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A Comparative Study of FIR and IIR Digital Equalization Techniques for Loudspeaker Systems

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1. INTRODUCTION

With the evolution of digital audio, greater possibilities are arising for the improvement of some of the most significant high level deficiencies in the audio chain, ie. those induced by loudspeakers and the listening environment. Indeed, this subject has attracted a great deal of work in recent years, a sample of which is given in references [1,..,4]. The technique for effecting loudspeaker and room response equalization generally involves deconvolving the said response with its inverse, generated by a digital filter. The type of filter used for the afore mentioned task most commonly falls into the category called Finite Impulse Response (FIR) filters. Although a number of methods for deriving the filter coefficients have been tried, the most widely used approach employs a Least Mean Squares (LMS) optimization process. Ultimately, providing that the FIR filter has a sufficient number of coefficients, all of the methods mentioned in the referenced papers will give equalization to a satisfactory degree. Unfortunately, owing to the finite amount of time allowed for the filter computations, the number of coefficients must be constrained. For example, current state of the art digital signal processors, such as the Texas Instruments TMS320C25, permit filter lengths of around 200 coefficients when operating at 44.1 kHz, the compact disc standard. This figure dictates that for most loudspeakers, and certainly all room responses, a large approximation to the true inverse response must be made for full range equalization of the audio bandwidth.

An alternative method for loudspeaker equalization suggested by the authors in an earlier paper, Greenfield *et al* [5], employs Infinite Impulse Response (IIR) filters for partial equalization of the loudspeaker's response. The use of IIR structures is limited to minimum-phase equalization as, the true inverse of the excess-phase response produces an unstable IIR filter. With many systems however, particularly loudspeakers where the major source of excess-phase distortion is attributable to the crossover, there remains a significant proportion of minimum-phase distortion. Thus in cases where there exists (minimum-phase) low to mid-band anomalies or high 'Q' resonances, both of which have slowly decaying impulse responses, IIR structures enable considerable computational savings to be made. Minimum-phase equalization, as carried out in [5], results in an all-pass response which for many applications may be satisfactory, or sometimes even preferable. If the response is desired to be linear-phase further equalization can be performed by an FIR filter which introduces overall delay into the system.

The complexity of acoustic sources invariably means that any attempted equalization will be approximate to some degree. This paper discusses and presents the results of a comparative study between FIR[1] and IIR equalization techniques. The emphasis is placed on the practical realization of the digital filters. Therefore the equalizers will be derived on the basis that the computational requirements are congruent with real time operation using the TMS320C25 DSP device operating at a sampling rate of 44.1 kHz. This implies a maximum of 180 and 200 coefficients in the cases of the IIR and FIR structures respectively.

In order to highlight the differences between the two equalization techniques we begin with a brief discussion of the optimization techniques used to derive the IIR and FIR filter coefficients. In section 3 the equalizers are derived from the responses of measured loudspeaker systems. Assessment of the equalization processes is given, based on time domain and frequency domain error criteria. A discrepancy between the error performance and observed character of response plots brings into question the relevance of such error criteria regarding perceived sound. A discussion of this and other aspects concerning differences in the equalization techniques follows in section 4.

2. EQUALIZER DERIVATION

Both the IIR and FIR coefficients are derived using a LMS algorithm operating on discrete time domain data. The algorithm, presented below, is essentially the same for both cases, although it is used to effect solutions to converse problems in each case, ie. that of modelling and inverse modelling for the IIR and FIR filters respectively.

2.1 LEAST MEAN SQUARE ALGORITHM

Consider the schematic of the optimization process shown in figure 1 where n is the discrete time variable, x(n) is the input signal, y(n) the output signal from the filter, d(n) the desired output signal and e(n) is the difference signal d(n)-y(n). Using a tapped delay line (FIR filter) of length M samples with tap weights h_j, the LMS algorithm attempts to minimize the squared error function

$$\sum_{n=0}^{N} e^2(n) = \sum_{n=0}^{N} \left(d(n) - \sum_{j=0}^{M} h_j x(n-j) \right)^2 \qquad 2.1$$

Writing equation 2.1 using vector notation with $X^T = [x(n-0), x(n-1), \ldots, x(n-M)]$ and the coefficient vector $H^T = [h_0, h_1, \ldots, h_M]$.

$$\sum_{n=0}^{N} e^2(n) = \sum_{n=0}^{N} \left(d(n) - H^T X(n) \right)^2 \qquad 2.2$$

where T denotes the transpose operator. Minimising Equation 2.2 by setting its derivative, with respect to H, equal to zero yields

[1] From this point on an FIR structure refers to one which has purely a feed-forward path and an IIR structure has both feed-forward and feed-back paths.

$$0=\sum_{n=0}^{N}\left(d(n)Y^{T}-H^{T}X(n)X^{T}(n)\right) \qquad 2.3$$

Letting $A=\sum_{n=0}^{N} d(n)X^{T}(n)$ *and* $B=\sum_{n=0}^{N} X(n)X^{T}(n)$

where A and are B known as the cross correlation vector and auto correlation matrix respectively. Equation 2.3 becomes

$$H^{T}B = A^{T} \qquad 2.4$$

which leads to the optimal solution, in the least mean squares sense

$$H_{opt} = B^{-1}A \qquad 2.5$$

There exist a number of methods for solving equation 2.5 some of which tend to be more robust than others. Further reference on this subject can be found in Press *et al* [6] chapters 2 and 14. The method found to be most practical was a straight forward gradient search algorithm, of which details are given in [5]. The algorithm allows an approximate solution to be found for a set of equations that number greater than the set of variables, in essence approximating a solution for the condition where the number of impulse samples is greater than the number of filter coefficients.

2.2 FIR EQUALIZER DESIGN

The strategy used here follows an approach similar to that of Mourjopoulos [2]. With respect to the block diagram of figure 1, here the input signal x(n) is the loudspeaker's impulse response and the desired signal is the unit sample delayed by some time τ. Thus the optimization process is based on the direct deconvolution of the loudspeaker's impulse response. The time delay τ is found by a side effect of the IIR optimization process where the excess-phase function is determined. Evaluating the excess-phase function in the time domain gives a clear indication of the necessary time delay. A study on the effect of differing the time delay by Mourjopoulos *et al* [7] showed that provided the minimum delay requirement was met, the mean squared error settled on a constant figure. A similar conclusion was confirmed in this research, where little deviation of the mean squared error was observed when using values of delay greater than the initial estimate. It will therefore be assumed that the use of the delay values obtained in the IIR optimization process will not significantly effect the results pertaining to the FIR solution.

2.3 IIR EQUALIZER DESIGN

The optimization procedure for IIR filters is not so straight forward as the preceding process. Attempting to produce the inverse of a mixed-phase signal directly, will almost certainly result in an unstable solution, as the process tries to converge on a function having poles outside of the unit circle. The method adopted therefore, is to form a model of the loudspeaker (which is known to be stable) from which the excess-phase zeros can be found and dealt with separately. Looking again at figure 1; with this strategy, the input signal x(n) is a unit sample (starting at t = 0) and the desired response d(n) is the impulse response of the loudspeaker. Full details of the algorithm are given in [5], however here it will suffice to say that the modelling process ultimately delivers the pole-zero locations commensurate with loudspeaker's parameters. Inspection of the zero locations reveals the excess-phase zeros, ie. those outside of the unit circle. By reflecting the excess-phase zeros about the unit circle and substituting these newly formed zeros for the originals in the model, a minimum-phase function exhibiting an identical amplitude characteristic to the original is obtained. Similarly an all-pass function exhibiting the excess-phase response of the system is generated by retaining the original excess-phase zeros and creating a denominator function from the newly found minimum-phase zeros. Cascading the all-pass function with the minimum-phase function results in the original mixed-phase function. This is readily seen in equation 2.6

$$H(z)=\frac{\prod_{i=1}^{N-L}(z-m_i).\prod_{k=1}^{L}(ze_k^*-1)}{\prod_{j=1}^{M}(z-d_j)}.\frac{\prod_{k=0}^{L}(z-e_k)}{\prod_{k=0}^{L}(ze_k^*-1)} \qquad 2.6$$

N, M and L are the lengths of the entire mixed-phase numerator, denominator and the number of excess-phase zeros respectively. The m_i and e_k are the original minimum-phase and excess-phase zeros respectively (* denotes the complex conjugate) and the d_j are its poles.

The minimum-phase equalizer is simply the reciprocal function of the minimum-phase model. The excess-phase (all-pass) equalizer is formed by evaluating the impulse response of the all-pass function up to the point beyond which it becomes negligible. In reversed time order, the values of the impulse response become the coefficients of an FIR filter, providing the excess-phase equalization. As mentioned earlier, the point at which the impulse is truncated provides a suitable estimate for the time delay required in the FIR optimization routine described in section 2.2.

3. COMPARATIVE TESTS PROCEDURE AND RESULTS

The FIR and IIR equalization techniques have been applied to a number of loudspeaker types. In this section the results are given for two of the systems which cover the extremes of loudspeaker equalization demands. The first to be considered is that of a high quality closed-box system which has both a flat frequency response and a reasonably sharp impulse response. Contrary to the desirable properties of the former, the second example is an experimental open-baffle loudspeaker system which has an irregular frequency response needing a significant amount of equalization. The data used in this research is obtained by an impulse measurement system. The impulses are truncated before the first reflection arrives at the microphone, effectively giving an anechoic response. Unfortunately this aspect has the effect of limiting low frequency resolution. Therefore, in order to provide a meaningful results, the responses have been artificially manipulated to give what is assumed the correct low frequency alignment. The frequency and impulse responses of the two systems are shown in figures 2. and 3. respectively.

Ideally the ultimate goal of any loudspeaker equalizer is such that the cascaded response of loudspeaker and equalizer produces an impulse of sufficiently small duration, having a flat spectral response over the audible frequencies. In digital systems this ideal will be considered the unit sample. As loudspeaker systems have zero output at 0 Hz, the matched equalizer would therefore require infinite gain at d.c., which is clearly unreasonable. A preferable choice of aiming response is that of a high-pass filter which retains the original's stop band roll-off rate, and cut-off frequency chosen not to overdrive the drive units. The aiming response for the systems under study are as follows: linear-phase high-pass filter with cut-off frequency 100 Hz and transition rates of 12 dB/octave and 6 dB/octave for the closed-box and open-baffle systems respectively.

With these design parameters, the FIR and IIR equalizers have been calculated and the corresponding equalized responses for the two loudspeaker systems are shown in figures 4. and 5. These responses are simulated on the TMS320C25 DSP device as the noise incurred in real-time measurement tends to obscure some of the results. Confirmation of the accuracy of the simulations is given in figure 6. showing the real-time measured responses of the high quality loudspeaker after IIR equalization.

4. DISCUSSION OF RESULTS

The assessment of equalizer performance brings into view some interesting points apropos audio equalization. Mourjopoulos *et al* [7], in their paper comparing LMS with Homomorphic techniques, suggest that suitable error criteria on which equalizer performance can be judged are; the mean squared error in the time domain and the standard deviation of the magnitude response in the frequency domain. Table 2.1 gives these error figures for the two loudspeaker systems. The time domain function is taken over a 10 ms window and is expressed as a percentage of the aiming response amplitude squared. The frequency domain function is taken from 100 Hz to 20 kHz using uniformly spaced samples. In both cases the IIR technique is superior based on the frequency domain error, however the differences are so minuscule that they would appear to be audibly imperceptible. Inspection of the magnitude plots clearly reveals serious (almost certainly audible) ripples in the low to low/mid band of the FIR equalized high quality loudspeaker's response. The discrepancy occurs because the ear is sensitive to a logarithmic frequency scale, thus the uniform frequency sampling used in the error function does not truly reflect a suitable assessment criteria. Whilst this deficiency is straight forward to rectify, there are other aspects which are not so apparent. For example the significance of shallow but broad-banded troughs, or narrow high 'Q' resonances; how are these features to be weighted in an error criteria ?. The FIR equalized open-baffle system, has similar difficulties in the low frequency regions. In this example the frequency response starts to roll-off steeply at 200 Hz, hence failing the design specification. Looking at the time domain error function, a surprising result is achieved. The IIR approach is again superior in the case of the high quality system, whilst the converse is true of the open baffle system. This result is most surprising as the impulse response of the open baffle equalizer (shown in figure 7.) is well over 1500 samples long. The increased error is due to the slightly boosted low frequency response below 100 Hz which sustains the duration of the impulse response of the IIR equalized system. This flatter bass response is more likely to be sonically preferable to the premature roll-off of the FIR equalized response.

As an aside, an hypothesis drawn from these results is that for audio applications the general criteria often applied to the appraisal of optimization algorithms should not be applied without reservation. Even where more sophisticated error functions are used, such as non-uniformly weighted or distributed samples, the most appropriate (measurable) indication of the systems performance will be obtained by visual inspection of the graphically represented data. Ultimately, of course, the best indicator must be what your ear tells you.

A further aspect encountered in the optimization processes is worth noting. When the demands are too great for the FIR filter, given a set number of coefficients, the optimization procedure tends to behave quite radically at critical points in the system's response. The IIR modelling approach on the other hand is reasonably well behaved, merely producing smoother and less detailed equalization than a higher order filter would otherwise achieve. An example of this situation is demonstrated in figures 8. and 9. where the number of coefficients used in the open-baffle equalizer is reduced to 100 and 80 for the FIR and IIR filters respectively.

5. CONCLUSION

A comparison between an FIR based and a hybrid FIR/IIR based solution to the problem of loudspeaker equalization has demonstrated that a superior performance can be attained using the latter solution. The gains are more significant where the systems to be equalized exhibit a predominantly minimum-phase response, therefore making IIR filters particularly attractive to loudspeaker equalization. A secondary issue made apparent by this study, is that care must be taken when using the standard statistical error criteria a means of assessing audio systems.

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	Frequency Error (dB)	Time Error
H/Q FIR	0.377611	0.002338
H/Q IIR	0.309677	0.001703
O/B FIR	0.759935	0.002128
O/B IIR	0.322964	0.002157

Table 1 Amplitude standard deviations and time domain errors pertaining to the High Quality (H/Q) and Open Baffle (O/B) loudspeaker systems for the FIR and IIR equalization techniques.

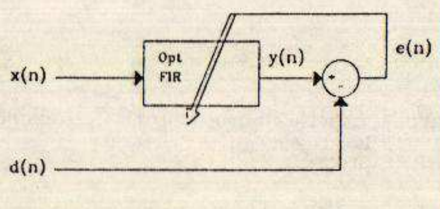

Figure 1. Block diagram of the filter optimization process.

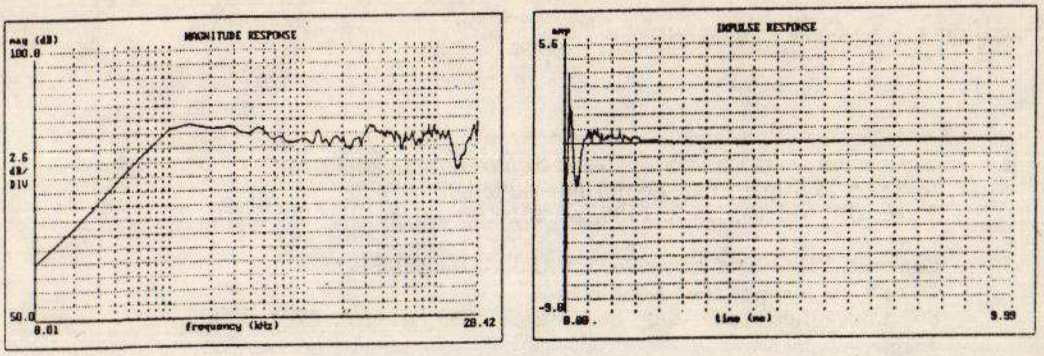

Figure 2. Responses of the High Quality Loudspeaker.

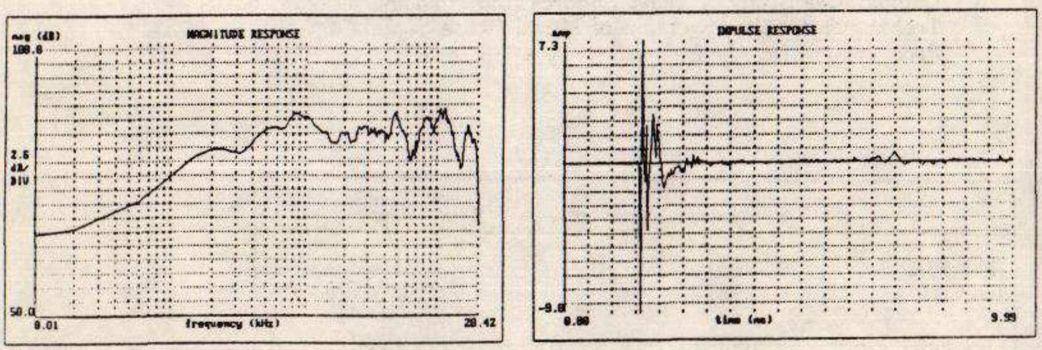

Figure 3. Responses of the Open-Baffle Loudspeaker.

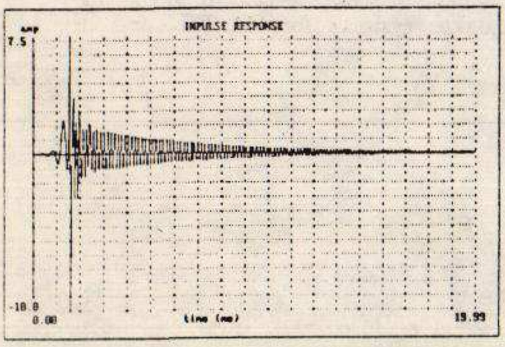

Figure 7. Impulse Response of the Open-Baffle Loudspeaker Equalizer.

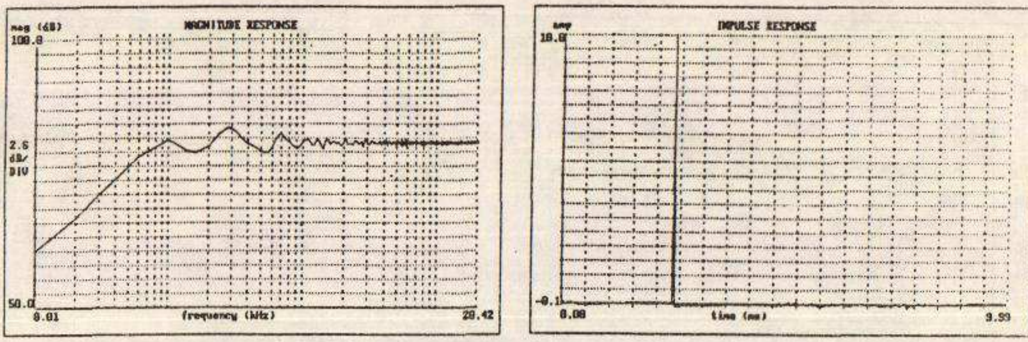

Figure 4. a) Responses of FIR Equalized High Quality Loudspeaker.

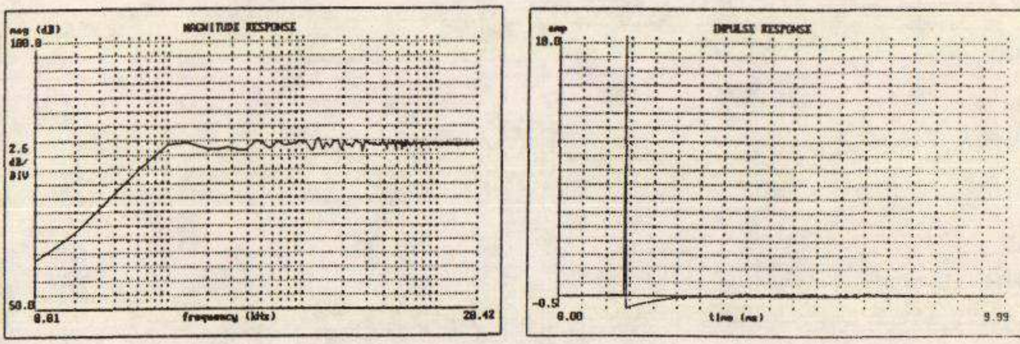

Figure 4. b) Responses of IIR Equalized High Quality Loudspeaker.

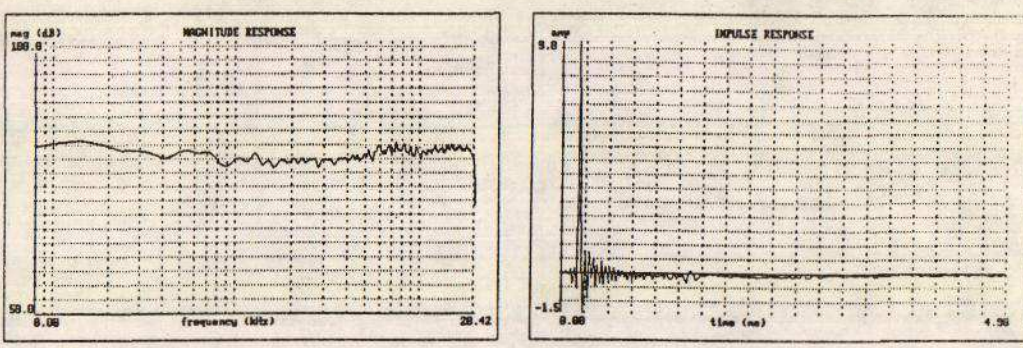

Figure 6. Real-Time Measurements of the IIR Equalized High Quality Loudspeaker.

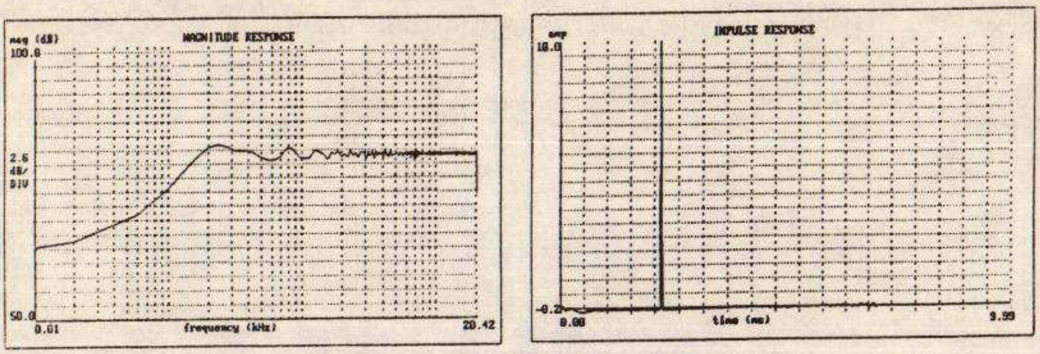

Figure 5. a) Responses of FIR Equalized Open-Baffle Loudspeaker.

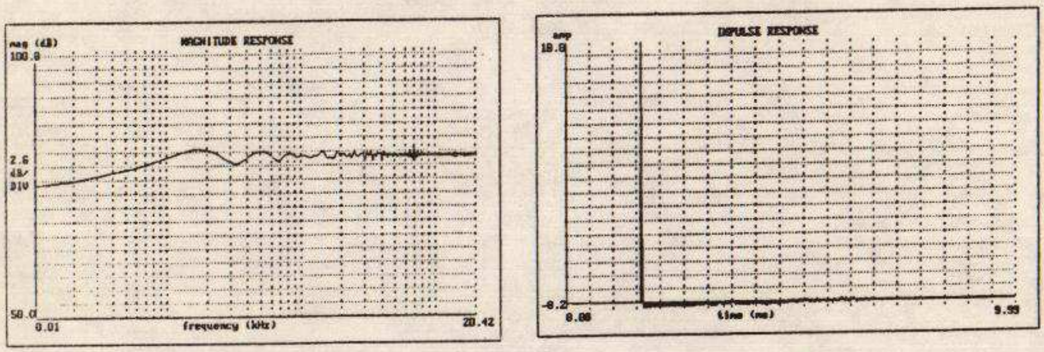

Figure 5. b) Responses of IIR Equalized Open-Baffle Loudspeaker.

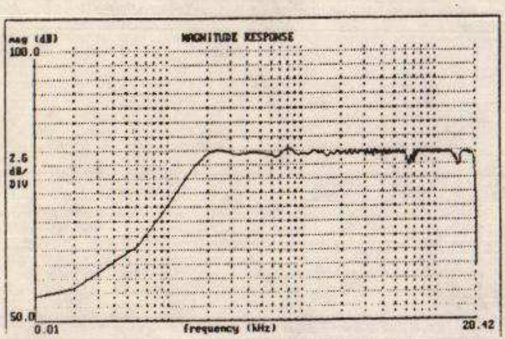

Figure 8. Frequency Response of Open-Baffle loudspeaker Equalised with a 100 Coefficient FIR Filter.

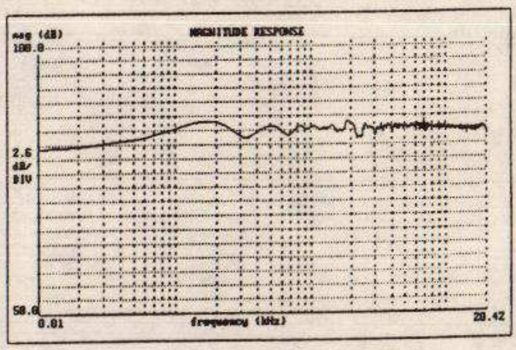

Figure 9. Frequency Response of Open-Baffle loudspeaker Equalised with a 80 Coefficient IIR Filter.