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# THE STRUCTURAL DYNAMICS PROBLEMS OF NAVAL ARCHITECTURE

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There is nothing particularly new in the observation that a ship is an elastic beam which is thrown into oscillation by waves on the sea surface. But only in the restricted sense of studying bodily motions can it be said that the literature reflects this form of idealisation. Bodily motions, in which the hull is assumed to be rigid, are examined under the title of 'seakeeping' and although considerable progress has been made over the years in that field, antisymmetric motion (i.e. coupled sway, yaw and roll) still raises very serious questions - notably in connection with capsizing of small and medium sized vessels. Seakeeping has been, is and will continue to be a flourishing area of theoretical and experimental research.

When we turn to dynamics of the elastic ship/beam we find a very different state of affairs. Sheer technological necessity ensured that ship structures were closely studied and that classification rules were drawn up long before it was possible to study them using all the paraphernalia of modern structural dynamics. The result is that an imposing edifice now exists in the form of semi-empirical knowledge based on many thousands of man-years of ship surveying and much specialist research. Generally speaking, ships do not founder as a result of faulty structural analysis.

This does not mean, of course, that all is well and that there is little that is useful to be done. The most obvious shortcoming of present techniques of structural analysis is that they impose a certain conservatism on design, so that a radically new departure is a matter for serious concern. Again it is apparently true that present assumptions of rigidity lead to inaccurate, and therefore necessarily conservative, estimates of loading. Perhaps most important, though, present techniques do not make it plain what the important issues are if a new problem has to be tackled. (To take a particular example from recent events, it would not tell one how to determine which section of a ship is most vulnerable if the ship is subjected to repeated slamming in a heavy sea.).

Attempts are now being made to put the structural side of naval architecture on a sounder footing. This has been made possible by modern techniques of computing, by progress in random process theory, by the accumulation of knowledge on random sea-states, by recent advances in the theory of non-conservative systems - and by the emergence of structural dynamics as something more than an appendage of the theory of mechanical vibration. It would be foolish to suggest that this departure has been greeted with undiluted glee by the naval architecture fraternity, however, because many new concepts have had to be applied to ships. Nevertheless progress does appear to have been made on the basis of linear theory.

Naturally this is not to suggest that all existing knowledge is useless. It means, rather, that contemporary structural dynamics and hydrodynamic theories have now to be adapted to the needs of naval architecture. As often as not, dynamical considerations determine the field conditions in which existing statical results are directly applicable. In other words, a subject which might be called 'Ship Hydroelasticity' is beginning to emerge, based on existing quasi-static theory. By its very nature this field is potentially of interest to structural analysts, dynamicists, hydro-dynamicists, physical oceanographers, applied mathematicians, ..... It is a meeting ground of several disciplines.

As one would expect the introduction of structural dynamics to naval architecture has produced a reaction. Needs have been revealed which could not be met by existing theory so that structural dynamics itself has seen a certain amount of development. Several examples of this are to be found in the need to formulate orthogonality conditions for symmetric and antisymmetric vibration of non-uniform Timoshenko beams; and when the antisymmetric distortion has involved coupled bending and twisting, allowance has had to be made for warping stiffness. Again, in the formulation of a practical extension of the Prohl-Myklestad method to the coupled bending and twisting of a non-uniform beam it was desirable to check the method against a suitable "exact" solution; accordingly the problem of a uniform beam has been solved. There remains much to be done in the dynamics of non-uniform thin-walled beams of open section.

Another outcome of all this is that some important gaps in our present knowledge of hydrodynamics have been revealed. Notable among these is the need of an adequate representation of antisymmetric fluid actions. Admittedly extreme motions (particularly in roll) will require a non-linear theory, but even the hydrodynamic actions of small motions appear to be inadequately modelled by existing theories.

One outcome of this work in ship hydroelasticity will be obvious to structural dynamicists but seems not to have been apparent from the outset. It is that seakeeping has been revealed as that special case of the general theory which arises when only rigid body modes are admitted.

Ship hulls are excited into oscillation by machinery as well as by the sea. The two problems are usually quite distinct, being effectively separated by excitation frequency. Generally speaking 2 Hz is a high frequency for serious wave-excited responses whereas it is a low one for serious vibration of mechanical origin. Now problems of the second type are very common indeed and the responses, if they occur, can be very expensive to cure. Accordingly this aspect of ship structural dynamics, too, receives considerable attention.

Historically, the world 'vibration' has been associated with mechanical excitation but not with wave excitation so that again an effectively separate subject has grown up. This distinction turns out to be less serious than the one that grew up between seakeeping and structural analysis for several reasons. In the first place the frequency range is far different as we have already noted and this places a strain in idealisation of the ship as a beam and greatly alters the influence of the surrounding sea, by diminishing hydrodynamic damping and making added mass and inertia effects independent of frequency. Secondly, the excitation (which is commonly 'at propeller blade rate' or caused by a gearbox) remains stationary within the hull instead of passing along it in the form of wave crests. Thirdly, far greater emphasis is placed on 'local vibration' - a notorious area of difficulty in structural dynamics.

For military reasons interest centres on 'quiet ships'. It is undesirable to radiate noise underwater, noise which originates in the form of mechanical vibration of the hull. If this is a consideration, the problems just mentioned are vastly increased since the frequencies of the offending responses may be as great as 100 kHz or more. In practice emphasis is placed more on stress propagation theory than on modal analysis when dealing with them and, predictably, nothing remotely like a general attack on the phenomena is in sight yet.

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# DERIVATION OF STRUCUTRAL DYNAMIC MODELS FROM MOBILITY DATA

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## INTRODUCTION

In the course of performing a vibration analysis for a typical engineering structure, it is often necessary to obtain a realistic mathematical description of the dynamic behaviour of the structure, or one of its components. In many cases, a purely theoretical analysis is not capable of providing the necessary degree of representation and if the structure or component exists then recourse may be made to experimental measurements on it in an attempt to derive the required model.

The structural dynamic model may be required for a variety of different applications, each of which may impose different constraints or priorities on the nature and precision of the model to be obtained. Principal amongst the applications currently being explored at Imperial College are :

(i) to provide a check on theoretical models of complex aerospace structures;

(ii) to define the properties of one (or more) of the components of an assembly for use in a substructure-coupling analysis;

(iii) to facilitate a prediction of the effects on the vibration properties of a given structure of making modifications to it; and

(iv) to permit the determination of dynamic forces exerted on a component under the complex excitation developed during normal operating or service conditions.

The approach used in all cases is that of 'modal testing' whereby the structure is submitted to the measurement of a series of mobility parameters (or other equivalent frequency response functions) which are then processed (by modal analysis) to yield the basic modal properties of the structure. Specifically, the properties which are obtained from this analysis process are : for each mode of vibration identified - the natural frequency ($\omega_r$), the modal damping factor ($\eta_r$) and a limited description of the mode shape, namely a vector of the relative amplitudes at each of the discrete points tested, $\{\phi\}_r$. Thus the complete modelling

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process uses the techniques of mobility measurement and modal analysis, in both of which areas there are many different procedures and methods currently available.

TYPES OF MODEL

In order to reduce the amount of data (a) to be measured and (b) to be stored as the 'model', three different types of model are used. These are referred to subsequently as 'mobility' models, 'modal' models and 'spatial' models. The mobility model consists simply of a single square matrix of order N (where N is the number of coordinates chosen to describe the structure's behaviour), in which each element is a mobility (or alternative frequency response) function, $Y_{ij}(w)$, relating response and excitation between two of the coordinates, i and j. Each such element is stored separately as the coefficients of the series :

$$Y_{ij}(\omega) = i\omega\left\{\sum_{r=1}^{m} \left\{{}_rA_{ij} / (1 - (\omega/\omega_r)^2 + i\eta_r)\right\} + R_{ij} - S_{ij}/\omega^2\right\}$$

and can only be fully defined by measuring all the mobility quantities individually, over a frequency range which encompasses M modes of vibration. (In fact, by using the symmetry of the matrix, only $\frac{1}{2}(N^2+N)$ different elements need be specified.)

The modal model provides a means of describing almost the same data in a more compact form, and requires considerably less measurements to be made. This model describes the system by two matrices, a diagonal (MxM) eigenvalue matrix (incorporating both the natural frequencies and the modal damping factors) and a rectangular (MxN) mode shape matrix. This type of model enables the (NxN) mobility matrix matrix to be computed frequency by frequency, with the exception of the residual terms R and S which cannot be readily accounted for in this case.

The third type of model - the spatial model - is basically an extension of the modal model which may be made when the number of modes included (M) is equal to the number of coordinates used (N). In this case, the eigenvalue and mode shape matrices can be rearranged so as to form a mass matrix and a complex stiffness matrix, which then provide a mathematical model of the structure in terms of the spatial parameters - mass, stiffness, damping. Within these categories of model, there is also a choice of damped and undamped models. Several applications, including the verification of the predictions from a theoretical (e.g. finite element) model and the modelling of a single component forming part of an assembled structure, demand only the fundamental properties of inertia and flexibility and for these, an undamped model is adequate, indeed optimum. Other applications may demand a more exact description of the structure as tested and in these cases a damped model is necessary.

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## MODAL ANALYSIS TECHNIQUES

In the construction of these various models, two modal analysis procedures are employed, one for the undamped type of model and the second for damped models.

The first of these methods demands only the accurate location of resonance and antiresonance frequencies plus the specification of a small number of off-resonant points for each mobility curve (and thus avoids the problems encountered in making accurate mobility measurements near resonance on lightly damped structures). The second method requires a small number of accurate mobility data points near each resonance, plus a few more off resonant points near the lower and upper bounds of the frequency range covered.

The performance of both analysis methods has been assessed using simple structures and theoretical models, and both have been applied to many practical engineering structures. Comparisons of the models derived from given experimental data by these and other methods, suggests that the process of modal analysis is not yet fully developed - there being several potential sources of error. In particular, the more commonly used modal analysis processes (for damped structures) may well be particularly sensitive to small nonlinearities in the system.

## SPECIFIC MODELS

A number of examples of the application of these modelling techniques to specific structures are now given. The cases cited all relate to relatively simple beam-type structures; these being chosen to permit a parallel theoretical analysis to be made.

<u>I-beam plus masses</u>. An I-section beam, about 1.5m long, with a number of concentrated masses added at points along its length, was used for a series of modelling studies. Mobility measurements were made at several points and undamped models of all three types were obtained - mobility, modal and spatial - using first a 5-coordinate model and secondly, one with 10 coordinates. The resulting mass and stiffness matrices (constituting the spatial model), while lacking the appearance of those derived in a theoretical analysis, do provide an adequate description of the structure's dynamic behaviour, as referred to the 5(or 10) chosen coordinates. The mass matrix in particular, is difficult to interpret physically as it is a full matrix with large (and often negative) off-diagonal elements. However, the sum total of all the elements is found to be exactly equal to the total mass of the structure. Furthermore, the form of these matrices is similar to that encountered with 'condensed' or 'economised' models used to reduce the size of large finite element descriptions of structures.

These mass and stiffness matrices may be used to examine the effects of introducing modifications to the structure (in this case; a rearrangement of the lumped masses attached to the beam) and the modified system's properties have been both predicted from the new mass / stiffness matrix model and also measured on a reassembled beam, the two sets of results showing good agreement.

Built-up Uniform Beam. One of the limitations of the modal and spatial models is the inability to take proper account of the effects of modes which exist on the structure, but are not included in the model. These effects are represented by the residue terms, R and S, in the mobility model. It can be shown that the more important* of these two (R, accounting for modes with frequencies above the range considered) is only significant for point mobilities, generally being an order of magnitude smaller for the transfer mobilities. It may also be shown that the addition of such a residue to a point mobility may be simulated in the model by considering that a simple spring is interposed between the basic structural model and the coordinate to which that point mobility relates. This results in a model which is of order 2N although it still only possesses N 'real' modes of vibration.

A second application of the modelling procedures is to a pair of uniform beams - one 1.4m long, the other 0.65m long. A spatial model (of the type just described) has been derived for each beam in turn using (a) theoretical and (b) experimental mobility data, and the two models then combined so as to represent a single long beam (of length 2.05m). The mobility properties of this third beam have thus been predicted and have also been evaluated directly and excellent agreement found between the two sets of results. A corresponding set of calculations made without the refinement of including the residual effects yielded decidedly inferior results.

* In freely-supported (ungrounded) structures, the second residual term S, which includes the rigid body modes, may well have a major influence and also be difficult to evaluate.