MODAL RESPONSES OF FLEXIBLE UNSYMMETRIC PRISMATIC BODIES IN STILL WATER

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Consider a prismatic beam, such as a ship's hull, which is symmetric, port and starboard. Let C' be the centroid of the cross section and at this point erect a right handed set of axes C'x'y'z' such that C'x'z' is the plane of symmetry. The product of area in the cross section is zero, so that

$$I'_{y'z'} = \int y'z'dA = 0.$$

Suppose now that a slice of the beam, having thickness Δx, is isolated. Provided the material is homogeneous, the centre of mass C of the slice coincides with C'. We may thus define a right handed frame of axes Cxyz, parallel to and coincident with C'x'y'z' such that the product of inertia in the Cyz plane is zero. That is to say

$$I_{yz} = \int \rho yz dA = \rho\int yz dA = 0,$$

where ρ is the density of the material. In this case the principal axes of area associated with the centroid C' coincide with the principal axes of inertia associated with the centre of mass C.

If this symmetric structure is loaded symmetrically, the centre of mass C moves away from C'. The latter point remains fixed in the structure since no more structure is added - only weight. Suppose the point C now lies at $(0,\bar{z}')$ in the frame of axes C'y'z'. The new product of inertia I_{yz}, with respect to the new centre of mass C, is still zero even though the cargo and structure have different densities. Thus we can identify principal axes of inertia through C and these are parallel to the principal axes of area at C'.

The centre of shear S is associated with the unloaded structure. It lies on an axis in the beam such that if the shear force at any section acts through S, there is no resultant twisting of the beam. Since C' is identified with the structural properties, it appears that the point S is best located by reference to the axes erected at C'.

Since all the systems of axes, erected at C, C' and (if needed) S, are parallel to one another in a symmetric beam there is no difficulty in relating the shear force, bending moment or twisting moment to distortions [1-3] defined by reference to these systems of axes. Whether the symmetric ship is in ballast or fully loaded, the analysis previously used in structural dynamics investigations [1-3] remains unaffected because the systems of axes always remain parallel to one another.

Questions arise, however, when it becomes necessary to consider <u>unsymmetric</u> flexible prismatic bodies. It then becomes necessary to ask what choice of axes should be made, for instance. Consider now a homogeneous beam

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having an unsymmetric section (and we shall continue to think of a "hull" so that we now contemplate lack of symmetry port and starboard). A slice of thickness δx has centre of mass at C while the cross-sectional centroid of area is at C', these two points being distinct. Again principal axes of area may be erected at C' such that the product of area is zero. (i.e. $I'_{y'z'}=0$). Similarly a right handed frame of axes may be erected at C such that the product of inertia is zero; these are the principal axes of inertia $Cx_oy_oz_o$ such that $I_{y_oz_o}=0$.

Use of these latter axes simplifies the equations of motion since $I_{y_oz_o}=0$. Provided the beam is uniform, distortions may be described by reference to these axes. But the product of area about parallel axes through C' is now non-zero and this leads to difficulty in defining the constitutive equations relating shear force, bending moment and twisting moment to the distortions.

An alternative is to erect axes at C which are parallel to C'x'y'z'. This is a fresh system of axes Cxyz in which the products of inertia are non-zero, i.e. $I_{yz}\neq0$. Thus C'x'y'z' remain the principal axes of area whereas Cxyz are no longer the principal axes of inertia. Distortions can now be defined in the directions C'x', Cx, C'y', Cy, C'z', Cz or about these axes.

Since the shear centre S is concerned with the structural properties, its position is defined with respect to C'. Thus we may define a component of shear force which is parallel to the axis Cy passing through S, and S may not lie on a principal axis of area.

These ideas may be adopted when the distortions of a flexible, unsymmetric prismatic body floating in still water are considered. Lack of symmetry may arise in several ways:

(i) The dry hull has port and starboard symmetry but the hydrostatic fluid actions are set up by an unsymmetrical underwater shape because the ship is heeled by steady wind forces.

(ii) The underwater hull shape is symmetric but the dry structure is unsymmetric, port and starboard; this is the case with an aircraft carrier or a roll on roll off ship.

(iii) Both the dry structure and the underwater "hull" shape are unsymmetric, port and starboard; this is the case of a "Salter duck" wave energy device, for instance.

The effects on the hydrodynamic coefficients of lack of symmetry in the underwater hull shape have been discussed elsewhere [4] and so the emphasis in this paper is mainly on lack of structural symmetry. Coupled equations of motion are developed by reference to the general right handed axes Cxyz. This involves defining coupled distortions along the axes Cy, Cz and about the axis Cx, those along Cy, Cz not being necessarily horizontal and vertical as is normally the case with a symmetrical section. Modal techniques are used to solve these equations. Orthogonality relationships are derived and the calculation of natural frequencies is discussed. The analysis is extended to the structure - fluid interaction problem, a brief discussion of which is included.

References

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