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THE DESIGN AND APPLICATION OF MODULAR, ACOUSTIC DIFFUSING ELEMENTS

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1. INTRODUCTION

The need for acoustic diffusing elements has been expressed almost since the beginning of the scientific approach to acoustic design. In concert halls, particularly in large modern ones, there has always been the necessity for minimising acoustic absorption (to make the most of what instrumental and vocal energy is available) whilst simultaneously avoiding discrete echoes or reflections. Designers have gone to great lengths to reduce the unfavourable aspects of this compromise.

Even in much smaller rooms, such as studios and control rooms, there is a need to avoid discrete reflections, although the need to maintain the sound energy is not so important. In the case of stereo monitoring this avoidance of discrete reflections, which can seriously affect the image localisation, has been achieved by such an extensive use of absorption that the rooms have become almost anechoic. Apart from being very oppressive to work in, this approach gives rise to other acoustic problems.

Historically, diffusion alone (that is, without very much accompanying absorption) has been attempted by the deliberately acoustic or incidental decorative dispositions of hard reflecting surfaces of various shapes. Concert halls, with large spaces and long distances have been reasonably well-served by this approach. BBC music studios, from the very early days of broadcasting, have incorporated some form of hard diffusing shapes. These have been based on part-cylindrical or sharp-edged prismatic solids and have been mostly only partially successful. Any such regular structures tend to introduce repetitive reflection structures which may be audible as colorations.

Fortunately, there exists a class of mathematical functions which, if implemented as solid physical objects, achieve some degree of diffuse reflection. These are derived from the theories of numerical sequences based on prime numbers. They were first developed and described by M.R. Schroeder in the mid-1970s [1,2]. They have been investigated by others since [eg, 3] and used in some Concert Halls [eg. 4]. Versions of these diffusers are marketed commercially.

2. PARTICULAR BROADCASTING STUDIO PROBLEMS

The solution of a number of different studio acoustic problems might be assisted by the availability of some form of wideband, diffusing modules. Large BBC music studios are now more likely than they were to contain an audience element. This gives them some of the same types of problems as real concert halls, for example, requirements to minimise both the total absorption and the sound energy which is vertically incident on the audience. Smaller studios and control rooms need to avoid significant specular reflections as mentioned above, principally to avoid distortion of the stereo perception.

However, the most immediately pressing problem is that of floor-ceiling multiple reflections (flutter echoes). This problem is exacerbated by the current trend to very 'dead' rooms which may to some extent itself be as a result of the non-availability of diffusing elements. This leads to an acoustic design in which all of the wall surfaces are heavily treated. The resulting discrepancy in mean absorption coefficient between the vertical and the two horizontal axes becomes very large. This is further worsened by the current demand for 'acoustic tile' ceilings and the practical requirements for carpet tiles rather than 'proper' carpet (at least in control rooms). The overall effect of this design philosophy is to make a room which, at frequencies up to about 1 kHz, has practically no absorption for sound travelling in the vertical direction and an average coefficient of about 0.5 for the two horizontal directions. It is currently thought that this asymmetry is responsible for sometimes very severe 'honks' or colorations, subjectively centred on about 500 Hz, although the investigation work is presently incomplete. It appears to be rendered acceptable (sometimes barely) by the use of carpet on underlay in studios (probably because of the higher absorption around 500 Hz) or by the presence of technical equipment, particularly the control desk, in control rooms (because of the increased diffusion). A ceiling which reflected little of the incident sound energy back in the vertical direction might eliminate the problem entirely.

For most acoustic treatment purposes, the BBC uses a system of modular absorbers, originally developed in the early 1960's but significantly added to since then [6,7,8]. This is a system of nominally 600mm square modules, of a number of different depths (in the range 50mm to 300mm). Variations in absorption coefficients as functions of frequency are obtained from different infill details and front-panel materials and finishes. The range presently includes 12 principal types, of which about five or six are in widespread and regular use. To fit in with this range of absorbers, a diffuser would need to be nominally 600mm square and around 200mm in depth.

3. THEORY

3.1 General.

The underlying theory is perfectly well set-out in the original papers[1,2] and analysed extensively in [3]. The main purpose of this paper is to describe adaptations and implementations of the underlying concepts to produce a design for a practical, modular system of diffusing elements which can be used in the acoustic treatment of studios and other rooms in the same way as modular absorbers. To this end, sufficient of the theory is repeated here to allow the arguments to be followed without the reader having necessarily to resort to the prime references.

3.2 Quadratic Residues.

Consider a sequence of complex numbers:

$$r_n = \exp(i\varphi_n) \qquad \ldots\ldots\ldots\ldots 1$$

where $\varphi_n = 2\pi n^2/p$, n is an integer, p is a prime number.

φ_n and r_n are both periodic with period p, and n^2 can be replaced by its residue modulo p. For example, for $p = 17$ the resulting sequence of residues is:

$$(n^2)_{mod 17} = 0,1,4,9,16,8,2,15,13,13,15,2,8,16,9,4,1,0,1,4,\ldots \qquad \ldots\ldots\ldots\ldots 2$$

The relevant property of the sequence r_n in the present context is that its discrete Fourier transform has constant magnitude. This transform may be interpreted in terms of the energy contained in a number of different spatial frequency components. In practical acoustic terms, if we can generate a reflection wavefront with phase jumps according to (1) at regular intervals then the acoustic energy will be reflected equally in 'all' of the spatial directions. (In fact, the number of different discrete directions (lobes) is related to the number of discrete Fourier components in the transform domain.)

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These phase jumps can be implemented by reflecting a plane wave from a reflection phase grating, as illustrated in Fig. 1. This represents a section through a solid structure which consists of number of 'wells' of different depths. The wells are regularly spaced and have rigid dividing walls and terminations. A sound pressure wave entering a well will propagate to the closed end, be reflected and return to the open end after a time delay $2d_n/c$, where d_n is the depth of the nth well and c is the velocity of sound. This corresponds to a phase change of:

$$\varphi_n = 2d_n \cdot (2\pi / \lambda_l) \qquad \ldots\ldots\ldots\ldots 3$$

where λ_l is the wavelength.

To construct the entire wavefront, the depths of the wells must be given by:

$$d_n = \lambda_l \varphi_n / 4\pi = (\lambda_l / 2p)(n^2)_{\mathrm{mod}\,p} \qquad \ldots\ldots\ldots\ldots 4$$

In the resulting phase grating, the maximum well depth is approximately equal to half λ_l, the longest wavelength to be diffused. The range of wavelengths diffused extends down to $\lambda_l / (p-1)$. The width of the well should be less than half the minimum wavelength to ensure reasonably plane propagation in the wells. Such structures are, for obvious reasons, called Quadratic Residue Diffusers.

Thus, to put some numerical values to these parameters, if the total module depth is 200mm then the lowest frequency for theoretical diffusion is 860 Hz. A sequence length of 17 (as in 2 above) will give an upper frequency limit of 13.76kHz. The well-width would have to be less than 12.5 mm and, thus, the overall repeat distance of the sequence would be a maximum of 212 mm.

Such diffusers are marketed, mainly for low frequencies. They are one-dimensional structures which diffuse only in one plane. In the orthogonal plane they are specular reflectors. They are useful (and fairly widely used) in circumstances where the lack of diffusion in the 'specular' direction may not be important, for example in rooms with a large plan-to-height ratio. They are also the type used in the rooms described in Reference 4.

3.3 Primitive roots.

The class of diffusers described above has the property of nominally uniform reflection. There is a second class which has the somewhat modified property of not reflecting any energy at all in the specular direction (the discrete Fourier transform has a null at -(angle of incidence)). These are based on a similar number-theoretic principle, that of primitive roots.

A theorem states "Every odd prime number p has primitive roots g defined by the fact that g^n is not congruent to $1_{(\mathrm{mod}\ p)}$ for all $0 < n < p-1$". In practical terms this means that, over that range of n, the expression g^n gives all $(p-1)$ different (non-zero) numbers. For example, 2 is a primitive root of 11 and:

$$(2^n)_{\mathrm{mod}\ 11} = 2,4,8,5,10,9,7,3,6,1 \quad \text{for } 0 < n < p \qquad \ldots\ldots\ldots\ldots 5$$

This sequence may be used in place of that given by (3) to generate a different type of diffusing structure, but with the same general principles. The resulting structures are called, not surprisingly, Primitive Root Diffusers.

4. PRACTICAL IMPLEMENTATIONS AND TWO-DIMENSIONAL DIFFUSERS.

4.1 General.

The general principles of implementing these one-dimensional diffusers has been outlined above. In practice, commercial ones are usually assembled from fairly large numbers of timber sections and are decoratively finished and architecturally featured to reflect their 'exotic' origins. They are usually heavy and expensive. In some cases, there may also be excess acoustic absorption and other problems because of the inherent resonances of the large areas of poorly restrained and relatively non-rigid wooden panels.

4.2 Omitting the dividing walls.

The theoretical basis of these diffusers assumes that each of the wells is divided from its neighbours by a partition. This is necessary for the ideal behaviour but creates complications in manufacture. The partitions must be fairly stiff and, especially for the Primitive Root Diffuser, very thin at the front edge to avoid re-generating the specular reflection component which the design seeks to eliminate. If the partitions were to be omitted, the performance would be theoretically unaffected for the special case of normal incidence. At oblique angles of incidence some departure from the ideal performance would have to be accepted.

For the particular problem of floor-to-ceiling multiple reflections (or, indeed, almost any occurrence of 'flutter echoes'), the condition of normal incidence is essentially assured. In other cases, the resulting theoretically inferior off-axis performance may still be adequate in practice. The benefits of this simplification are especially great if mass-production is to be considered. It is to be noted that a photograph in reference 3 shows the omission of the dividing walls, although the text makes no reference to it.

4.3 Two-dimensional structures.

In what follows, the term "one-dimensional" refers to a diffuser, which although actually being a solid object, has a section which is a Primitive Root or Quadratic Residue sequence in only one direction. In the other face direction, the section is invariant. The term "two-dimensional" is used to describe a diffuser in which the sections of both plan directions are parts of number-theoretic sequences.

The one-dimensional modules described so far diffuse only in one plane (strictly two intersecting planes - of the incident sound 'beam' and of the reflected 180 diffusion respectively). Extending the diffuser into two dimensions so that it reflects an incident sound beam into a solid angle of 2π steradians requires that the sequence is in some way extended or folded away from the straight linear form.

Ref. 3 shows, without explanation, such a two-dimensional Quadratic Residue Diffuser. It is based on a sequence length of 7. By inspection, the construction method has been determined as first generating the 7-step sequence horizontally and vertically along two adjacent edges to the required length (ie. numbers of sequence repetitions). The remainder of the sequence surface is generated by taking the sum of the vertical (column) and horizontal (row) indices modulo 7. No justifications are given for such a construction, but it may be assumed that such a structure can be shown to have, in two dimensions, the same kind of properties as the one-dimensional structures have in one dimension.

A second two-dimensional construction is shown in reference 3 for Primitive Root Diffusers. The primitive root sequence for a prime number p has p-1 elements. The number (p-1) will have a pair of factors, say x and y. For example, for p = 13, 12 = 4 . 3. These two factors could represent a two-dimensional matrix which is just large enough to accommodate all p -1 elements. If the sequence elements are written down diagonally in the matrix in horizontal and vertical positions modulo x and modulo y, then an array may be generated:

$$\begin{matrix} a & j & g & d \\ e & b & k & h \\ i & f & c & l \end{matrix}$$

where $a, b, c, ...$ represent the successive elements of the sequence.

It can be shown that the Discrete Fourier Transform of this structure has the same properties in two-dimensions as the one-dimensional transform of the linear array. A restriction, again not given in the reference, is that x and y must themselves have no common factors.

If required, approximations to Quadratic Residue-based two-dimensional diffusers could also be made in the same way, by dropping one term from the sequence. By analogy with pseudo-random sequence noise generators (based on maximal-length binary sequences), the loss of one term from a long sequence should give negligibly small errors.

5. PRACTICAL CONSIDERATIONS AND SIMPLIFICATIONS.

The theory so far has shown how number theory might lead to the design of a diffusing module. There are some practical considerations which may lead to compromises.

The first, the elimination of the dividing walls between the cells, has already been dealt with. It would be quite impractical to construct significant areas of two-dimensional diffusers in an economical way if it were essential to retain the dividing walls. In general, except in the special case of normal incidence, the omission of the walls will cause some departure from the ideal spatial response. It was necessary to establish whether, in general applications, such simplified modules would be sufficiently useful to justify their development.

The modules are also designed to work over a definite frequency band but their performance outside those theoretical limits might be important. The floor-ceiling, multiple reflection problem is a particular case. It is unlikely that a total depth could be accommodated which allowed the theoretical performance down to the necessary low-frequency limit (say 400-500 Hz). Therefore, in this important frequency range the diffusers would be 'working' below their theoretical lower frequency limit. It remained to be demonstrated whether the out-of-band performance would be useful.

One very important consideration for large areas is the effect of repeating the diffuser pattern by stacking identical modules side-by-side. Ref. 3 gives an extensive analysis of this factor for one-dimensional diffusers and shows that the concentration of energy into discrete spatial frequency components increases with the number of pattern repeats. This is an undesirable feature which ultimately results in a small number of sharply-defined specular reflections instead of the even diffusion which is the objective. In two-dimensional diffusers with much longer sequence lengths and correspondingly greater numbers of discrete spatial components, this may not be such a serious problem but a means of controlling it could be useful. Figs. 2 and 3 show comparable 'slices' through the reflection pattern for a particular diffuser design, with one module and an array of 4x4 modules respectively. The problem arises because of the adjacent repeats of the sequence. It was thought that a module could be designed which actually incorporated a pattern of different-length sequences, each of which would have its own 'lobe' patterns, then the sequences would not be adjacent and the overall pattern less repetitive - in fact it would be a super-imposition of several diffraction patterns. (It will be seen later that a Primitive Root diffuser cannot be made in this way, but that fact had not been established before the manufacture of prototypes and the first field trials).

Another problem which is clearly relevant but which so far has not received any theoretical consideration is that of the near-field performance. The simple theory considers only the far-field response. In large spaces this is probably sufficient. However, most applications in smaller rooms involve near-field conditions. Clearly, there will be some modification of the theoretical performance if the the incident sound wave is not plane, as assumed. There will also be differences if the observer or microphone is close to the diffuser. Whether these factors will be practically significant remained to be demonstrated. Initially, it was assumed that even if the performance were not 'theoretical' some significant practical benefits would be obtained.

6. THE DESIGN OF A PRACTICAL PRIMITIVE ROOT DIFFUSER.

An example may be taken of the design of a diffusing module to replace a standard A3 modular absorber. Such a module will have a face dimension of 580 mm x 580 mm and a depth of about 185 mm. Because only some parts of the module project to the maximum extent, it may be acceptable to increase the depth slightly, to 200 mm, to obtain a slightly lower cut-off frequency. This corresponds to a lowest frequency of 860 Hz. An upper frequency limit of 12 times that may be thought adequate - about 10 kHz. This corresponds to a cell width of about 17 mm. The total width of the module divided by this cell width gives about 34, that is, the whole module is composed of about 1156 square cells of about 17 mm sides.

At this point two fundamental constraints need to be considered:

a) a square array is not possible because of the common factor limitation

b) a single sequence should not fill the whole area because of the repetition effects.

Thus, the problem reduces to one of finding an arrangement of rectangular arrays, each of which contains (a prime number-1) cells, with sides with no common factors and whose total size is 34 or greater in each direction. In fact, this is not easy for 34 x 34 but a solution for 34 x 35 is shown in Fig. 4.

The high frequency limit needs some further consideration. Sequence lengths of the order of hundreds would theoretically work up to hundreds of kilohertz. The cell size (and the complete diffuser size) would be correspondingly minute. This is obviously both unnecessary and unwanted. The size of the cell has already been set approximately by an *a priori* decision about the upper frequency limit. The additional detail in the hundreds of different cell depths given by the sequence is unwanted. If the well depth sequence were to be quantised into a suitably small number of different depths then, up to some frequency, it would make no difference. Frequencies above that will, by definition, be above the frequency range of interest. Thus, by quantising the cell depths into, say, 16 different values, the upper frequency limit may be made into the same as it would have been for a sequence length of 16 - that is, for this example, 16 . 860 Hz (=13.76 kHz).

The modular diffuser described above is a very complicated structure, especially for hand fabrication of prototype units. With nearly 1200 pieces per module, the prospect of constructing one was fairly daunting. The idea of fabricating a batch of 16 for field trials was unthinkable. A simplified version, based on a design upper frequency limit of 3.4kHz was developed. In a similar way to the more detailed version, this was based on a square array of 12 x 12 elements, subdivided into smaller sub-arrays and quantised into four different depths. The elements were nominally 50mm square. Fig. 5 shows the layout of the simplified module and Fig. 6 shows, as an example, the results a computer calculation of the reflection pattern of this diffuser for a frequency of 1720 Hz, an angle of incidence of $0,-\pi/4$ and a reflection plane at an angle of 0. This diffraction pattern is for the theoretical diffuser with well walls and with quantised depths. The absence of the expected null at -(angle of incidence) should be noted. Further study of this theoretical model showed that the principle of assembling smaller PR diffusers into a larger module did not work - the result was the same type of behaviour as a QR diffuser. At the time of the experimental work and the field trials, this had not been established.

7. PRACTICAL RESULTS AND FIELD TRIALS.

A sample batch of 16 modules of the simplified design was constructed using hand-assembly of lengths of pvc tubing with moulded pvc end caps. Available stock sizes did not include the 48.3mm size necessary to achieve the nominal 580mm overall dimensions. Instead, 45mm, square-section tubing gave an overall size of 540mm and a border of 20mm all round. Fig. 7 shows a photograph of the completed prototype module.

Attempts were made to measure the reflection properties of a square array of four of these modules in an anechoic chamber at frequencies close to the lower cut-off frequency (860Hz). At these frequencies, a small number of lobes would be expected (about 4) and attempts were made to identify them. In practice, even in an anechoic room, the small spurious reflections from all of the other surfaces add up until the wanted response is swamped. Even using time-domain filtering, no more than the broad peaks of the main lobes could be positively identified. These appeared to be approximately in the expected locations.

Several test installations have been carried out using the prototype set. In one test room (which was being used to investigate the acoustic problems of control rooms and in which the vertical 'honk' was evident when using carpet tiles on the floor), the installation of 16 modules on the ceiling eliminated the problem totally.

The second test was a field trial in the Annexe to the new Television Music Studio at Television Centre. This room had been originally designed with the usual, nearly-dead acoustic. Subsequently, the users asked for the room to be brightened by replacing some of the absorbent ceiling tiles with plain sheets of plasterboard and removing the carpet underlay. Predictably, because of the heavy wall treatment, this led to severe 'honking'. Replacing 16 out of the total of about 92 ceiling tiles by modular diffusers significantly improved the situation, to the extent that the users then suggested the removal of the carpet altogether. At the time of writing, this had not yet been done. The prototype set of diffusers has remained in the room ever since (Fig. 8). It was necessary to construct another set of prototypes in order to continue the evaluation!

In both of these cases the improvement could be identified subjectively as well as objectively by measurements of the decay characteristics using directional microphones. The prolonged decay in the vertical direction associated with the subjective problem is usually quite clearly seen.

Both of these tests gave quite surprising and unexpectedly good results. Because of the low density at which the modules could be installed, less than 20% of the area in both cases, it had been expected that the results would be uncertain. The concept and the theory had been based on the premise that the surfaces would be treated fully with diffusers. In fact, it was shown that significantly less than that amount produced complete subjective and objective cures of the vertical 'honk' problem.

8. CONCLUSIONS.

The well-established design principles of acoustic diffusers based on number theory have been outlined, together with some extensions to modules which can diffuse into a solid half-space rather than into a half-plane.

The result is a module which reasonably fits into the normal BBC scheme of acoustic treatment and which may be used to control the sound energy flow patterns within a room by means other than absorption. In field trials, these modular diffusers appear to be effective in curing some acoustic problems. If the initial promise is maintained throughout further development and field trials and a reasonably economic manufacturing process can be developed, the result will be a useful addition to the range of acoustic control products.

Some further work remains to be done - on theoretical analysis, on laboratory measurements, on the development of a more economical manufacturing method and on their use to solve other acoustic design problems - perhaps ultimately reducing the need for so much acoustic treatment in studios and control rooms.

9. REFERENCES.

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10. ACKNOWLEDGEMENTS

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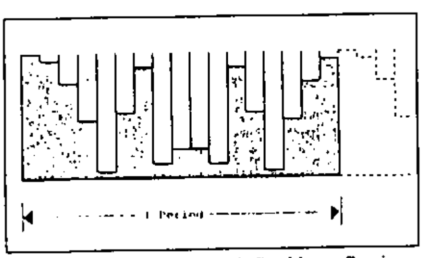

Fig. 1. Section of Quadratic Residue reflection grating.

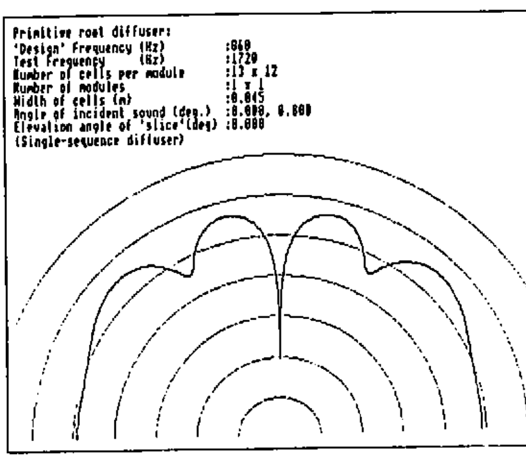

Fig. 2. Calculated spatial response for single Primitive root 2-D diffuser.

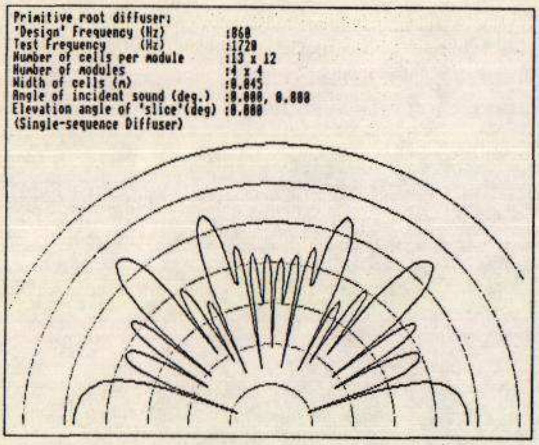

Fig. 3. Calculated spatial response for 4x4 array of Primitive root 2-D diffusers.

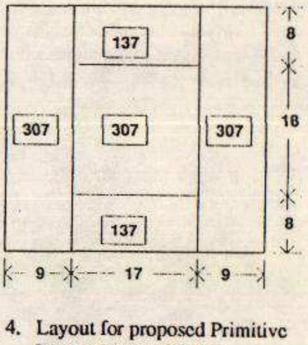

Fig. 4. Layout for proposed Primitive Root modular diffuser.

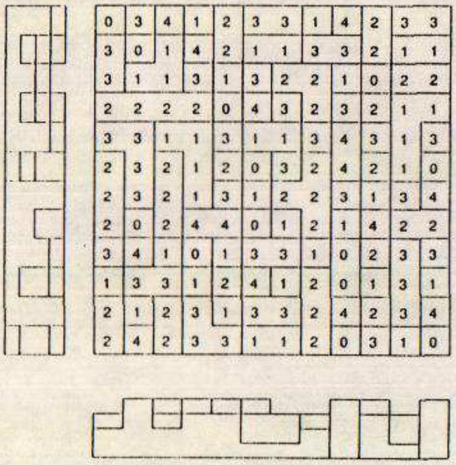

Fig. 5. Layout of prototype diffuser.

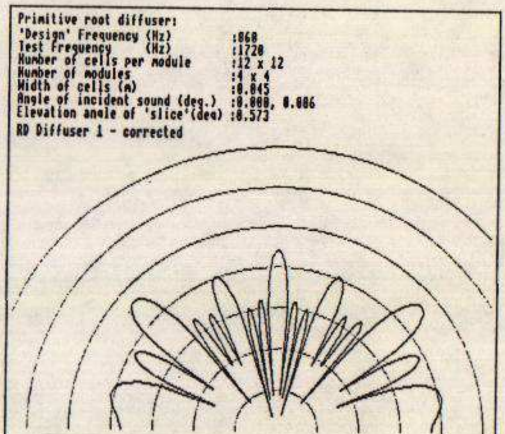

Fig. 6. Calculated spatial response of prototype diffuser.

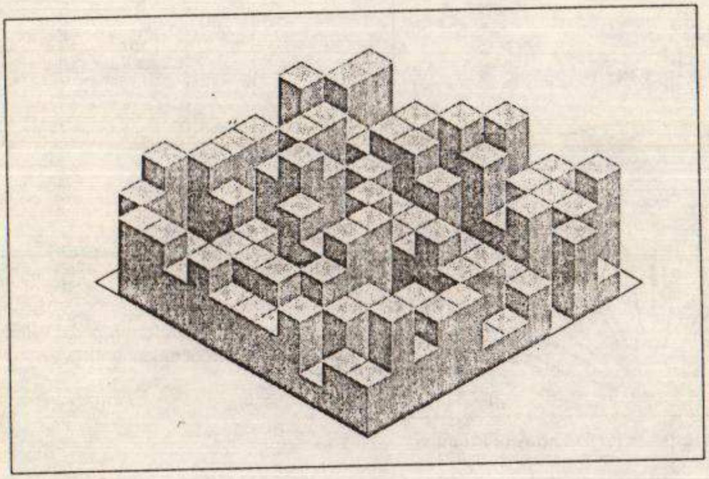

Fig. 7. Prototype modular diffuser.

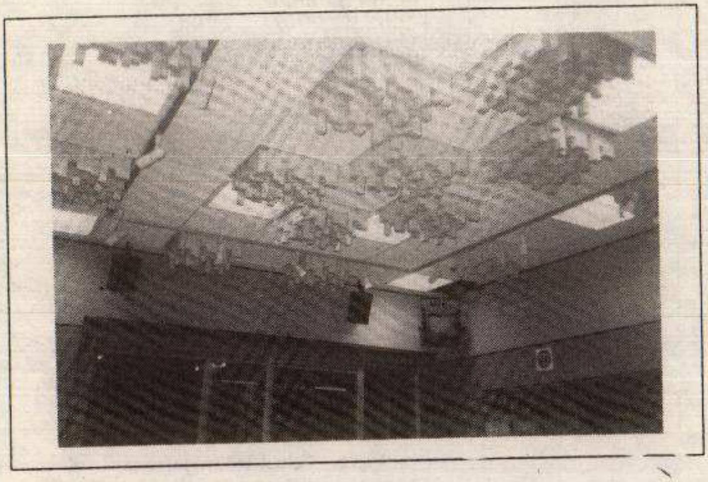

Fig. 8. Photograph of test installation.