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AN INVESTIGATION ON THE SOURCES OF JET NOISE

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The picture of jet noise sources as (usually convected) quadrupolar sources, supplemented by dipoles when temperature gradients are significant, acting on moving fluid is discussed and compared to experimental far-field data, including single and two-microphone measurements. Alternative descriptions for the sources are also considered in the discussion. It is pointed out that two-point far field correlations of the sound pressure, which represent when normalized the average phase angle of instantaneous emissions in different directions, permit obtaining important insight into details of the source structure. As the sensitivity of these data to variations in source properties is different from that of single microphone data, the use of both types of data is essential in unraveling details associated with non-compactness and on the nature of the sources.
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1. Introduction

Since the appearance of Lightill's analogy [1], jet noise sources are mostly regarded as an equivalent distribution of quadrupoles or, more precisely, as an equivalent stress field. Ffowcs William [2] was possibly the first to point out that, in a hot flow, the Lighthill stresses do not cancel out exactly, so that dipole-like sources are also present. Although in the original analogy, the sources are seen as (usually moving and) acting on an a reference medium at rest, from Lilley's work [3] onward, the picture of moving sources acting on moving fluid showed to be able to provide better agreement with experiment by permitting to remove a good deal of mean flow shrouding effects (including refraction) from the equivalent source terms, at the expense of the necessity of dealing with a more complex Green's function, expressing these effects. More recently, the extension of Lilley's approach permitted impressive agreement with experiment [4, 5], obtained with no adjustable constants. The simulations performed by Goldstein and Lieb [4] and by Karabasov et al [5], however, differ in some aspects, notably in the hypothesis concerning source non compactness. In fact, there are so many factors that influence the final result that it is far from trivial to know with certainty which features of the source are essential for a good prediction and which have only minor relevance.   An important type of measurement, which can shed light into some of the details of the source structure, is that of the correlation of the sound pressure in two far field points, which, when properly normalized, express the average phase of the simultaneous emissions in the directions of the observers considered. These correlations, measured with azimuthal separation for a set of fixed values of the polar angle, where first published in the 1970's and 19080's, by Maestrello [6] and Juvé and co-workers [7-8] and discussed by Ribner, Fuchs and others [9-12]. More recently, Cavalieri et al [13] produced a significant amount of new data, which included also the directivity of the first 3 modes.

While Ribner [9, 10] and Musafir [12, 14] discussed the experimental findings in terms of a non-compact distribution of point sources, with good general agreement, Fuchs [11] and Cavalieri et al.[13] interpreted the results in terms of coherent sources. Notably, Cavalieri et al consider axially extended wavepackets and study the evolution of the different modes.

This paper discusses some of the features of the experimental data considering both the point (or incoherent) source model and the wave packet model.

2. The sound field of a point quadruple in a medium at rest

Let $\mathbf{T}(t)$ represent a point quadrupole located at $\mathbf{y}$, with components $T_{ij}(t)$.The far field pressure $p(\mathbf{x}, t)$ is given, for $|\mathbf{x}| >> |\mathbf{y}|$, by

$$p(\mathbf{x},t)=\frac{\ddot{T}_{xx}\left(t-\left(|\mathbf{x}|-\mathbf{y}\cdot\mathbf{n}\right)/c_0\right)}{4\pi|\mathbf{x}|c_0^2} \tag{1}$$

where $\mathbf{n}$ is the unit vector along the direction of $\mathbf{x}$, $T_{xx} = \mathbf{n}.\mathbf{T}.\mathbf{n}$ and c_0 is the sound speed.

The correlation of pressure signals at two far field points, $\mathbf{x}$ and $\mathbf{x}'$, with $|\mathbf{x}'| = |\mathbf{x}|$, considering a time delay $\tau = t' - t$ (where t' is the reception time at $\mathbf{x}'$) will be noted as $R(\mathbf{x},\mathbf{x}',\tau)$ and is given by

$$R(\mathbf{x},\mathbf{x}',\tau)=\frac{1}{\left(4\pi|\mathbf{x}|c_0^2\right)^2}\frac{\partial^4}{\partial\tau^4}\overline{T_{xx}T}_{x'x'}(\tau^*) \tag{2}$$

where τ^* is the emission time difference of the signals considered, being given, in the far field approximation (with $\mathbf{n}'$ being the unit vector of $\mathbf{x}'$), as $\tau^* = \tau + \mathbf{y}\cdot(\mathbf{n}' - \mathbf{n})/c_0$.

It is simple to show that for the case of a statistically axisymmetric point quadrupole, $\overline{T_{xx}T}_{x'x'}$ depends only on 5 different components $\overline{T_{ij}T}_{kl}$ (those for which the indexes $ijkl$ are equals in pairs) and that, for the 2 observers at the same polar angle θ and with the azimuthal separation $\Delta\phi$, the correlation for $\tau^* = 0$ when plotted as a function of $\Delta\phi$ contains only the azimuthal modes 0, 1 and 2, their proportion depending also on θ. For $\theta = 90^o$,for instance, only the azimuthal modes 0 and 2 are present.

3. Sources embedded in a jet flow

3.1 Point quadrupole

If a point quadrupole is placed inside a parallel flow, the far sound field can be expressed by

$$p(\mathbf{x},t)=\frac{F_{ij}\left(T_{ij}([t])\right)}{4\pi|\mathbf{x}|} \tag{3}$$

where the F_{ij} represent operators acting on the T_{ij}, which depend on frequency, source position, flow velocity profile and Mach number, while $[t]$ stands for the appropriate retarded time. The F_{ij} have closed form expressions in the limits of low and high frequencies, when, for a pug flow they have the form $F_{ij} = c_0^{-2}D_{ij}\partial^2/\partial t^2$, where the D_{ij} are Mach number dependent flow factors, which express the directivity of the individual T_{ij} components; they are given in the low frequency (LF) limit by Dowling et al [15] and in the high frequency (HF) one by Goldstein [16]. For these cases and considering sources moving along the flow centerline (an hypothesis unnecessary for the LF case), the corresponding flow factors affect the proportion of azimuthal modes (0, 1, 2) radiated to the far field at each polar angle θ but do not introduce other modes. As discussed in [12, 14], it follows form the form of the D_{ij} that, for the lower frequencies, the axisymmetric mode is more affected by the flow than the others, being significantly amplified for low values of θ and reduced for $\theta > 90^o$.

3.2 Distributed sources

To model a real jet, one has to go beyond the point source model and consider distributed sources, eventually with axial and/or azimuthal coherence. The simplest model would be a ring or a line of incoherent sources as used e.g., by Ribner [9]. In this case, as each individual source is no more located at the same distance from the 2 microphones, it is expected that the peak value of the correlation coefficient, $r(\theta, \theta', \Delta\phi)$, will decrease in magnitude since now the signals received at time t at **x** and **x**´ where not emitted simultaneously, what will show essentially as a loss of correlation which increases with the total angular separation. In extreme cases (i.e., for sufficiently high values of τ^*, for which $|r|$ will be small) the effect may also show as a change in the sign of r. If we except these cases, the plots of r for fixed values of θ considering incoherent sources will be similar to those of a point source, but showing a loss of correlation which increases with $\Delta\phi$.

Coherence effects may further modify the 'point source' correlation patterns, depending on the coherence type (azymuthal or axial) and 'intensity'. A study on the effect of azimuthal coherence on ring sources at the $\theta = 90^o$ plane is given in [17]. As can be expected, the increase in azimuthal coherence increases the relevance of the axisymmetric mode. As for axial coherence, it is possible it affects equally the different T_{ij} components and, in this case, for observers with a fixed θ (i.e., for azimuthal correlations) its effects would not show in the plots of r (in this case, of $r_1(\theta, \Delta\phi) = r(\theta, \theta, \Delta\phi)$), being cancelled by normalization. But the directivity, of course, would be modified and this modification may be quite significant as shown in the experiments, e.g. of Laufer and Yen [18] and Juvé [8] for excited jets. In these cases, vortex rings seem to be the source of sound at certain frequencies and it can be shown that they can be modelled by the diagonal components of **T**.

4. Discussion

A good jet noise model should be able to reproduce both single (i.e., amplitude) and two-point (phase) measurements. While the far field correlation data available is somewhat limited, most of it is not accompanied of the corresponding amplitude data. The welcome exceptions are the excited jet case discussed by Juvé [8] and the measurements of Cavalieri et al [13], which include the overall directivity and that of the first 3 azimuthal modes.

The broad band data of Maestrello [6] and Juvé and Suniach [7] are consistent with a model of incoherent point quadrupoles embedded in plug flow, as the corresponding flow effects are seen in the azimuthal correlations plots. These effects seem to be adequately modelled by the low frequency D_{ij} since, far from the jet axis, the LF and HF D_{ij} become similar while for low θ, the higher frequencies are refracted away while the lower ones are amplified. While the data of Maestrello for $M = 0{,}75$ suggests that $r_1(90, 90)$ is zero, that of Juvé and Suniach [7] and Juvé [8], for $M = 0.4$ is consistent with a negative $r_1(90, 90)$, The same occurs with the data of Cavalieri et al [13] for $M = 0{,}6$. As discussed in [12, 14, 19], $r_1(90, 90)$ is associated essentially with the cross-correlation of the longitudinal source components in the $\theta = 90^o$ plane, $\overline{\ddot{T}_{22}\ddot{T}_{33}}$. This quantity is rather difficult to measure in the flow field, but can be inferred from far field acoustic measurements. It is relevant to mention that most detailed models or simulations for $\overline{\ddot{T}_{22}\ddot{T}_{33}}$ (e.g., [4, 5, 9]) lead to non-negative values, while a simple reasoning based in an incompressible flow [12, 14] shows that a negative $\overline{\ddot{T}_{22}\ddot{T}_{33}}$ can be expected, at least for low Mach numbers.

Examination of the directivity of the first subharmonic of the excitation frequency, measured by Juvé [8] suggests the data is affected by strong axial coherence effects [19]. As for the directivity data provided by Cavalieri et al [13], although that of the axisymmetric mode is consistent with a strong axial coherence, it is likewise consistent with the incoherent sources' model. Since the directivities of modes 1 and 2 are also consistent with this model, it is more likely that the jet noise sources behave like axially coherent wave packets mostly for excited jets.

Considering heated jets, the temperature gradient will introduce new sources (possibly of both dipole and quadrupole type), with respect to those in the cold jet, and also affect the proportion of old and new sources [20]. The measurement of far field correlations could help in assessing details of the modelling for this and other cases.

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