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A NEW FAMILY OF CURVILINEAR PLATE BENDING ELEMENTS FOR VIBRATION AND STABILITY

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NOMENCLATURE

$\{\alpha\}, \{\beta\}, \{\delta\}$	=	vector of constants.
$[B]$	=	structural geometric stiffness matrix.
$\{\delta\}$	=	structural degrees of freedom.
λ	=	eigenvalue (= (frequency x $2\pi)^2$:- for non-buckling calculations)
$[M]$	=	structural mass matrix.
$[P]$	=	transform polynomial.
$[P^*]$	=	displacement polynomial.
$[S]$	=	structural stiffness matrix.
$[w]$	=	assumed displacement function.

INTRODUCTION

This work reports an investigation of a family of non-conforming, thin, plate bending elements. The elements have four, eight or twelve nodes (Fig. 1), and three degrees of freedom at each node. The freedoms present, at each node, consist of: a single transverse displacement and two rotations about lines which are perpendicular and lie in the plane of the element.

The choice of geometric nodal displacements implies that the elements may be used in a complex practical engineering problem. Stiffeners and in-plane elements may be superimposed, easily, to idealise very general, stiffened[1], folded plate structures. The results presented here refer only to plates of constant thickness but the program suite used[2] allows the use of elements of variable thickness. The four node element may take a non rectangular shape and the eight and twelve node elements may have curved sides. One of the objects of this work is to determine variations in accuracy as the elements are distorted from a basic rectangular shape.

FORMULATION

Element axes, x, y, are set up in the plane of each element in a structure (Fig. 1). To facilitate subsequent integrations, the element is mapped into a unit square in the $\xi\eta$ plane. The transformed nodes are equispaced on the periphery of the unit square. The $\xi\eta$ to x y transform is defined by: $[x, y] = [P][\{\alpha\},\{\beta\}]$[3]. For the four node element: $P = [1 \;\; \xi \;\; \eta \;\; \xi\eta]$. Additional terms: $\xi^2, \eta^2, \xi^2\eta$ and $\eta^2\xi$ are used for the eight node element, and further terms: $\xi^3, \eta^3, \xi^3\eta$ and $\eta^3\xi$ are used for the twelve node element.

An assumption is made, in terms of ξ and η, for the transverse deflection of the element as follows: $w = [P^*]\{\delta\}$ where $\{\delta\}$ is a set of constants which may be expressed in terms of the nodal displacements. For the four node element:

$[P^*] = [1 \; \xi \; \eta \; \xi\eta \; \xi^2 \; \eta^2 \; \xi^2\eta \; \xi\eta^2 \; \xi^3 \; \eta^3 \; \xi^3\eta \; \eta^3\xi$. For the eight node element the following additional terms are used: $\eta^4, \xi^4, \eta^5, \xi^5, \eta^3\xi^2, \eta^2\xi^3, \eta^4\xi, \eta\xi^4, \eta^4\xi^2, \eta^2\xi^4, \eta^5\xi, \eta\xi^5$ and for the twelve node element we include the further terms: $\xi^6, \eta^6, \xi^7, \eta^7, \xi^6\eta^3, \xi^3\eta^6, \xi^7\eta, \xi\eta^7, \xi^4\eta^3, \xi^3\eta^4, \xi^5\eta^3, \xi^3\eta^5$. The bending strain energy, energy due to applied in-plane loads and kinetic energy are calculated at points on the element using the assumed displacement, w. Integration, using the Gaussian numerical method[4], provides the three energies, above, for the whole element. The energies are now differentiated, with respect to the nodal displacements upon which they depend, to yield stiffness, geometric stiffness and mass matrices for the element. The element matrices obtained are merged into three matrices which refer to the whole structure. Vibration problems reduce to the eigenvalue problem:

$$[[S] + [B] + \lambda[M]]\{\delta\} = 0$$

[4], which is solved to yield λ and normalised

A large structure may have thousands of degrees of freedom and be impractical to solve, on present computers, in that form. In such a case continuous reduction[5] of degrees of freedom is necessary to keep the number of degrees of freedom to fewer than one hundred.

The displacement function used is not capable of defining all constant strain states[4]. To remedy this omission it is necessary to include: the terms: $\xi^2\eta^2$ in the four node case; $\xi^2\eta^2$ and $\xi^3\eta^3$ in the eight node case and $\xi^2\eta^2, \xi^6\eta^2, \xi^2\eta^6, \xi^3\eta^3$ and $\xi^4\eta^4$ in the twelve node displacement function. The inclusion of these terms will imply extra degrees of freedom in each element. These might be removed by reduction[5] before the element is merged into the structure.

RESULTS

The following list gives some of the tests made. A selection of the results obtained is included in this summary.

1. All elements were tested, in various meshes, on a square, simply-supported plate. For examples see Figs. 2 and 3.
2. The four node element was used to obtain the monoaxial buckling stress of a square, simply-supported plate with various meshes.
3. The frequencies of a square simply-supported plate were found for different meshes; with the plate subject to an in-plane load. The results are given in Fig. 4.
4. The natural frequencies of a circular plate were found using the twelve node element.

REFERENCES

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4. The Finite Element Method in Structural and Continuum Mechanics. O.C. Zienkiewicz and Y.K. Cheung. McGraw-Hill (1967).

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Fig. 1

The Elements in the x, y Plane showing the Axes

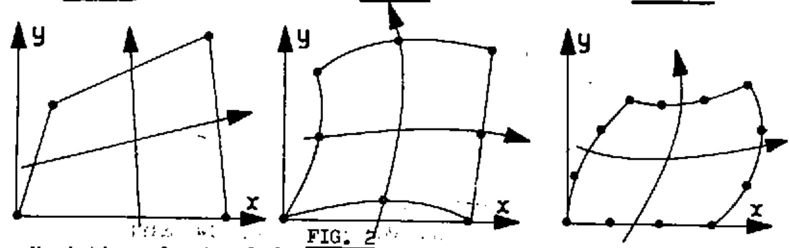
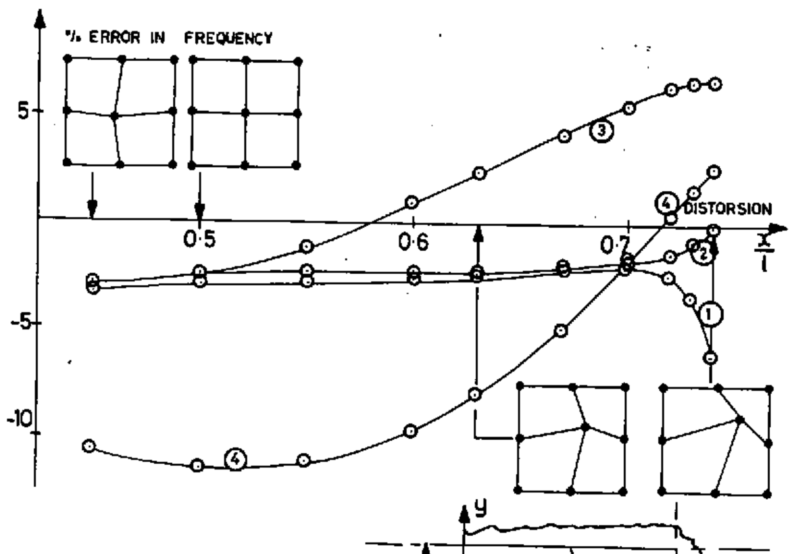


FIG. 2

Variation of natural frequencies, in symmetric modes of a square, simply-supported plate as the mesh is varied. The idealisations use four, four node bending elements and twelve degrees of freedom in a quarter of the plate.

M and N are the number of nodal lines perpendicular to the x and y directions, respectively.



CURVE	M	N
1	2	2
2	2 OR 4	4 OR 2
3		
4	4	4
5	2 OR 6	6 OR 2
6		
7	4 OR 6	6 OR 4

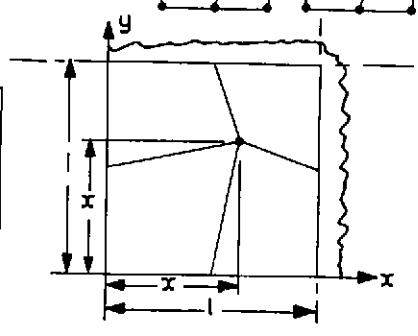


Fig. 3

VARIATION OF NATURAL FREQUENCIES, IN SYMMETRIC MODES, OF A SQUARE, SIMPLY-SUPPORTED PLATE AS THE MESH IS VARIED. THE IDEALISATIONS USE FOUR, TWELVE NODE BENDING ELEMENTS AND SIXTY DEGREES OF FREEDOM. ALL ROTATIONAL FREEDOMS WERE REDUCED OUT LEAVING TWENTY TRANSVERSE FREEDOMS AT THE SOLUTION STAGE.

TESTS WERE MADE ON A QUARTER OF THE PLATE. THE CURVE-MODE RELATION IS DEFINED IN FIG. 2.

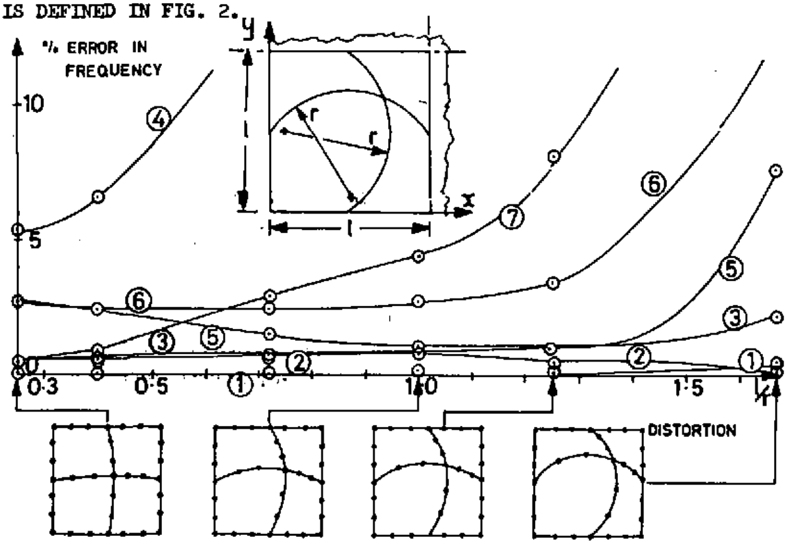


Fig. 4

RELATION BETWEEN FIRST FREQUENCY AND THE UNIFORM, MONOAXIAL (X-DIRECTION), IN-PLANE STRESS, ON A SQUARE, SIMPLY-SUPPORTED PLATE. THREE DIFFERENT MESHES WERE TRIED: EACH USING FOUR, FOUR NODE BENDING ELEMENTS AND TWELVE DEGREES OF FREEDOM

THE DISTORSION IS DEFINED IN FIG. 2.