Proceedings of the Institute of Acoustics

VIBRATIONAL POWER IN MULTI-POINT-CONNECTED SYSTEMS

R.A.Fulford & B.M.Gibbs

Acoustics Research Unit, University of Liverpool, P.O.Box 147, Liverpool L69, 3BX

1. INTRODUCTION

The engineering trend towards stiff/lightweight components has accentuated the problems associated with vibration. Many of these arise when a vibrating machine is mounted rigidly upon a base or floor and there is a growing need for accurate and accessible methods of estimating the structure-borne sound transmission as a prelude to its control. Ideally a design method based upon a source-receiver model - the machine being the source and the base or floor the receiver - should be available. Then an engineer could 'rank' sources and receivers with regard to their vibrational characteristics and select sources and receivers such that the likely vibrational power would be below any set limit. This paper is concerned with applying the source-receiver model to structure-borne sound transmission from multi-point sources.

2. MULTI-POINT-CONNECTED SYSTEMS - A REVIEW

It is well understood that in a general multi-point-connected system coupling exists between all points and between all components of motion. Hence when solving for the response of the general system all such coupling must be considered. A common analysis approach is to describe, and then solve for, the whole system via the formulation of the complete mobility matrix [1]. This method is well understood and accurate but the matrices are usually large and offer little immediate engineering insight.

A solution to this comes from expressing the matrices as linear combinations of terms. The effective mobility concept [2] is based on the premise that a point can be considered individually if the effect of all other points on this point are taken into account. In a general multi-point-connected system it can be written;

$$Y_{ii}^{nn\Sigma}=Y_{ii}^{nn}+\sum_{m=1,m\neq n}^{N} Y_{ii}^{mn}\frac{F_j^m}{F_i^n}+\sum_{j=1,j\neq i}^{6} Y_{ij}^{nn}\frac{F_j^n}{F_i^n}+\sum_{m=1,m\neq n}^{N}\sum_{j=1,j\neq i}^{6} Y_{ij}^{mn}\frac{F_j^m}{F_i^n} \quad (1)$$

where Y is a general system complex mobility term. The first set of indices are associated with the generalised velocity and the second with the generalised force where n, m denote position and i, j direction. F is the force at point n and in direction j resulting in the excited system.

The first term is the direct contribution at the point being considered (the point mobility), the second term (the first summation) is the contribution from the other points in the direction of motion being considered (transfer terms), the third term the contributions from the other directions at the point being considered (cross terms) and the fourth term the contribution from the other points in directions other than that being considered (cross-transfer terms).

It remains to apply the effective mobility concept to the source-receiver problem.

In a single point uni-directional system, the power at the interface is given from;

$$Q=\frac{(\tilde{V}_{sf})^2}{|Ys+Yr|^2}Yr \qquad (2)$$

where Vsf is the free velocity of the source, Ys the complex source point mobility and Yr the complex receiver point mobility.

The effective mobility allows a point in a multi-point-connected system to be considered individually. Hence it can be applied directly to equation (2) and the power at point n and direction i in a multi-point-connected system can be written as;

$$Q_i^n=\frac{(\tilde{V}_{sf_i}^{\,n})}{|Ys_{ii}^{nn\Sigma}+Yr_{ii}^{nn\Sigma}|^2}Yr_{ii}^{nn\Sigma} \qquad (3)$$

Such formulisation is very illustrative. Firstly - and excitingly - it shows that if the effective mobility of both the source and receiver can be independently calculated then the power at the source-receiver interface can be obtained before the source and receiver are in contact. However, it shows also that separated source and receiver parameters are difficult to extract.

On closer inspection, it is seen that we can consider, not a source parameter, but instead a potential source parameter [3]. If in addition an interface, rather than a receiver, parameter is developed the power can be given by the product of the two.

Define the phase difference $\Delta\Phi$ and the magnitude ratio α of the source and receiver effective mobilities and apply these to equation (3) to give the power as;

VIBRATIONAL POWER IN MULTI-POINT-CONNECTED SYSTEMS

$$Q_i^n=\frac{(\tilde{V}_{sf_i}^{\;n})^2}{(Ys_{ii}^{nn\Sigma})^*}\cdot\frac{\alpha^{n\Sigma}e^{i\Phi_r^{n\Sigma}}}{(\Phi^{n\Sigma})^2+2\alpha^{n\Sigma}\cos\Delta\Phi^{n\Sigma}+1} \tag{4}$$

which is seen as a two term product. The first of which;

$$S_i^n=\frac{(\tilde{V}_{sf_i}^{\;n})^2}{(Ys_{ii}^{nn\Sigma})^*} \tag{5}$$

is a function of source parameters and has the units of power. The significance being that vibration in each of the six components of motion can be compared directly with each other. How much of this 'potential power' is actually 'realised' is governed by the second term;

$$Cf_i^n=\frac{\alpha^{n\Sigma}e^{i\Delta\Phi n\Sigma}}{(\alpha^{n\Sigma})^2+2\alpha^{n\Sigma}\cos\Delta\Phi^{n\Sigma}+1} \tag{6}$$

which is a function of both source and receiver parameters. This is known as the 'coupling function'. The power then is given simply as the product of the two terms.

This formulisation is attractive in its simplicity compared with the mobility matrix method.

It is clear that the applicability of the source descriptor and coupling function concepts depends upon the practicalities of estimating the effective mobilities. In particular, if the number of terms in the effective mobility which require consideration could be reduced then the resultant data reduction will lead to practical application.

A preliminary study is now described in which the relative importance of the transfer, cross and cross-transfer terms in equation (1) are assessed for a simple multi-point-connected source-receiver system.

3. THEORETICAL EFFECTIVE MOBILITY

Consider a lossless 5mm thick finite beam attached via eight randomly positioned perfect (massless and rigid) point connecters to a lossless 10mm thick infinite beam (see fig.1). The finite beam is excited at one end with a vertical point force having frequency invariant magnitude and phase of unity and zero respectively. The finite beam is the source and the infinite beam is the receiver.

Consider the effective mobility at one point (point one) for vertical translation. (The model allows consideration of vertical translation and of rotation only).

A solution to the model based upon the Eulerian bending wave equation yields sixteen terms in the effective mobility - a point term, a cross term, seven transfer terms and seven cross-transfer terms. A study of associated mobilities and generalised force ratios in each of these terms is hence possible. The latter are particularly important, since it is the force ratios which are unknown prior to contact between source and receiver.

3.1 Magnitudes of vertical force ratios.

Using force F1 as reference, a set of force magnitude ratios are shown in fig.2. The ratios are presented as functions of frequency. Also indicated are the source beam bending wavelength and the associated Helmholtz number kx where x is the distance (1m) between the two extreme (1 and 8) contacts.

There appears to be two distinct regions. Below kx=25, is a region of stability where all bar one (F2/F1) of the ratios are the same with a magnitude of about 10^{-1}. Above kx=25 the ratios fluctuate with respect to frequency within a band which varies between 10^{+1} and 10^{-3}. The transition corresponds to the condition that there is an average of one half wavelength between the contact points. The first region corresponds to a rigid body motion and the second to a modal response.

The large force ratio (F2/F1) results from the close proximity of the point force exciting the total system. A 'rain on the roof' force distribution on the source beam will reduce such large deviations but will increase the overall value of all ratios.

Above kx=25 any increase in modal overlap will reduce the extremes in value. The model is lossless and the results are therefore a 'worst case'. It is expected that in a real system where damping is apparent the extremes in this region will reduce and the band be compressed.

It was thought too that a third region would be seen in real damped systems where the modal overlap becomes high and the force magnitude ratios converge to a stable, uniform value.

3.2 Magnitude of Moment to Vertical Force Ratios

Consideration must also be given to excitation by the other components of motion; in this case moments about an axis into the plane of the page. Consider the magnitude of the moment to force ratios used in the cross and cross-transfer terms. The ratio associated with the cross mobility (M1/F1) along with the ratios associated with the two extreme cross-transfer mobilities (M2/F1) and (M8/F1) are shown in fig.3. It is seen that in the first region (M1/F1) and (M2/F1) both have an approximate value of 10^{-1} and that (M8/F1) has an approximate value of 10^{-2}. In the second region (M2/F1) and (M8/F1) seem to be banded between 10^{-1} and 10^{-5}. (M1/F1) however displays a steady kx dependant decrease.

3.3 Phase of the Generalised Force Ratios

To complete the analysis of the generalised force ratios consideration needs to be given to their phase. Fig.4 shows the phase of a typical force ratio (F2/F1) and the phase of a typical moment to force ratio (M4/F1). Again two regions are seen. The second region encourages the idea that a random phase assumption is possible whereas in the first region a zero phase assumption is possible.

3.4 Evaluating the Significance of the Terms

To simplify the formation of the effective mobility it is useful to try to normalise the contribution of the transfer, cross and cross-transfer terms with respect to the point mobility.

Transfer mobilities are dimensionally compatible to the point mobility and this remains so when they are multiplied by the dimensionless force ratio to form the transfer terms. For the cross and cross-transfer terms however compatibility with the point mobility only exists if the product of mobility and force ratio is taken. The cross (and cross-transfer) mobilities and associated ratios having dimensions m^{-1} and m, respectively.

Some insight into the significance of the transfer terms can be gained then from consideration of the ratio of transfer mobility to point mobility. In fig.5 two such functions are plotted. Below kx=25 the values are constant at less than unity. If the force ratio is assumed unity then it can be seen that the transfer terms contribution to the effective mobility is comparable to that of the point mobility. Above kx=25 the values vary within 10^{-2} and 10^{+1}. The force ratios vary too in this region and all that can in general be said is that above kx=25 the transfer terms contribution is variable.

Assessment of the cross and cross-transfer terms relative to the point mobility is made by first taking the product of the mobilities and associated moment to force ratios. This is shown in fig.6. for two typical cross-transfer terms. It is seen that below kx=25 the contributions of the cross-transfer terms are about a factor of 10 below that for the point mobility. Above kx=25 the contributions are variable and are significant at some frequencies.

5. CONCLUDING REMARKS

This paper has indicated the potential use of the source descriptor and coupling function concepts to multi-point structure-borne sound sources. The use is dependant upon the application of the effective mobility concept. For a simple model an effective mobility has been obtained and it is shown that an assessment of transfer terms can be approached with knowledge of either the mobilities or associated force ratios whereas for cross and cross-transfer terms it is the product of the mobilities and associated force ratios which are required.

Estimation of the force ratios prior to source-receiver connection has been approached. In regions of rigid body motion possibilities exist for unity magnitude and zero phase approximations. In modal regions the generalised force ratios are somewhat variable. For the magnitude tentative suggestions are towards defining a mean value and extreme upper and lower limits. For the phase a random approximation could be made.

6. REFERENCES

[1] V.I.POPKOV, 'Vibroacoustic Diagnostics and the Reduction of the Vibration of Shipboard Machinery', Book by V.I.POPKOV, 'Izdatel'stvo Sudostroyeniye', UDC 629.12.03-8:628.517, 1974. Reproduced by National Technical Information Service.

[2] B.PETERSSON & J.PLUNT, 'On effective mobilities in the prediction of structure-borne sound transmission between a source and a receiver structure, Part 1: Theoretical background and basic experimental studies', J.S.V. 82(4), 517-529, 1982

[3] J.M.MONDOT & B.PETERSSON, 'Characterisation of structure-borne sound sources. The source descriptor and the coupling function', J.S.V. 114(3), 507-518, 1987

VIBRATIONAL POWER IN MULTI-POINT-CONNECTED SYSTEMS

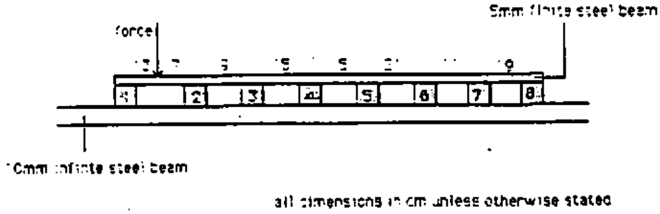

Figure 1 : A finite beam source and infinite beam receiver

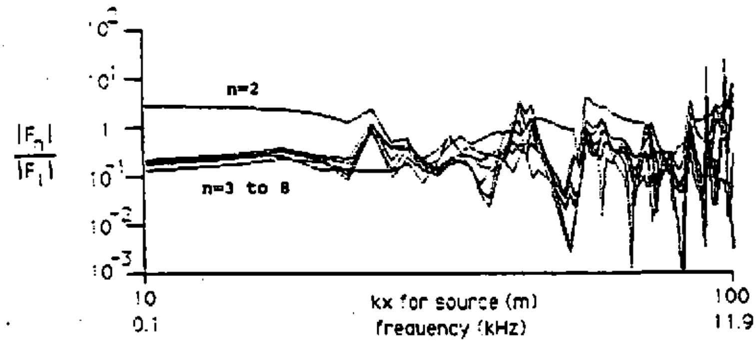

Figure 2 : The Force Ratios

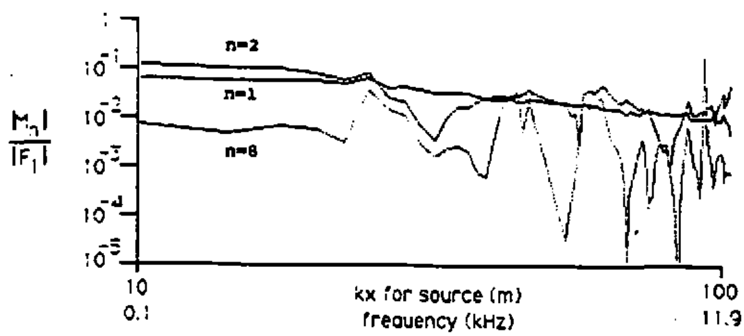

Figure 3 : Moment to Force Ratios

VIBRATIONAL POWER IN-MULTI-POINT-CONNECTED SYSTEMS

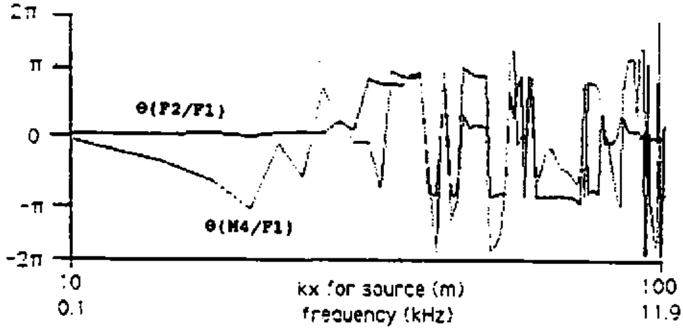

Figure 4 : Force Ratio Phase

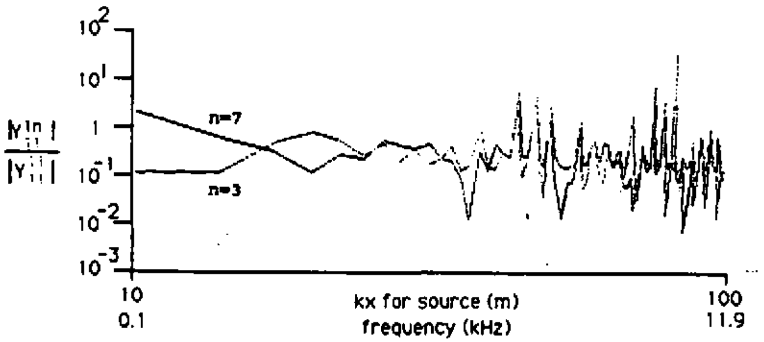

Figure 5 : Transfer to Point Mobility Ratios

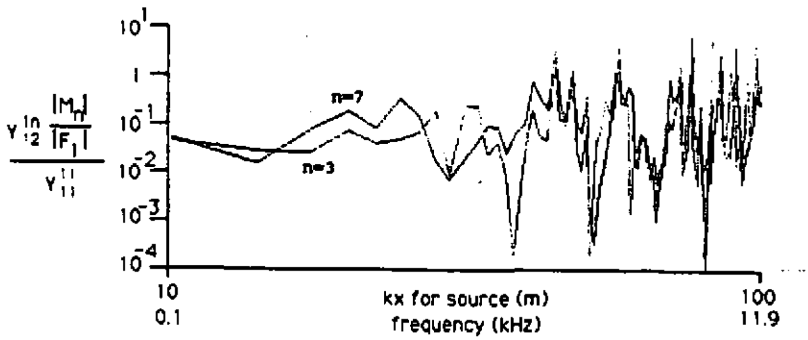

Figure 6 : Cross-Transfer Terms to Point Mobility Ratios