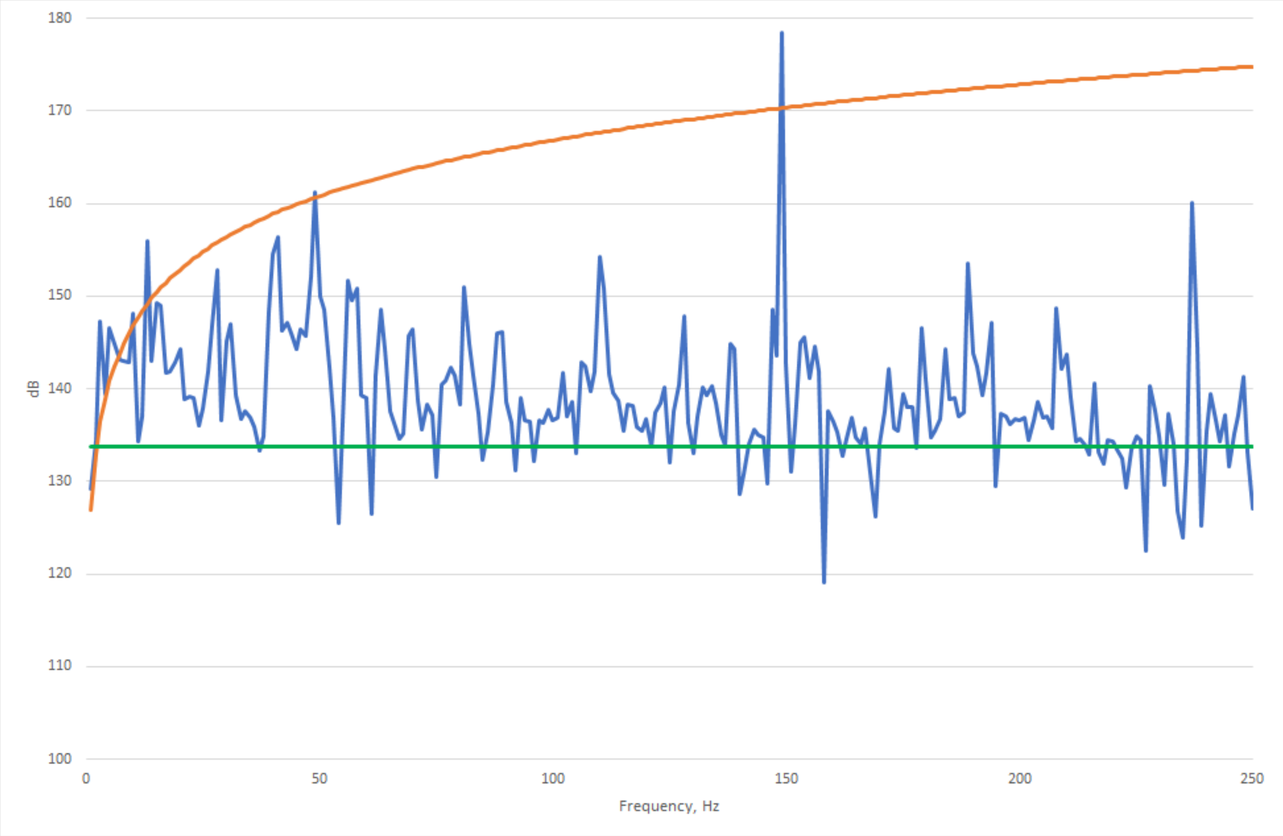

Figure 3. Driving point impedance of (1) column above bearing (middle curve) together with (2) an equivalent lumped mass (upper curve) and (3) an infinite column.

Figure 4 shows the velocity transmissibility of the bearings along with the SDOF transmissibility curve. Figure 4 shows, in 1/3 octave bands, the velocity transmissibility relative to the base velocity of the building with the bearings in place, and Figure 5 is the velocity transmissibility relative to the base velocity of the building with rigid support. So that it takes account of the effect of the bearings on the source below as well as the structure above. Of particular note is the absence of a peak at the SDOF bearing natural frequency, and the presence of two coupled peaks, one above and the other below the SDOF natural frequency, resulting from the coupling between the structure above the bearings and the foundations below. Comparison of Figure 5 with Figure 4 shows that when the change in the foundation velocity is taken into account, the actual transmissibility more closely resembles that predicted using the driving point impedance of an infinite column than a lumped mass.

Figure 4. Velocity transmissibility of the 10Hz bearings (1) from the FDTD model (middle curve) together with (2) an equivalent lumped mass (lower curve) and (3) an infinite column (upper curve).

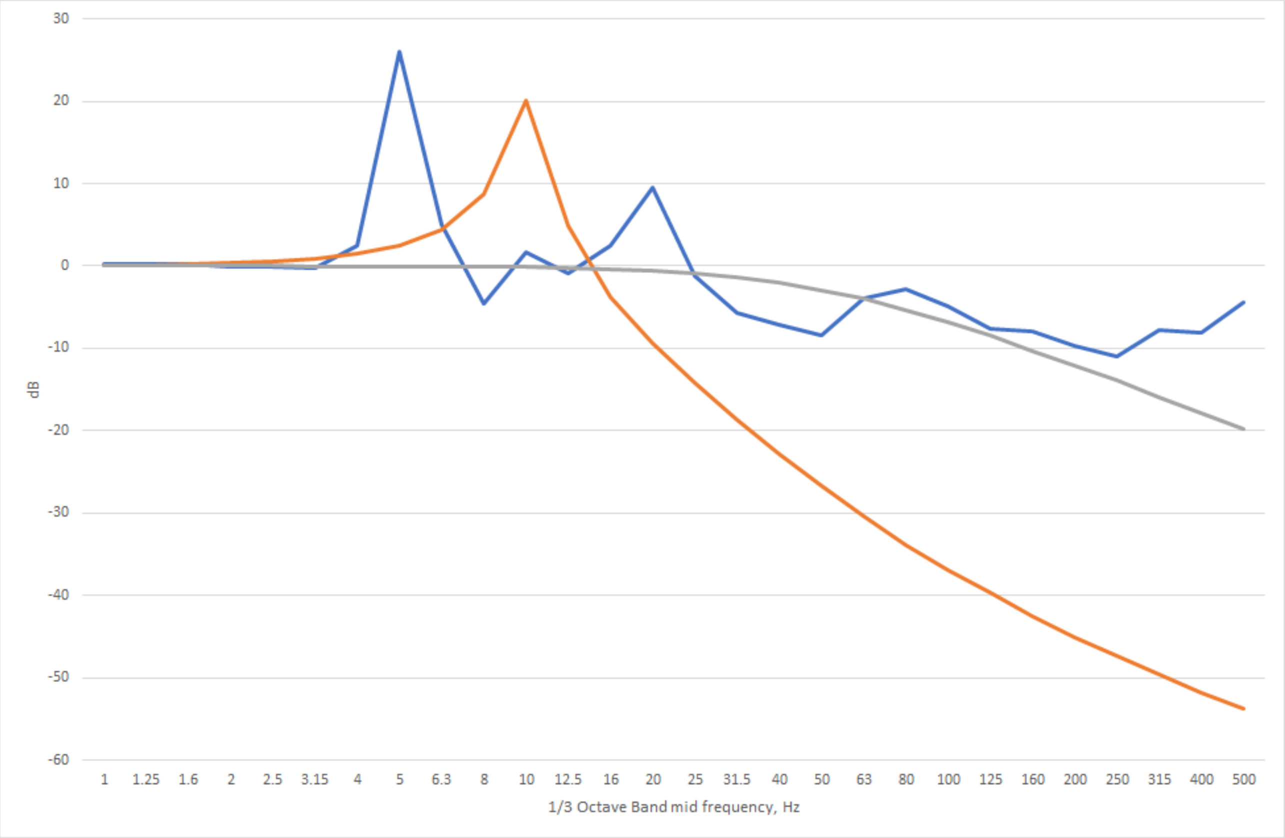

Figure 5. Velocity transmissibility of the 10Hz bearings relative to the base in the unisolated building(1) from the FDTD model (middle curve) together with (2) an equivalent lumped mass (lower curve) and (3) an infinite column (upper curve).

3. Conclusions

It can be concluded the predicting the performance of a base-isolation system over the frequency range relevant to the reduction of groundborne noise using a SDOF model is likely to lead to an over-optimistic result is the base of large, tall or complex buildings. The reasons for this have been discussed and primarily relate to the dynamic response of such buildings which will limit the driving point impedance of the structure “seen” by the top of the bearings. The additional degrees of freedom of a lumped mass are only a partial explanation. Additionally, the use of a simple velocity transmissibility equation can be misleading, as the velocity of the base is affected by the insertion of the bearings. The SDOF model also fails to allow for coupling between the mass-on-a-spring of the building and its bearings and the mass and spring system which exists in the foundation.

Numerical modelling is capable of taking all these matters into account, and provides the most detailed method of predicting the performance of base isolation systems.

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