STRUCTURAL ANALYSIS FOR VIBRATION CONTROL USING MOBILITY AND POWER FLOW METHODS

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1.0 INTRODUCTION

Machinery-induced vibration is often a cause of concern or annoyance in structures such as buildings, ships, aircraft etc., and vibration control is therefore a topic of continuing study. In any such problem, the first approach is usually to modify the source and to isolate it from the supporting structure via resilient vibration isolators. The vibration transmission characteristics of the structure between the source and the area of unacceptable vibration levels must then be examined and appropriate vibration control procedures carried out. It can thus be seen that, from the vibration control point of view, there is continuing interest in vibration isolation and structural vibration transmission. The coupled source, isolator, receiver problem may be studied in detail and such quantities as force or motion transmissibilities evaluated with consideration of the dynamic characteristics of each element. The mobility method is often used for this purpose, however, there is also a need for a simplified approach to enable transmissibility studies to be made for isolators coupled to structures for which detailed frequency response data are not available. In the case of vibration transmission between points on a distributed structure, measurement of transfer functions alone will not yield enough information to enable transmission paths to be identified and suitable remedial action, ie. vibration control procedures, to be carried out. For this reason, attention has turned in recent years to the development of methods for measuring the power flow through structures. This approach to the understanding of transmission mechanisms with the objective of transmission reduction has also been complemented by study of the power flow through isolator systems. The power flow concept can thus be seen to unify vibration control methods by seeking to minimise the power input to the structure and then to minimise the power transmission through the structure.

This paper reviews some developments in the approximate mobility approach to the study of vibration isolation problems and describes the use of power flow methods for identifying source contributions and vibration transmission mechanisms.

2.0 THE APPROXIMATE MOBILITY APPROACH TO THE COUPLED SOURCE - ISOLATOR - RECEIVER PROBLEM

The frequency response approach to structural analysis and estimation of coupled system performance has been well reviewed together with descriptions of suitable measurement techniques [1,2,3]. The mobility approach to the vibration isolation problem is discussed in [4] and for example, the effectiveness (E) of a massless isolator coupling a source to a receiver is given by:-

$$E = \left| 1 + \frac{M_I}{M_S + M_R} \right|$$

where M_I = the isolator mobility, M_S = the source mobility, M_R = the receiver mobility — which are generally complex functions

It is clear that for good isolation it is required that $|M_I| >> |M_S + M_R|$ and obviously for good design and behaviour prediction, source and receiver mobilities must be estimated. At the present time there is little information concerning the mobilities of sources such as machines and this is a subject of continuing research. However, although some simplifications may be made concerning the nature of sources by assuming that they are of either of the force or velocity type, the perhaps most significant problem at present concerns mobility characteristics to be ascribed to receivers (substructures) such as floors, bulkheads, machinery seatings etc., composed of beams, plates or beam and plate combinations.

It can therefore be seen that if a complete set of measured or predicted data is not available, assessment methods will be necessarily approximate . Because of these problems, and the need for very simple isolation system assessment procedures, which may be applied rapidly to a wide range of structures without recourse to the computer, very simple, approximate formulae have been developed for estimating the point mobilities of common structural elements.

The approach is based upon the fact that at high frequenc es, typical sub-structures composed of beam and plate-type elements do not exhibit clearly defined resonances and anti-resonances in their point mobility. In such cases, the finite practical structure may be approximated to by an equivalent infinite structure with a very simple curve for the point mobility characteristic which well represents the average behaviour of the point mobility of the finite structure [5,6]. This approach may be used for the prediction of point mobilities of practical structures at high frequencies where modal contributions have coalesced. At low frequencies, where resonances are well separated, this approximation is much less accurate although it is still valuable since it gives an average level of the frequency response magnitude. If sources with discrete frequency content are being considered, the upper magnitude limits of mobility due to resonant behaviour must be considered but if broad band excitation only is of interest, mean square responses can be estimated to an acceptable degree of accuracy using the simple formulation.

The driving point mobilities are given for force and moment excitation of various structural elements in [7]. The mobility formulae given apply to infinite structures within which no resonant behaviour can occur. The point mobilities of finite structures will approximate to these values at high frequencies above the frequency at which the frequency interval between resonances is less than the bandwidth of a resonance; criteria for this condition are also tabulated in [7]. When a finite structure is being represented by an equivalent infinite structure the largest error in the estimated mobility will occur at a resonance frequency in the low frequency region. The largest peaks in the mobility of finite beams and plates have been calculated; these are included in [7] together with a list of the moduli of the ratios of the peak point mobility of the finite structure to the point mobility of the infinite structure which in most cases, is inversely proportional to the loss factor. The formulae developed represent a very simple guide for the design of machinery seatings and isolation systems in the absence of experimental data and without the use of large computational facilities which might only, if necessary, be subsequently used in a limited way.

3.0 POWER FLOW THROUGH ISOLATORS AND STRUCTURES

The objective in applying a vibration control procedure, such as isolation, should be to reduce the power flow from a source to the receiver. The power flow (P) into a structure may be expressed via the point mobility according to the nature of the source in the following way:-

For a force (F) source, $P = \frac{1}{2}|F|^2 \; \mathrm{Re}\,\{M\}$

for the velocity (V) source, $P = \frac{1}{2}|V|^2 \; \frac{\mathrm{Re}\,\{M\}}{|M|^2}$

where Re{M} denotes the real part of the mobility of the structure. In the isolation problem, in its most simple form, the source could perhaps, for example, be represented by a rigid mass (m) supported on a simple massless spring isolator of stiffness (k) which isolates the source from the receiver. Now, the power flow into a substructure may be estimated by using one of the simple expressions for the receiver mobility given by the infinite structural representation. For example, if a mass-spring system is coupled to a plate then:-

$$P(\omega) = \frac{|F|^2}{2m\omega_o} \; \frac{\alpha}{|1 - \Omega^2 + i\Omega\alpha|^2}$$

where $\alpha = \frac{m\omega_o}{\sqrt{8 B_p \rho h}} = m\omega_o \times$ (foundation mobility)

where $\omega_o^2 = k/m$ is the undamped natural frequency of the system with the flexible foundation clamped

$\Omega = \frac{\omega}{\omega_o}$ where $\omega =$ frequency in radians/sec

B_p = bending stiffness of a plate = $Eh^3 / 12(1-\gamma^2)$

E = modulus of elasticity　　ρ = density
γ = Poisson's ratio　　h = plate thickness

Similar expressions can be derived for velocity sources and the various sub-structures, [7,8]. Moment excitation as well as force excitation of foundations is an important consideration in this type of analysis. This is because the method in which a machine is mounted often results in moments being the mechanism by which the foundation is excited. For example, moment excitation occurs when there is rocking of a machine on a horizontal foundation or alternatively when a machine is mounted on a cantilevered support, creating flexural wave motion in a structural member.

Studies have been made [9] of methods for measuring the power flow through vibration isolators. Several methods have been proposed for measuring the power input to a structure; for example measurement may be made via accurate measurement or estimation of the real component of the point mobility of the structure. It has been shown [9] that for frequency averaged power calculations, the real component of mobility of a finite structure may be represented adequately by that of an equivalent infinite structure.

Both theoretical and experimental approaches have been made in the study of structural power flow in order to determine the relative importance of various power transmission mechanisms. Theoretical studies carried out so far have been on necessarily simple models. Power transmission in grillage-type structures has not been examined. The wave propagation and power flow due to force and moment excitation applied to the beam has been studied at the driving point and in the far field of an infinite plate with a simple, line stiffener [10]. It was found that the motion at the driving point is largely controlled by the beam. For example, if the beam is excited by a force or moment so that flexural wave motion is induced, then the power transmitted by these waves will initially be associated with the beam. As the waves travel away from the source they radiate into the plate so that in the far field more power is transmitted by the plate than the beam.

Experimental techniques have also been developed for the measurement of power flow (intensity) in one and two dimensional structures [11]. The methods involve mounting transducer arrays on the structure and estimation of time and spatial derivatives of the motion, the latter involving the use of finite difference methods. The technique has been used to estimate the power flow in a plate [12] excited by a point source, a digital computer having been used to perform the necessary data manipulation. It is hoped that in the future, as a result of work currently underway, simplified multipoint measurement methods for intensity determination will be developed which will form the basis of an approach to the problem of vibration transmission mechanism identification in built-up structures.

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