BRITISH ACOUSTICAL SOCIETY

71/18

Symposium on
Finite Element Techniques in
Structural Vibration
24th - 25th March 1971

A NON-CONFORMING HYBRID CYLINDRICAL SHELL FINITE ELEMENT

R. D. Henshell, B. K. Neale and G. B. Warburton

Introduction

In formulating the element it was important that it should be compatible with the PAFEC 70 programs [1]. These programs consist of a suite of subroutines capable of analysing one, two and three dimensional structures with a wide range of different elements in the same problem. If there are variations in shell thickness between elements or if various element types are to be mixed, it is difficult to accommodate more than six degrees of freedom at each nodal point, that is three translations and three rotations. Olson and Lindberg [2] show that it is possible to include higher order nodal variables in mixed element structures. Although PAFEC 70 does include this facility, the present authors believe that the method is too cumbersome for the automatic analysis of practical structures. The element presented in this paper has five of the six possible degrees of freedom at each node. The rotation about the normal to the element base plane is omitted.

Element geometry The element size and position is defined by the radius r, length l, subtended angle α and node numbering as shown in Fig. 1. The five degrees of freedom at each node are u_x, u_y and u_z, displacements in the x, y and z co-ordinate directions respectively, and θ_x and θ_y, rotations (using a right handed convention) about the x and y co-ordinate axes respectively.

Theory Since numerical integration is used throughout, the element co-ordinate system is scaled into the ξ, η plane in a similar manner to reference [3].

The u_x and u_y displacements are linear polynomials in ξ, η and the u_z displacement is the Birkhoff-Garabedian polynomial which has been successfully used by Deák and Pian [4] for flat plate bending analysis. For a flat plate element the displacement functions chosen would be conforming. However, when the functions are modified to include rigid body modes, using the method described by Cantin and Clough [5], the shell element becomes non-conforming. The θ_x and θ_y rotations are obtained as functions of the u_y and u_z displacements by considering small rotations of the middle surface of the shell and neglecting all second order and higher terms.

Terms in the stress assumption given in Table 1 are chosen to satisfy the equilibrium relationships derived for an infinitesimal portion of the shell. Assuming Novozhilov's relationships [6] between the forces, moments $\{N\}$ (see Fig. 2) and the strains of the middle surface of the shell $\{\epsilon\}$, the strain energy of the element can be written down in terms of the $\{\beta\}$ constants of the stress assumption. The $\{\beta\}$ constants are then expressed in terms of the constants of the displacement function by equating the internal strain energy of the element to the external work done by the forces $\{N\}$, after a suitable transformation, at the interelement boundaries and by minimising the complementary strain energy with respect to each β constant. The boundary displacements are evaluated from the displacement functions. Finally the element stiffness matrix is obtained by substituting for the $\{\beta\}$ constants.

The element mass matrix, which allows for the effects of transverse and in-plane inertias, is obtained by substituting the displacement functions in the kinetic energy expression for the element.

The structural stiffness and mass matrices are assembled from the element matrices and the eigenvalue problem solved with the relevant standard PAFEC 70 subroutines.

Results 1. The natural frequencies of the clamped curved fan blade shown in Fig. 3 were analysed with 2 x 2 and 3 x 3 meshes. Advantage was taken of the symmetry and the boundary conditions along the centre line varied for the symmetric and anti-symmetric modes. The results obtained were compared with the best results from Olson and Lindberg's displacement finite element [7], where in-plane inertia was neglected; and an upper bound solution [8] using the power series approach. The experimental results are to be found in reference [7]. It is clear from Table 2 that the hybrid shell element gives very good results. A full discussion of the modes of vibration of the fan blade are given by Olson and Lindberg [7].

2. For a 1 x 1 mesh in the free-free pinched cylinder [5] the displacement under the point load was within 2.4 per cent of Cantin's [9] converged result.

Conclusion From the above results and many others which have been obtained it is apparent that the hybrid element presented is extremely useful for practical engineering structures especially when the mesh is coarse.

Acknowledgements The authors wish to acknowledge the financial support of the Cape Engineering Co. Ltd., Warwick.

REFERENCES

1. PAFEC 70, Users Manual 1970 Dept. of Mech. Eng. Nottingham University.

2. M.D. Olson and G.M. Lindberg 1970. Conference on sonic fatigue, Southampton University, Free Vibration and random response of an integrally stiffened panel.

3. I. Ergatoudis, B.M. Irons and O.C. Zienkiewicz 1968. Int. J. S.Structures, 3, 31 Curved isoparametric quadrilateral elements for finite element analysis.

4. A.L. Deák and T.H.H. Pian 1967 AIAA J. 5, 187 Application of the smooth surface interpolation to the finite element analysis.

5. G. Cantin and R.W. Clough 1968 AIAA J. 6, 1057 A curved cylindrical shell finite element.

6. V.V. Novozhilov 1959 The theory of thin shells, GRONINGEN: P. Noordhoff.

7. M.D. Olson and G.M. Lindberg 1968 NRCC No. 10198, Report LR-497 A finite cylindrical shell element and the vibrations of a curved fan blade.

8. M.D.J. Bridle Ph.D. Thesis, Nottingham University, Vibrations of thick plates and shells (to be published).

9. G. Cantin 1970 AIAA J. 8, 1252 Rigid body modes in curved finite elements.

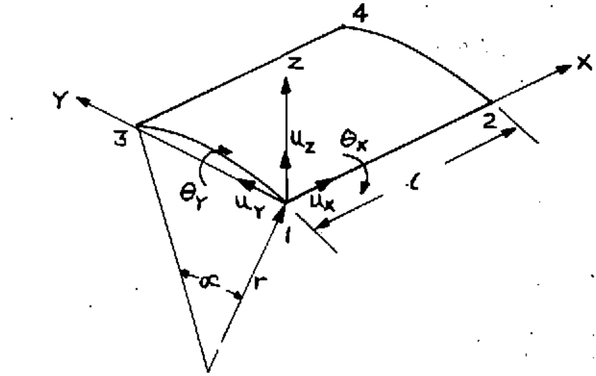
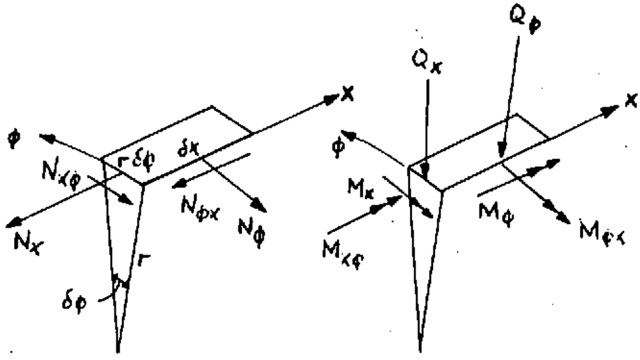


Figure 1. Element Geometry

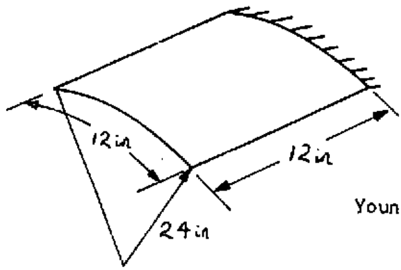


In-plane forces　　　　Bending forces

It is assumed that $N_{\phi x} = N_{x\phi}$

and $M_{\phi x} = M_{x\phi}$

Figure 2. Forces acting on an infinitesimal portion of the shell



t = 0.12"

Density = 0.283 lb/in^3

Poisson's ratio = 0.3

Young's Modulus = 30 x 10^6 lbf/in^2

Figure 3. Clamped curved fan blade showing 3 x 3 mesh

Table 1. Stress assumption

$$
\begin{Bmatrix} N_x \\ N_\phi \\ N_{x\phi} \\ M_x \\ M_\phi \\ M_{x\phi} \\ Q_x \\ Q_\phi \end{Bmatrix}
=
\begin{bmatrix}
1 & \phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r & x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{x}{r} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{x^2}{2} & \frac{x^3}{6r} & 0 & 1 & x & \phi & x\phi & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & r\phi & 0 & 0 & 0 & 0 & 0 & 1 & x & \pm\phi & r\phi x & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -x & -\frac{x^2}{2} & 1 \\
0 & 0 & x & \frac{x^2}{2r} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{Bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{14} \end{Bmatrix}
$$

$$\{N\} = [Q]\{\beta\}$$

Table 2. Natural Frequencies of the curved clamped fan blade

Type	Frequency Hz				
	Olson & Lindberg[7]		Hybrid shell element		Power Series[8]
	Finite element	Experimental			
Degrees of freedom / Mode No.	140	-	50	105	108
1	93.5	86.6	85.0	86.3	85.8
2	147.7	135.5	131.3	137.0	138.3
3	255.1	258.9	232.9	244.9	246.7
4	393.1	350.6	323.5	340.2	342.5
5	423.5	395.2	326.2	366.8	388.8