THE RESPONSE OF VEHICLES ON ROUGH GROUND

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1.0 Introduction

The motion of vehicles traversing rough ground at variable velocity is non-stationary, and in a recent paper [1] the authors described a method for the computation of the propagation of vehicle response variance, using the concept of a spatial shaping filter to model the ground profile. The formulation is for linear systems, leading to a state-space form for the combined vehicle and excitation, and allows any deterministic velocity history to be accommodated.

In this paper the above problem is briefly reviewed and extended to include vehicles described by non-linear, differential equations. It is generally true that randomly excited, non-linear, differential equations require a Monte-Carlo simulation approach for their solution, but here the analytical technique of statistical linearization allows the problem to be cast in linear, state-space form. This use of describing functions to approximate the non-linear elements within the state-space framework leads to the so called C.A.D.E.T. (Covariance Analysis by DEscribing function Technique) of Gelb and Warren [4]. The concept of the spatial shaping filter is again used to expedite the non-stationary analysis of non-linear systems without costly simulation.

We demonstrate the validity of the method via a worked example and comparison with Monte-Carlo simulations.

2.0 Linear Systems Theory

2.1 General Theory

A finite dimensional, linear system driven by white-noise may be modelled by:

$$\underline{\dot{x}} = A\underline{x} + B\underline{w} \; ; \quad \underline{x}(t_0) \qquad (1)$$

where $\underline{w}$ is a white-noise vector with zero-mean and $E[\underline{w}(t_1)\underline{w}^T(t_2)] = Q\delta(t_1-t_2)$. If $P(t)$ denotes the zero-lag covariance matrix $E[\underline{x}(t)\underline{x}^T(t)]$, then it is well known (eg. [2]) that P satisfies:

$$\dot{P} = AP \pm PA^T + BQB^T \; ; \quad P(t_0) \qquad (2)$$

For our purpose the white-noise vector is a function of space, which in turn, is regarded as a function of time, $\underline{w}(s(t))$. Restricting $s(t)$ such that $\dot{s}(t) > 0$ and $\dot{s}(t)$ vanishes only at isolated points, t_i say, then, from [5], we get:

$$E[\underline{w}(s(t_1))\, \underline{w}^T(s(t))] = Q\delta(t_1-t)/\dot{s}(t) \; ; \quad \dot{s}(t) \geq 0$$

We now postulate the existence of a white process (in time) such that

$$w(s(t)) = \underline{w}_1(t) \,/\, \sqrt{\dot{s}(t)} \qquad (3)$$

where $\underline{w}$ and $\underline{w}_1/\sqrt{\dot{s}}$ are 'covariance equivalent'. This allows us to incorporate the excitation, under the shaping filter hypothesis of [1], into the state-space form of equation (1), where A and B are now time varying. Equation (2) may now be applied to yield the time varying, zero-lag covariance matrix, P(t).

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2.2 Application to the vehicle problem (linear dynamics)

It is necessary at the outset to describe the vehicle plus ground combination (as a function of time) by a first order, linear, vector differential equation:

$$\dot{\underline{y}} = F\underline{y} + G\tilde{\underline{h}} \qquad (4)$$

$\underline{y}$ denotes vehicle response vector; $\underline{h}$ denotes the input (ground profile) vector; $\sim$ implies that the variable is regarded as a function of time.

To obtain the form of (4) in cases where the input is not given explicitly in terms of the state variables, it is necessary to substitute from the (transformed) excitation differential equation, directly into the vehicle equation. This point will be made clearer in the worked example.

Now, if the spatial covariance matrix for $\underline{h}(s)$ is known, then it is possible, for a certain class of processes, to regard $\underline{h}(s)$ as the output of a white excited linear filter, i.e.

$$\underline{h}(s) = H\underline{g}(s) \quad \text{where} \quad \frac{d}{ds}\underline{g}(s) = J\underline{g}(s) + M\underline{w}(s) \qquad (5) \text{ and } (6)$$

By applying the transformation $\frac{d}{ds} = \frac{1}{\dot{s}(t)} \cdot \frac{d}{dt}$, (3), (4), (5) and (6) may be combined to give:

$$\frac{d}{dt}\begin{bmatrix}\underline{y}\\ \tilde{\underline{g}}\end{bmatrix} = \begin{bmatrix}F & GH\\ 0 & \dot{s}(t)J\end{bmatrix}\begin{bmatrix}\underline{y}\\ \tilde{\underline{g}}\end{bmatrix} + \begin{bmatrix}0\\ \sqrt{\dot{s}}\ M\end{bmatrix}\underline{w}_1(t) \qquad (7)$$

which is of the form (1) so that (2) applies.

3. The Inclusion of Non-linear elements

The Random Input Describing Function is fully covered elsewhere [3]. It is sufficient here to note that the RIDF results from the replacement of the true non-linear element by a linear 'gain' such that the mean square output error is minimised. This procedure requires an assumption to be made regarding the nature of the input process, which we assume is Gaussian. The RIDF is generally a function of the mean and variance of its input process i.e. RIDF = $D(\mu,P)$.

3.1 The CADET approach

The Covariance Analysis by Describing function Technique (CADET) utilizes the properties of RIDF's and equations (1) and (2) to yield the zero-lag covariance matrix, P, for non-linear systems. So if the system plus excitation can be modelled thus:

$$\dot{\underline{x}} = A\underline{x} + \underline{f}(\underline{x}) + B\underline{w} \quad ; \quad \underline{x}(t_o) \qquad (8)$$

then, under CADET, the elements $f_i(x_i)$ are replaced by $D_i(\mu_{xi},P_{xi}).x_i$

Equation (8) may now be rearranged to give:

$$\dot{\underline{x}} = (A+D)\underline{x} + B\underline{w} \quad ; \quad \underline{x}(t_o) \qquad (9)$$

which is of the form (1) so that (2) applies, except that the coefficient matrix is itself dependent on P and μ, and is a non-linear, differential equation with variable coefficients, and must in general be solved numerically. A similar argument applies to the bias propagation. In general then, the bias and covariance equations are coupled and must be solved simultaneously.

3.2 An example

A symmetrical non-linearity was chosen to remove the problem of bias propagation Consider the vehicle shown in Fig 1 with damping that is an odd function of the square of its relative vertical velocity (i.e. $\dot{x}|\dot{x}|$). Its equation of motion is

$$\ddot{x} + \gamma\dot{x}|\dot{x}| + \omega_o^2 x = -\ddot{\tilde{h}} \quad ; \; (x = y-\tilde{h}) \qquad (10)$$

So we replace the function $\dot{x}|\dot{x}|$ by its RIDF, [3]; $D(P) = \sqrt{8P_{\dot{x}}/\pi}$ (11)

The RHS of (10) is not explicitly in terms of state variables $\tilde{h}$ or $\tilde{h}'$ so the substitution referred to in section 2.2 must be performed.

A second order spatial shaping filter is chosen to describe the ground profile:

$$\frac{d^2h}{ds^2} + 2\alpha\beta\frac{dh}{ds} + \beta^2 h = Kw(s) \qquad (12)$$

where $K = \sqrt{4\alpha\beta^3}\,\sigma$ which gives the variance of h as σ^2. The states for the shaping filter are h and h' (which regarded as a function of time are written $\tilde{h}$ and $\tilde{h}'$). Now combining (10),(11) and (12), taking care with the transformation of the independent variable, we finally arrive at the augmented system in state-space form.

$$\frac{d}{dt}\begin{bmatrix} x \\ \dot{x} \\ \tilde{h} \\ \tilde{h}' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\omega_o^2 & -\gamma\sqrt{8P_{\dot{x}}/\pi} & \beta^2\dot{s}^2 & (2\alpha\beta\dot{s}^2-\ddot{s}) \\ 0 & 0 & 0 & \dot{s} \\ 0 & 0 & -\beta^2\dot{s} & -2\alpha\beta\dot{s} \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \\ \tilde{h} \\ \tilde{h}' \end{bmatrix} + \begin{bmatrix} 0 \\ -\sqrt{4\alpha\beta^3\dot{s}^3}\,\sigma \\ 0 \\ \sqrt{4\alpha\beta^3\dot{s}}\,\sigma \end{bmatrix} w_1(t)$$

which yields, via equation (2), ten, first-order, ordinary differential equations in P_{ij}, which can be solved with a simple, fourth-order, Runge-Kutta numerical integration routine. Initial conditions are set according to the arguments put forward in [1].

4. Brief Discussion and Conclusions

Here, only P_{11} is discussed but CADET produces the other covariances simultaneously.

Fig 2 shows the propagation of P_{11} whilst the vehicle undergoes a constant acceleration from rest. The response displays certain features of linear systems (see also [1]) but also exhibits a very exaggerated initial transient which does not occur with linear dynamics.

Fig 3 corresponds to the situation where a vehicle, travelling at constant velocity on a perfectly smooth surface, suddenly encounters rough ground. The response is again similar to that for a linear system but the rippling is more pronounced.

The results from these Monte-Carlo simulations (ensemble size 10,000), show that the approximate method is extremely effective in spite of the assumptions inherent in the approach. The tendency of the method to underestimate is a natural consequence of statistical linearization, as shown in [3] but this discrepancy is small when compared to the cost of simulation (a factor on the order of the ensemble size). More importantly, the non-stationary CADET approach mimics the behaviour of the true system accurately enough to facilitate its study, although it is advisable (see also [3]) that simulation be employed as a final check.

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Illustrations

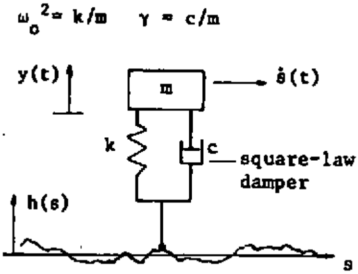

Fig.1

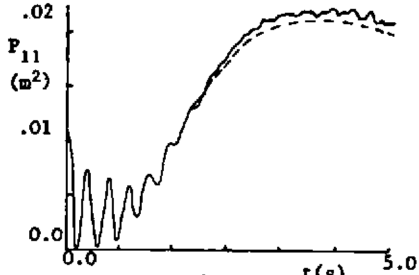

Fig.2

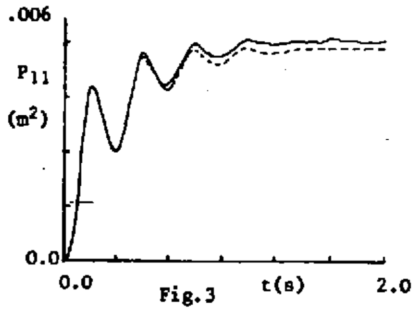

Fig.3

——— 10000 Run Simulation
- - - - C.A.D.E.T.

References

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