FAR FIELD CORRELATION PATTERNS OF JET NOISE

Ricardo E. Musafir

PEM/COPPE and DHS/EE - Universidade Federal do Rio de Janeiro
C.P. 68503 - 21945 - R.J., Brazil

INTRODUCTION

Since the publication by Maestrello [1,2] of sound pressure two-point correlation measurements in the far field of a turbulent jet, which were soon followed by those of Juvé et al. [3-6], a number of works appeared, either providing a conection between the experimental data and existing jet noise source models or aiming at the inverse problem of finding, from the measurements, an equivalent source distribution, e.g. [6-9]. Although many of these models have a high degree of sophistication, it was found that they were not general enough and that, in some cases, the sophistication itself prevented the identification of the relevant parameters. So, in order to explicitate these parameters, it was found most useful to develop a model for the radiation field, starting from the simplest possible model of source distribution - a single point source - that still retained the needed basic characteristics [10-12]. This model yelds point source correlation patterns whose study, while rendering easier the interpretation of the experimental data, points out, by the comparison against these data, to which extent improvements should be added. This investigation is reviewed and extended in this article.

CORRELATION PATTERNS

Basic Model - The quadrupolar nature of jet noise sources having been pointed out by Lighthill [13], the natural choice for the basic source is a point quadrupole $\underline{\underline{T}}$, located at $\underline{y}$, whose principal directions vary stationary randomly in time. This can be taken as an initial model for a compact jet. $\underline{\underline{T}}$ can also be assumed to be statistically isotropic or, as is more suitable for round jets, to have this property restricted to the x_2x_3 plane.

The pressure due to a point quadrupole in an otherwise homogeneous medium at rest at far field point $\underline{x}$, whith $x = |\underline{x}| >> y$, is given by

$$(4\pi c_0^2 x)\; p(\underline{x},t) = [\ddot{T}_{xx}] \tag{1}$$

where $T_{xx} = \underline{n}.\underline{\underline{T}}.\underline{n}$, $\underline{n} = \underline{x}/x$ and [ ] means evaluation at the apropriate retarded time, given here by $t - (\underline{x} - \underline{y}.\underline{n})/c_0$. Thus, the correlation of pressure signals at two far field points with time delay τ in reception is written as

$$R(\underline{x},\underline{x}',\tau) = \frac{1}{(4\pi c_0^2)^2 xx'} \frac{\partial^4}{\partial\tau^4} \{\overline{T_{xx}T_{x'x'}}\}(\tau^*) \tag{2}$$

where $\tau^* - \tau = \sigma$, the difference in the emission times of signals received simultaneously, which is given here by $c_0\sigma = x - x' - \underline{y}.(\underline{n}-\underline{n}')$.

For a single point source, the normalised correlation

$$r = \frac{R(\underline{x},\underline{x}',\tau)}{(R(\underline{x},\underline{x},0)R(\underline{x}',\underline{x}',0))^{1/2}} \tag{3}$$

which can be interpreted as the cosinus of the average phase angle between the signals $p(\underline{x},t)$ and $p(\underline{x}',t+\tau)$, will be influenced both by the differences in the simultaneous emission in the $\underline{n}$ and $\underline{n}'$ directions and by that in their reception time. The pattern due to source directivity alone, which will be noted by r_0, is obtainable for $\tau^* = 0$ (for a non-compact source, the distribution of strength and coherence will also affect the measurements).

In order to visualise the influence of the angular separation $\psi = \cos^{-1}(\underline{n}.\underline{n}')$ in $\overline{T_{xx}T_{x'x'}}$, it is most convenient to use an orthogonal carthesian system, with unit vectors $\underline{e}_\alpha // \underline{n}$, $\underline{e}_\beta.(\underline{n} \times \underline{n}') = 0$, $\underline{e}_\gamma$. Then,

$$\overline{T_{xx}T_{x'x'}} = \overline{T^2_{xx}} \cos^2\psi + \overline{T_{xx}T_{\beta\beta}} \sin^2\psi + \overline{T_{xx}T_{x\beta}} \sin(2\psi) \qquad (4)$$

Neglecting the third term in (4), which is zero for the statistically isotropic case (when the right hand side is an even function of ψ), r_0 can be written as

$$r_0 = \cos^2\psi + r_1 \sin^2\psi \qquad (5)$$

where $r_1 = \overline{\ddot{T}_{xx}\ddot{T}_{\beta\beta}} / \{\overline{(\ddot{T}_{xx})^2}\,\overline{(\ddot{T}_{x'x'})^2}\}^{1/2}$.

Equation (5) is a general expression for correlation patterns due to a statistically isotropic point quadrupole. Of course, as ψ represents total separation, for azimutal correlations (both observers having the same angle θ with the jet axis), only part of the curve will be seen as the azimutal separation varies from 0 to π.

In the simplified model above, only a normalized cross correlation of diagonal components is needed to model r_0 for any separation angle. For the statistically axisymmetric model, it was verified in [12] that, in the expansion

$$\overline{T_{xx}T_{x'x'}} = n_i n_j n'_k n'_\ell \overline{T_{ij}T_{k\ell}} \qquad (6)$$

where the axes system x_1, x_2, x_3 is used, only five different terms contribute to r, three auto-correlations (or amplitude terms), $\overline{T^2_{11}}$, $\overline{T^2_{22}}$, $\overline{T^2_{12}}$, and two cross correlations of diagonal terms, $\overline{T_{11}T_{22}}$, $\overline{T_{22}T_{33}}$.

Also, if the adimensional directivity $D(\theta) = \overline{(\ddot{T}_{xx})^2}/\overline{(\ddot{T}_{22})^2}$, which has the general form

$$D(\theta) = 1 + B\cos^4\theta + A\sin^2 2\theta \qquad (7)$$

is known, the cross correlation terms can be easily modelled if $tr(\ddot{\underline{T}}) = 0$, i.e., if the stress system induces only isovolumetric deformation, what is expected in the incompressible flow limit, when the source field can be thought as composed of lateral quadrupoles only. Then, by writing

$$\ddot{T}_{\alpha\alpha} + \ddot{T}_{\beta\beta} = -\ddot{T}_{\gamma\gamma} \qquad (8)$$

an expression for the needed cross correlation terms is obtained as

$$\overline{\ddot{T}_{\alpha\alpha}\ddot{T}_{\beta\beta}} = \frac{1}{2}\{\overline{(\ddot{T}_{\gamma\gamma})^2} - \overline{(\ddot{T}_{\alpha\alpha})^2} - \overline{(\ddot{T}_{\beta\beta})^2}\} \qquad (9)$$

Modified Model - Before discussing the limits of a single source model for comparison against experimental data, it is convenient to discuss the limits of the model itself.

Up to now, source convection has been completely neglected. In Lighthill's analogy, this effect appears through the movement of the stress field only, which results in an amplitude multiplication factor, which is eliminated by normalization, (i.e., it does not alter phase). The remaining effect is the alteration of the actual value of τ^*, which does not affect r_0.

Since it is known that a much better agreemen with one microphone data is obtained when the analogy is modified to take into account the effect of the surrounding flow in the sound generation process, this effect must be also included in the model. It is tantamount to consider also the movement of the distorting fluid elements. In this case, as the radiation of different quadrupole components to the same far field point will be differently affected by movement, convection will alter the measured phase of pressure signal which were simultaneously emitted in different directions. So, the patterns obtained from $D(\theta)$ alone can only represent the jet in the zero Mach number limit.

Adding the existence of flow in the vicinity of the stationary point quadrupole (since the displacement of $\underset{=}{T}$ will not affect r_0), the far field pressure can be written under the general form

$$(4\pi c_0^2 x)\, p(\underset{\sim}{x},t) = \mathcal{D}_{ij}\,[T_{ij}] \qquad (10)$$

where the $\mathcal{D}_{ij}$ depend on the Green's function of the problem, i.e., on the model for the jet flow, and involve partial time derivative operators. For the no-flow case, $\mathcal{D}_{ij} = n_i n_j\, \partial^2/\partial t^2$. Analitical solutions exist only for unidirectional flow depending only on the transverse coordinates, in the low (LF) and high (HF) frequency limits, that is, when the sound wavelength is large or small compared to the jet diameter. For a plug flow in the x_1 direction, (10) reduces to

$$(4\pi c_0^2 x)\, p(\underset{\sim}{x},t) = D_{ij}\,[\ddot{T}_{ij}] \qquad (11)$$

where the coeficients D_{ij} depend on Mach number M_0 and on the coordinates of $\underset{\sim}{n}$. What is needed now for the modelling of r_0 is

$$\overline{T_{ij}T_{k\ell}}\; D_{ij}D'_{k\ell} \qquad (12)$$

Taking the whole problem to be axissymmetric - note that the flow does not affect the correlations $\overline{T_{ij}T_{k\ell}}$ - with the D_{ij} given in the low and high frequency limits in [14] and [15], the resulting expressions can be summarized, if the jet flow density and sound speed are taken to be uniform and identical to those of the surrounding medium, by

$$\overline{T_{ij}T_{k\ell}}\; D_{ij}D_{k\ell}\; C_0(\theta)C_0(\theta') = \overline{T_{11}^2}\cos^2\theta\cos^2\theta' + \frac{\overline{T_{22}^2}+\overline{T_{22}T_{33}}}{2}\, q_0^2(\theta)q_0^2(\theta')$$
$$+ \overline{T_{11}T_{22}}\,(\cos^2\theta\, q_0^2(\theta') + \cos^2\theta'\, q_0^2(\theta)) +$$
$$\{4\overline{T_{12}^2}\, q_1(\theta)q_1(\theta')\cos\theta\cos\theta'\cos\Delta\phi + \frac{\overline{T_{22}^2}-\overline{T_{22}T_{33}}}{2}\, q_1^2(\theta)q_1^2(\theta')\cos(2\Delta\phi)\}E(\theta)E(\theta') \qquad (13)$$

where, with $C(\theta) = (1 - M\cos\theta)^2$, M being the local Mach number, $q_0(\theta) = C(\theta) - \cos^2\theta$ and, in the LF limit, $C_0(\theta) = C(\theta)$, $q_1(\theta) = \sin\theta$, $E(\theta) = 2C(\theta)/(1+C(\theta))$, while in the HF limit, $C_0 = C^{1/2}$, $q_1 = q_0$, $E = 1$, equation (13) applying only outside the cone of relative silence ($q_1^2 > 0$). The no-flow situation is easily recovered by setting $M = 0$.

If $\underset{=}{T}$ is assumed to be traceless, again the knowledge of $D(\theta)$ permits the modeling, for each M, of the LF and HF r_0.

Comments on the aplicability of (9) and other assumptions - In the domain of

FAR FIELD CORRELATION PATTERNS OF JET NOISE

Lighthill's analogy, for the compact jet ($M_0 \to 0$), T_{ij} stands basically for the total kynetic energy and so, the use of (9) is justified. As M_0 increases, the jet becomes non compact but, as long as one can think of compact independent emitters, the equivalence

$$\frac{\partial}{\partial t}(\rho v_i v_j) = p\,(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}) \;;\; \frac{\partial}{\partial t}(\rho v^2) = 2\,p\,\nabla . \underline{v} \tag{14}$$

holds locally for an inviscid fluid. Since the compressibility correction apllying to (9) is $O(M_c^4)$, M_c being the convection Mach number, and so negligible, the net effect will be a reduction in correlation amplitude due to retarded time effects. Although viscous effects were thought to produce relevant isotropic waves, Obermeier was able to show that for high Reynolds number flow, these are also expected to be neglible [16].

When a modified analogy is adopted, one must be very careful since then, even for the compact jet

$$T_{ij} = \int \rho v_i v_j \; dV \tag{15}$$

does not hold anymore. Indeed, the choice of a refference situation - a plug flow or a shear flow model being the most commons - affects differently both the $\mathcal{D}_{ij}$ and the actual T_{ij}, which is modified, say to T'_{ij}.

For the plug flow model, T'_{ij} is given by

$$T'_{ij} = \int \rho u_i u_j \; dV \tag{16}$$

where $\underline{u}$ stands for the time dependent part of $\underline{v}$. Then, it appears reasonable to set $tr(\underline{\underline{\ddot{T}}}) = 0$ and $D(\theta) \cong 1$. But (16) neglects stress components due to mean flow which actually deform the fluid, being also responsible for sound radiation.

They are accounted for in the shear flow model (Lilley's equation), when the effect of shear appears both through these terms and the $\mathcal{D}_{ij}$ (see [17]). The right hand side of Lilley's equation is given, in the isothermal case by [17]

$$\rho\,\{\frac{D}{Dt}\,(\frac{\partial^2 u_i u_j}{\partial x_i \partial x_j}) - 2\,\nabla U . \frac{\partial^2 \underline{u}\, u_j}{\partial x_1 \partial x_j}\} \tag{17}$$

with $U = v_1 - u_1 = U(x_2, x_3)$. This is equivalent, in the incompressible flow limit, to

$$\rho\,\frac{D}{Dt}\,\{\frac{\partial^2 v_i v_j}{\partial x_i \partial x_j}\} - 2\,\frac{\partial}{\partial x_1}\;(\nabla U . \nabla p) \tag{18}$$

The term proportional to space derivatives of the pressure appears to have a negligible contribution to the sound field [11,18] - as is expected of a term that is more properly described as a propagation term. Thus, the use of the same T_{ij} of the no-flow case and of equations (8-9) appears to be justified.

The use of the plug flow $\mathcal{D}_{ij}$ to model the real jet is justified as follows: due to the existence of shear, the main difference in the D_{ij} is the addition of extra terms proportional to $\cos\theta\,|\nabla U|\,\partial/\partial t$, which are sometimes taken to be dominant in the L.F. solution. Since there is no experimental evidence of the dominance of these terms (effectively, extra components T_{1j}, similar to the second term in (17), which increase the radiation in the jet flow direction) and theoretical calculations show them to be responsible for no more than a 3 dB increase ih the sound pressure level [19], it is thought that the effect of shear can be roughly taken into account in the directivity $D(\theta)$.

A point that may seem awkward, is the use of the plug flow model with $M = M_c = 0.6M_0$ to represent a jet with exit Mach number M_0. This amounts, in the shear flow model, to neglect in the L.F. solution, besides the terms proportional to $|\nabla U|$, a jet column scattering term due to the mismatch of M and M_0 (see [19]). This neglect is justified since the L.F. sound comes mostly from the adjustment region onward, where shear and maximum velocity decrease progressively.

COMPARISON WITH EXPERIMENT

Effect of time delay and choice of theoretical pattern - The single source model is useful for direct comparison as long as its characteristics are dominant in the experimental data. Otherwise, time delay effects have to be modelled. For broad-band data, an approximation is given by introducing a function $f(\tau)$ defined by

$$r(\theta,\theta',\Delta\phi,\tau) = r_0(\theta,\theta',\Delta\phi)\ f(\tau) \qquad (19)$$

Each compact independent emitter is supposed to have the same statistical properties, f standing basically for a reduction in correlation amplitude which aproximates the average over the source region, of the effect of asymmetry of positioning of these individual emitters in respect to two observers equally distant from the jet exit.

Since for azimuthal correlations,only the transverse dimension of the source region will influence the values of σ, retarded time effects are expected to be less important than for polar correlations, when the longitudinal dimension is relevant. As long as as it can be asserted that broad band negative correlations are not due to $f(\tau)$, a comparison of both the form and the regions of positive and negative values of the measured r (peak values) with those of r_0 can be done.

Since this condition was met for the azimuthal correlations of [1], for $M_0 = 0.75$ (see discussion in [12]),it is also met by the data of [3], for $M_0 = 0.4$.

The measured patterns are show in Figure 1. A typical narrow-band curve for $\theta = 45^0$ was included with the data of [3].

Figures 2 shows the theoretical L.F. azimuthal correlation patterns for $M_0 = 0.4$, $D(\theta) = 1$ and $M_0 = 0.75$, $D(\theta) = 1 + \cos^4\theta + 1/8 \sin^2 2\theta$. These directivities compare well with one point measurements and where chosen because of the value of $r(90,90)$ ($r(\theta,\Delta\phi)$ is used for peak $r(\theta,\theta',\Delta\phi,\tau)$).

The L.F. patterns where chosen to represent the jet since the existence of flow influences mostly emission at low angles θ, where the L.F. sound is dominant. It should be noted that the high frequencies will be highly influenced by retarded time effects and that the H.F. solution (14) is strongly dependent on the axisymmetry of source positioning inside the jet.

FAR FIELD CORRELATION PATTERNS OF JET NOISE

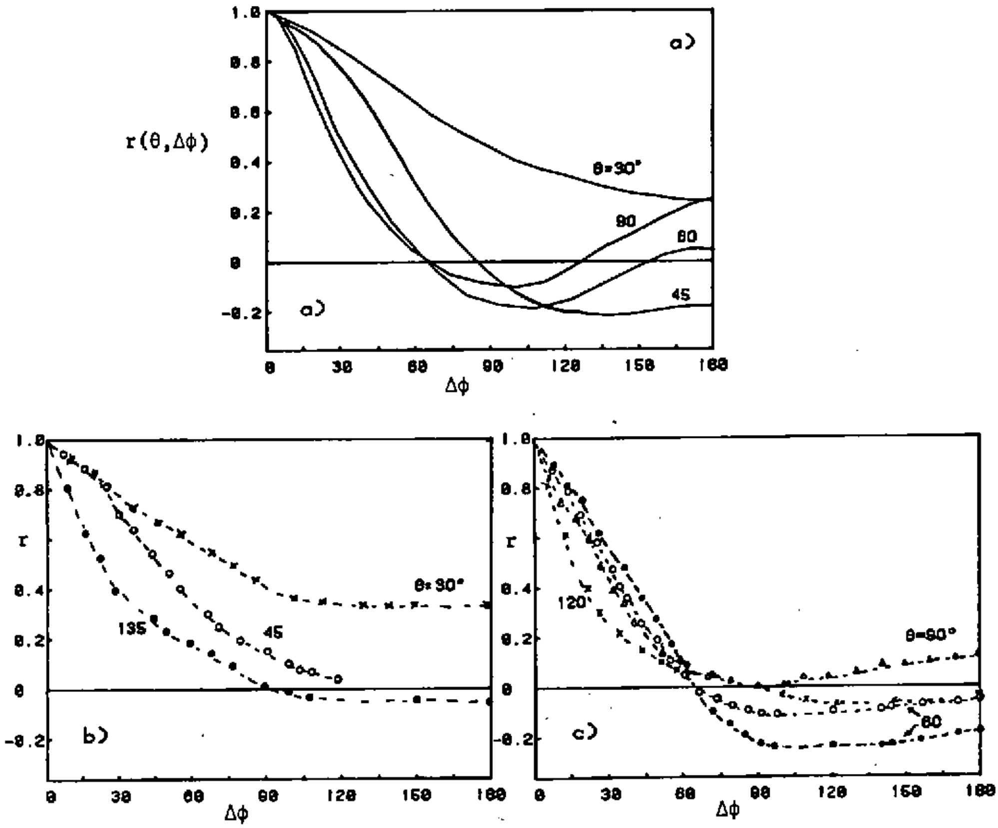

Fig.1 - Azimuthal correlations a)M_0=0.4, b)M_0=0.75. Broad-band data except a) θ=45° [6], for jet diameter Helmoltz number He=0.24. In c) •,=maximum He reduced from 3 to 0.28

The only remarkable characteristic of the theoretical H.F. solution is that near the outer edge of the cone of silence, $r_0 \to 1$ since the influence of all components other than T_{11} is much reduced as $q_1 \to 0$.

Discussion (broad-band data) - It is remarkable that, although there is no apparent reason why different D(θ) should be used, the matching of the regions of positive and negative r is quite good, the one exception being the curve for θ=60°, M_0=0,4 which, for large enough Δφ becomes positive. Nevertheless, the modification of the general trend for θ=60° with D(θ) is satisfactory. Indeed, if $\mathcal{D}(\theta)$ is suplementende by $-1/4\ \sin^2 2\theta$ the discrepance is removed.
For such a simple model, the agreement is encouraging, if one thinks on the number of factors involved. The picture of independent emitters is corroborated by the experimental data, azimuthal source coherence playing no relevant part in the broad band sound generation. Also, the effect of diferenciated amplification due to the existence of flow, emphasizing

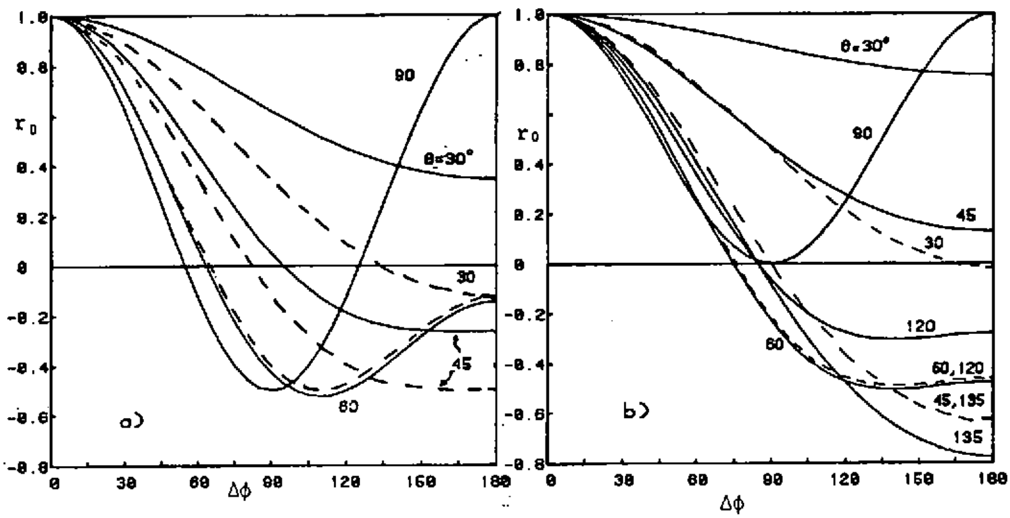

Fig.2 - Theoretical patterns: —— a) $M_0 = 0.4$, $D = 1$, b) $M_0 = 0.75$, $D = 1 + \cos^4\theta + 1/8 \sin^2 2\theta$; --- $M = 0$, D as before

the axisymmetric azimuthal mode as M cosθ increases, is clearly perceived. Unfortunately, only the data of [1] includes correlations for $\theta > 90^0$, but the differences in the curves for θ and $180^0 - \theta$ cannot be explained only in terms of the effect of convection on the actual value of τ^*. The increase of r with M cosθ is confirmed by aditional data of [1], for $\theta = 30^0$ and M_0 up 1.0.

It should be remembered that, althouth this may not be evident from observation of eq. (13), all negative correlations in the theoretical model, if D(θ) does not deviate too much from unity, are due to the $tr(\underline{T}) = 0$ hipothesis (see eq. (5)). The qualitative agreement also shows that the sometimes used hypothesis $T_{ij} = Q_0 \delta_{ij}$ is not appropriate.

Comments on Narrow Band Analysis - The narrow band azimuthal correlation data present the same characteristics of the broad-band data. In the data of Juvé et. al [4.6] for the unexcited jet, the zero crossing points of $\tilde{r}(\theta,\Delta\phi)$ ($\tilde{r}$ stands for the normalised cross-espectral density of the pressure signals) are roughly independent of frequency, except for $\theta = 60^0$, where for the lower frequencies, a second zero crossing exists. For the data of Maestrello [2], restricted to $\theta = 90^0$ only, this is not verified. Indeed, the data of both experiments present marked differences, suggesting that they were operated under different conditions.

For narrow band signals, the single source model can be easily extended if hypothesis on the source distribution are made. For azimuthal correlations, the natural model is a ring distribution of quadrupoles $\underline{T}(\nu,t)$, where ν is source azimuthal angle. The analysis was developed in [20] for $\theta = 90^0$, in terms of the Fourier transforms of $\overline{T_{ij}(\nu,t)T_{k\ell}(\nu,t+\tau)}$ and the source azimuthal coherence function $g(\Delta\nu)$. It was verified that discrepancies existed with respect to the incoherent ring source model ($g(\Delta\nu) \to \delta(\Delta\nu)$), but that introduction of coherence alone would not improve the comparison. To explain the low measured values of $\bar{r}(90,180)$ for very low frequencies the existence of some dipole noise was suggested [20,21], what is consistent with theoretical predictions [22].

A situation that deserves atention is that of excited jets. Excitation can fix the position of vortex pairing and the phase characteristics of developing instability waves causing, in certain cases, a considerable increase in the radiation at a discrete frequency (e.g. [6,23-24]. Again, divers models exist, predicting sometimes, similar features due to different reasons. An example is the prediction of a node in the sound field directivity, that is observed in some experiments [6,23], and that can be due either to azimuthal coherence effects in the developing instability waves [25] or to point source characteristics, in the vortex pairing model (indeed, to the embodiment of eq. (8) in the source model) [24]. In both models, the sound field is instantaneously axisymmetric and, although the azimuthal correlation data of [5,6] is rougly consistent with this for low angles θ, the pure tone increase of 3 dB observed for $\theta = 90^0$ [6], corresponds to a negligible modification of $\bar{r}(90,\Delta\phi)$ at this frequency, suggesting that source models must be improved upon.

The fact that in polar correlations, time delay effects are generally preponderant favours their use for source location, the influence of the "quadrupolar pattern" r_0 being usually neglected. For low enough frequencies, it will be discernible in the measurements and may, due to its zero, originate a spurious peak in the computed source image [26]. The extraction, whenever possible of $\bar{r}_0$ from the measured data before computing the source image was suggested, it being shown that the analysis of the angular separation at which a zero of $|\bar{r}|$ occurred, can provide, in some cases, a good guess on the nature of a peak in the computed source image.

CONCLUSION

It is thought that the searching for answers for many of the existing controversies on sound generation mechanisms in jets could proffit much from the study of far field two point correlation measurement data, which constitutes a tool still underexplored.

The comparison of the existing broad-band data with the "lateral quadrupole" single source model appears very promising, although there is no apparent reason why the source directivity should be different in the two experiments.

The reported analysis on narrow-band data shows that the measurements are influenced by factors not originally accounted for and that the establishment of reference correlation patterns, Mach and Helmoltz number dependent, for quantitative comparison is still very far - it may even not be possible, since the domain of variation of these factors is not yet known. This difficulty is due to the sensitivity of two point correlation measurements to details of the sound generation meachanisms and source distribution which have a lesser effect on one microphone measurements, a sensitivity which can be used advantageously. It should be noted that, even for unexcited jets, the existing published data is very scarce, and should be extended before firm conclusions can be drawn.

REFERENCES

[1] Maestrello, L., Two point correlations of sound pressure in the far field of a jet: Experiment, NASA TM X-72835, 1976.

[2] Maestrello, L., AIAA Paper 77-1267, 1977.

[3] Juvé, D., Sunyach, M., Structure azimutale du champ acoustique lointain d'un jet subsonique, C.R. Acad. Sc., Paris, t. 287, pp. 187-190, 1978.

[4] Juvé, D., Sunyach, M., Comte-Bellot, G., AIAA J., 17(1), pp. 112-113, 1979.

[5] Juvé, D., Sunyach, M., AIAA Paper 81-2011, 1981.

[6] Juvé, D., Aéroacoustique des jets excités D.Sc. Thesis, Lyon, 1986.

[7] Ribner, H.S., J.Sound Vib., 56 (1), pp. 1-19, 1978.

[8] Maestrello, L.,J. Acoust. Soc. Amer. 66 (6), pp. 1876-1890, 1979.

[9] Michalke, A., J. Sound Vibr. 87(1), pp. 1-17, 1983.

[10] Musafir, R.E., Slama, J.G., Zindeluk, M., Proc. Inter-Noise 84, pp. 257-260, Honolulu, 1984.

[11] Musafir, R.E., Geração de ruido por jatos turbulentos, M.Sc. Thesis, Rio de Janeiro, 1984.

[12] Musafir, R.E., Zindeluk, M., Slama, J.G., On the acoustic radiation of turbulence, Proc. VIII Braz. Congr. Mech. Eng., pp. 637-640, 1985.

[13] Lighthill, M.J., Proc. Roy. Soc., A211, pp. 564-587, 1952.

[14] Dowling, A.P., Ffowcs Williams, J.E., Goldstein, M.E., Phil. Trans. Roy. Soc., A288, pp. 321-389, 1978.

[15] Goldstein, M.E., J.Sound Vibr. 80(4), pp. 499-522, 1982.

[16] Obermeier, F., J. Sound Vibr. 99(1), pp. 111-120, 1985.

[17] Balsa, T.F., J.Fluid Mech. 79(1), pp. 33-47, 1977.

[18] Musafir, R.E., Zindeluk, M., to appear in Revue d'Acoustique, 1988.

[19] Tester, B.J., Morfey, C.L., J.Sound. Vibr. 46(1) pp. 79-103, 1976.

[20] Musafir, R.E., The sound field of a quadrupole ring distribution, Proc. IX Braz. Congr. Mech. Eng., pp. 829-832, 1987.

[21] Musafir, R.E., Zindeluk, M., Proc. 12 ICA, Paper J3-5, Toronto, 1986.

[22] Crigthon, D.G., J.Fluid Mech. 56(4), pp. 683-684, 1972.

[23] Laufer, J., Yen, T.C., J.Fluid Mech., 134(1), pp. 1-31, 1983.

[24] Bridges, J.E., Hussain, A.K.M.F.,J.Sound Vibr.117(2) pp.289-311, 1987.

[25] Huerre, P., Crighton, D.G., AIAA Paper 83-0661, 1983.

[26] Musafir, R.E., Proc. Inter-Noise 86, pp. 1335-1340, Cambridge, Ma., 1986.