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TRANSIENT RESPONSE OF PIEZOELECTRIC TRANSDUCER ELEMENTS DERIVED BY Z-TRANSFORM TECHNIQUE.

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INTRODUCTION

Transient analysis of piezoelectric elements has hitherto been achieved by the application of the Laplace transform to the wave equations for piezoelectric materials [1, 2]. The time domain transient solution is obtained by inverse transformation of algebraically complicated Laplace transforms. The method requires much algebraic manipulation and is only useful for simple input functions such as the impulse, step or ramp. In this paper we present a technique which overcomes these problems and provides for calculation of the transient response recursively. The z-transform is applied to the circuit model of a piezoelectric element to yield a discrete time model of the device. The model yields a recurrence relationship which represents the device as a digital filter and this can be applied to any real input function, irrespective of the complexity of its Laplace transform.

CIRCUIT MODELS OF PIEZOELECTRIC ELEMENTS

Equivalent circuits of piezoelectric elements may be based on real frequency (3) or the Laplace variable, s (1). Fig.1 shows the equivalent circuit of a plate transducer; it is a three port model and relates the variables at the electrical terminals to the forces and velocities of particle motion at the acoustic faces of the device. The ideal transformer converts electrical to mechanical energy and the impedances Z_1 and Z_2 result from the time delays associated with acoustic reverberations in the element.

$$Z_1 = \frac{2Z_0}{e^{sT_p} - e^{-sT_p}} \qquad Z_2 = Z_0\,\frac{1 - e^{-sT_p}}{1 + e^{-sT_p}} \tag{1}$$

T_p is the element reverberation time. By conventional circuit analysis techniques the relationships between the variables at the three ports can be described by a 3 x 3 impedance matrix.

$$\begin{bmatrix} F_1 \\ F_2 \\ V \end{bmatrix} = \begin{bmatrix} (Z_1 + Z_2) & Z_1 & \frac{N}{sC_0} \\ Z_1 & (Z_1 + Z_2) & \frac{N}{sC_0} \\ \frac{N}{sC_0} & \frac{N}{sC_0} & \frac{1}{sC_0} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ I \end{bmatrix} \tag{2}$$

Where C_0 is the static capacitance of the device and $N = C_0 h_{33}$. The relationship between any pair of variables is obtained by application of appropriate boundary conditions.

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As an example we consider a free plate connected to a voltage source by a terminated line (fig.2). The boundary conditions are $F_1 = F_2 = 0$. The voltage at the transducer terminals as a function of source voltage is, in its Laplace transformation,

$$V(s) = \frac{\left[sC_0Z_0(1 + e^{-sT_p}) - 2N^2(1 - e^{-sT_p})\right].V_g(s)}{sC_0Z_0(1 + sC_0R)(1 + e^{-sT_p}) - 2N^2(1 - e^{-sT_p})} \tag{3}$$

The continuous time solution could, with very much algebraic manipulation, be derived by inverse transformation of equation 3 for particular input functions. However a very much more general solution is obtained using discrete time modelling techniques, based on the z-transform.

THE DISCRETE TIME MODEL

The z-transform variable is defined as $z = e^{sT}$, where T is the sampling interval in a series of discrete time samples of a real continuous function of time. In order to apply the z-transform to equ.3 we choose a sampling interval which is a small fraction of the device reverberation time, $T = T_p/m$. Whence

$$e^{-sT_p} = e^{-smT} = z^{-m} \tag{4}$$

For functions of time which are band limited to 1/4T the Laplace variable, s can be represented by a z-transform obtained by bilinear transformation

$$s = 2(1 - z^{-1})/T(1 + z^{-1}) \tag{5}$$

Substitution of equs. 4, 5 into 3 yields the z-transform of the voltage at the transducer terminals

$$V(z) = \frac{\left[(A-B)(1-z^{-m-2}) + (A+B)(z^{-m}-z^{-2}) + 2B(z^{-m-1}-z^{-1})\right] V_g(z)}{\left[(AC+A-B) - 2(AC+B)z^{-1} + (AC-A-B)z^{-2} + (A+AC+B)z^{-m} + 2(B-AC)z^{-m-1} + (AC-A+B)z^{-m-2}\right]} \tag{6}$$

Where $A = 2C_0Z_0/T$, $B = 2N^2$ and $C = 2C_0R/T$. The transform of equ.6 is equivalent to a recurrence relationship between the sampled time domain forms of v and v_g. it is

$$v(n) = K_0\Big\{K_1v(n-1) + K_2v(n-2) + K_3v(n-m) + K_4v(n-m-1) + K_5v(n-m-2) + K_6\left[v_g(n) - v_g(n-m-2)\right] + K_7\left[v_g(n-m) - v_g(n-2)\right] + K_8\left[v_g(n-m-1) - v_g(n-1)\right]\Big\} \tag{7}$$

Where $K_0 = 1/(A+AC-B)$, $K_1 = 2(AC+B)$, $K_2 = (A-AC+B)$, $K_3 = -(A+AC+B)$, $K_4 = 2(AC-B)$, $K_5 = (A-AC-B)$, $K_6 = (A-B)$, $K_7 = (A+B)$, and $K_8 = 2B$.

Equation 7 represents the circuit behaviour of the piezoelectric element as a digital filter which evaluates the nth sample of the output waveform in terms of the nth and previous samples of the input waveform and previously calculated samples of the output waveform.

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CALCULATION AND EXPERIMENTS

Equation 7 was used to calculate the impulse response of a PZT5 plate (25mm dia. x 2 mm thick) when exicted in the circuit of fig.2 with 50 Ω source and line impedances (fig.3). Experimentally, the circuit was constructed using a standard pulse generator (Lyons PG73N) to supply a short impulse (20ns width) and the waveform at the transducer terminals was observed on an oscilloscope (Telequipment D83)) and photographed (fig.4).

DISCUSSION

The impulse response obtained by measurement corresponds closely to the form predicted by calculation. In other experiments (to be published) we have shown that the calculated response to more complex input functions is of comparable accuracy. The method of calculation has the great advantage that it can be applied to any real input function. The same technique could easily be applied to calculate electro-mechanical transduction responses, again for any real input functions. We have used the technique extensively to test new specimens of transducer material.

REFERENCES

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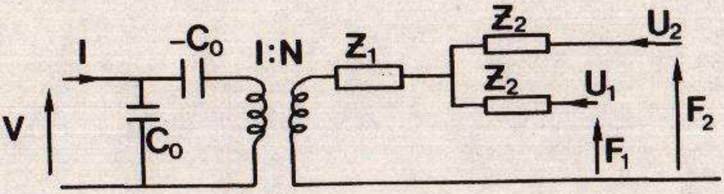

Fig 1 3-port equivalent circuit of piezoelectric plate.

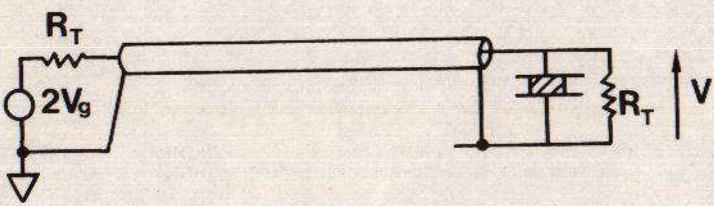

Fig 2 Piezoelectric plate excited by voltage source 2Vg, connected by matched line.

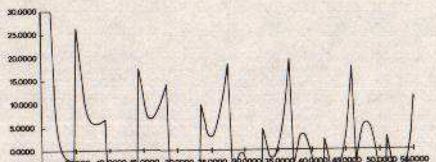

Fig 3 Calculated impulse response of PXE5 disc, 25 mm dia 2.0 mm thick.

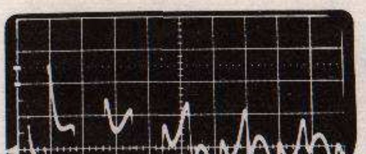

Fig 4 Measured impulse response of PXE5 disc, 10 mv cm^{-1}, 0.5 μs cm^{-1}.