Proceedings of The Institute of Acoustics

A Numerical Model of the Parametric Radiator

R. H. Mellen and M. B. Moffett
Naval Underwater Systems Center
New London, Connecticut 06320

Abstract: The field of a parametric radiator can be approximated as a complex volume integral over a spherically diverging source function. Rapid numerical convergence is obtained by modifying the contour of integration. The method also permits inclusion of an amplitude taper function which accounts for saturation effects. Results of a computer study which illustrate various nearfield-farfield transition phenomena are presented.

Because parametric radiators can behave like end-fire arrays of considerable length, field models using infinite range approximation are often insufficiently accurate.[1] We have devised a numerical model[2] that appears to give good account of effects observed at ranges greater than the primary Rayleigh distance but still effectively finite. In this region, we find simply that apparent source levels tend to be higher and beamwidths narrower than the infinite range model predicts. These discrepancies can be thought of mainly as a range error wherein the range to the projector is used rather than the range to the effective "center of gravity" of the array.

In our finite range model we approximate the source function as a spherically divergent wave of arbitrary beam pattern. The difference frequency pressure can then be written in dimensionless terms as the volume integral

$$P(\vec{R}) = \int_V dV\, Q(\vec{r})\,|\vec{\zeta}|^{-1} \exp(-ik\,|\vec{\zeta} + \vec{r}|) \qquad (1)$$

where $\vec{R}$ is measured from origin to field point, $\vec{r}$ from origin to source point and $\vec{\zeta}$ from source point to field point with k the secondary wavenumber. The source function, for simplicity, is approximated as

$$Q(\vec{r}) \approx Q_0\, D^2(\theta,\phi) \exp(-2\alpha r)\, E^2(r) \qquad (2)$$

where $r = |\vec{r}|$, D is the directional pattern of the primary wave with azimuthal angle ϕ and polar angle θ and E^2 describes the extra loss due to finite amplitude effects.

The range part of Equation 1 can be written as an integral in the complex variable z

$$F = \int_{z_0}^{\infty} dz\, E^2(z)\, \exp(-z)\, (z^2 + B^2)^{-\frac{1}{2}} \qquad (3)$$

where $z = u + iv = \alpha r + ik\,((r^2 + a^2)^{\frac{1}{2}} + r)$, $(z_0 = u_0 + iv_0$ for $r = 0)$, $B^2 = ik\alpha a^2$ and a is the perpendicular distance from field point to line of integration. Rapid numerical convergence of Equation 3 can be accomplished by changing the path of integration to $z = u + iv_0$ where u ranges from u_0 to infinity. Since the two contours can be connected at infinity without encircling poles or cutting branch lines, the results are the same. Finally we write Equation 1 as the two dimensional angular convolution

$$P(R,\theta,\phi) = D_0(\theta,\phi)\; D^2(\theta',\phi') \circledast F(\cos\nu) \qquad (4)$$

where $\cos\nu = \cos\theta' \cos\theta + \sin\theta' \cos(\phi' - \phi)$ and $D_0(\theta,\phi)$ is the aperture correction.[3]

Our procedure is to calculate and store a table of complex values of F for given values of the parameters k, α and R. Since F has a logarithmic singularity at $\nu = o$ we set the initial value of ν small compared to the primary beamwidth. Convolution with the primary beam pattern is accomplished by two-dimensional angular integration with tabular interpolation to provide required values of F. The field point angle is then changed and the procedure repeated.

Besides the usual difficulties in getting complicated computer codes to work automatically we found two major problems. First, the saturation taper has a pole in the complex plane that can fall fairly close to the modified path of integration. When this happens we have an abnormally large contribution to F in the neighborhood. This problem was solved by devising an adaptive Simpson's rule method in which the number of points is increased automatically in the critical region. The second problem involves the convolution program in which premature dropout may occur. On the other hand, it may bog down in noise if convergence criteria are too severe. This problem was overcome by various refinements of the integration procedure and convergence criteria. Since the adaptive Simpson's method proved too slow for the convolution, a piecewise 48 point Gaussian quadrature method was finally adopted. This gives good results for secondary beamwidths as small as 1°.

The major effects of finite range are illustrated in figure 1 which shows the difference frequency beam patterns calculated for a hypothetical conical source of half-width θ_0 and $2\alpha R = 1$. The apparant source level is the dB//Westervelt's source level and the beam angle re Westervelt half-angle θ_w. Small values of the parameter θ_0/θ_w then correspond to nearfield type sources in which the primary wave acts as if it were collimated. In this regime source levels are higher and beamwidths narrower than for the Westervelt case which is shown by the dashed line.

For large values of the parameter θ_0/θ_w the source acts spherically divergent. Apparent source levels are lower and beamwidths wider than for the Westervelt case. In the limit the beam tends toward the "square" of the conical primary beam pattern. Figure 2 shows a comparison of the calculated and experimental beam patterns for 90 cm diameter pistons of mean primary frequency 65 kHz and 3.5 kHz difference frequency. The range was 80 m or 2.9 Rayleigh distances. The dotted curve is calculated for 100 Rayleigh distances which is effectively infinite. The apparent source level is seen to be 2 dB higher and the beam pattern correspondingly narrower than for the "infinite" range conditions. Agreement between experimental points and the calculated curve is seen to be reasonably good.

In figure 3, we show the effect of primary wave saturation. The projector in this case is a 25 cm diameter piston with 250 kHz mean primary frequency and 5 kHz difference frequency. The curves are calculated for the appropriate primary levels. (Only one matching adjustment of absolute level was required) Agreement between theory and experimental points is also good. The effects of nonlinear attenuation appear in a leveling off of secondary source level and widening of the beam as the primary level is increased.

We conclude from these results that, within experimental error, the model accounts for finite range and finite amplitude phenomena.

References

1.) M. B. Moffett and R. H. Mellen, "Model for Parametric Acoustic Sources," J. Acoust. Soc. Am. 61; 325-337. (1977)

2.) R. H. Mellen and M. B. Moffett; "A Numerical Model for Calculating the Nearfield of a Parametric Acoustic Source." J. Acoust. Soc. Am. 63; 1622-1624. (1978)

3.) M. B. Moffett and R. H. Mellen; "On Parametric Source Aperture Factors." J. Acoust. Soc. Am. 60; 581-583. (1976)

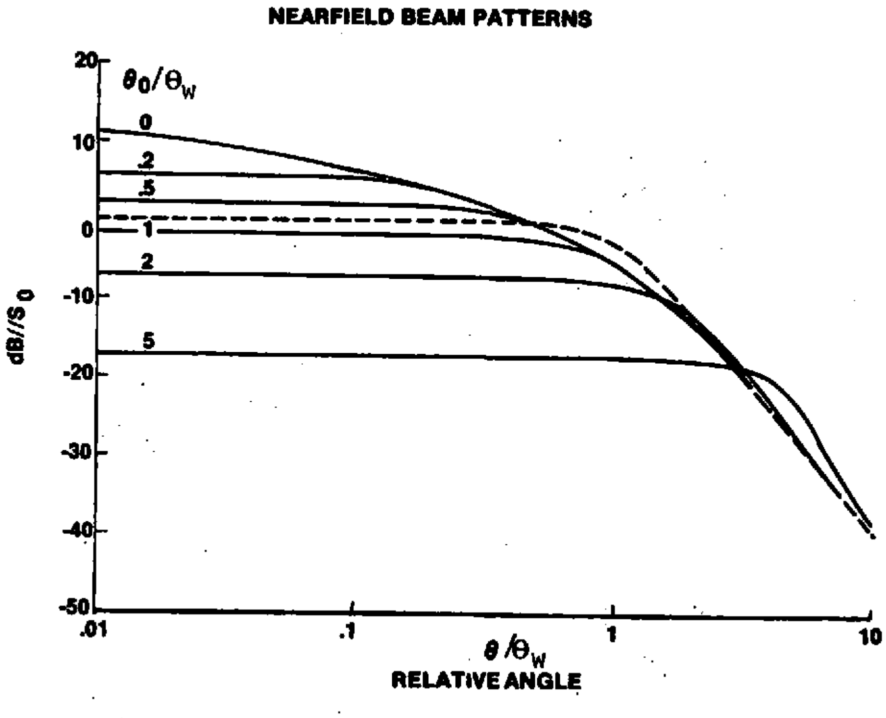

Figure 1. Secondary beam patterns for a conical source of half-angle θ_0 and $2\alpha R = 1$. Source levels are dB//Westervelt source level and beam angle re Westervelt's half-angle θ_W.

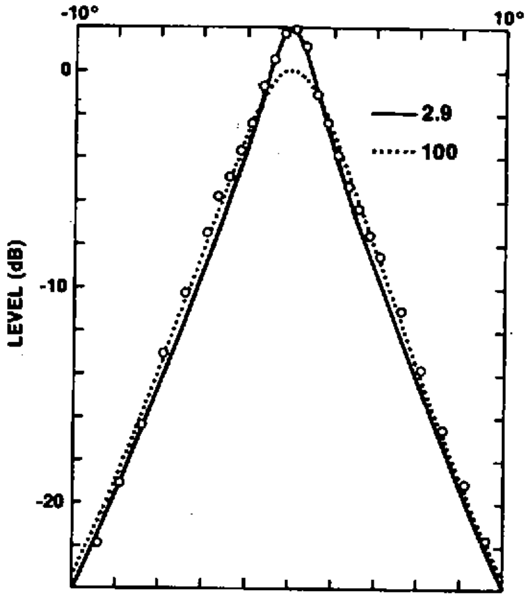

Figure 2. Experimental and theoretical beam patterns for 90 cm diameter piston with mean primary frequency 65 kHz difference frequency 3.5 kHz and R/Ro = 2.9.

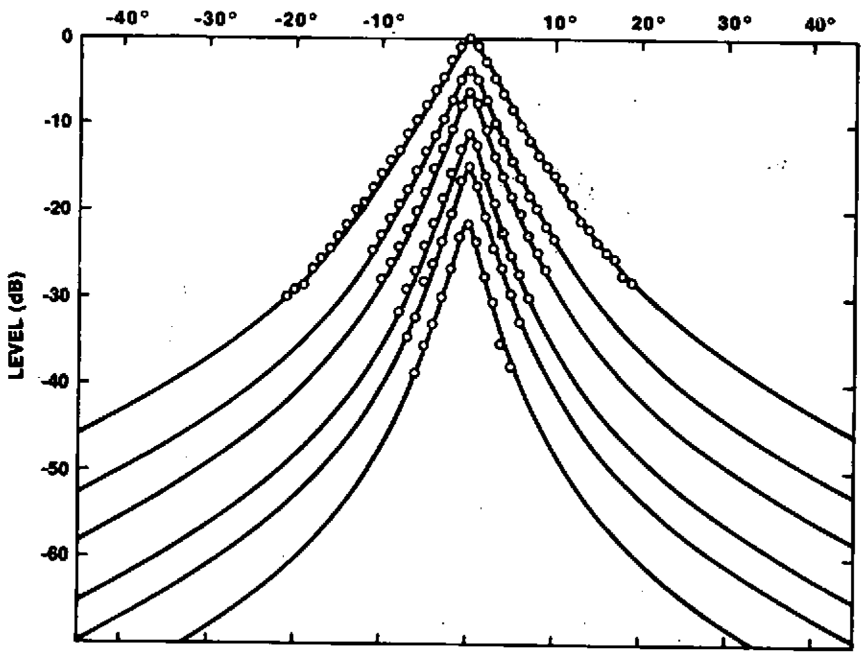

Figure 3. Experimental and theoretical beam patterns for 25 cm diameter piston with mean primary frequency 250 kHz difference frequency 5 kHz and R/Ro = 10 showing effects of primary wave saturation on source level and beamwidth.