ACTIVE VIBRATION ISOLATION BY CANCELLING BENDING WAVES

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ABSTRACT

Two active point forces applied to a vibrating beam may be used to cancel propagating and evanescent bending waves in a particular region of the beam, thereby isolating that region from the effects of disturbing forces acting elsewhere on the beam. We investigate implications of applying this infinite-beam theory to a finite system, where discontinuities cause reflections of both bending wave types, and conclude that a simple controller may be implemented without the need for high-order approximation of the dispersive transmission characteristics of bending waves. The high sensitivity of the wave-cancellation approach to error demands high accuracy in the identification of the system to be controlled, and in the implementation of the resulting controller. As idealised models would be inadequate for this purpose, a frequency domain technique for realising such a controller from measured response data is outlined. The same response data may be used to predict the maximum attenuation levels attainable in a noisy system.

This work culminated in the application of these techniques to the problem of isolating a cantilever beam from vibrations caused by a random, broadband disturbing force near the fixed end. Although component inaccuracies in the particular apparatus used in the experiment allowed only 6 dB of attenuation to be achieved over a bandwidth of 200 Hz, good agreement between predicted and experimental results was shown.

1. INTRODUCTION

When vibration level restrictions exist in a sensitive region of a structure subjected to external excitation, active control may be used to achieve isolation of that region from the effects of the disturbance. Isolation systems described to date have either used feedback-driven active elements to physically separate disturbance and isolated regions [1], or use knowledge of the disturbance itself to generate control forces to cancel the effects of that disturbance [2–4].

An alternative to the above methods of isolation is offered by the application of the principles of anti-sound as used in the prevention of sound transmission along a duct [5]. Any continuous element in a vibrating structure may be regarded as a waveguide for elastic waves. The geometry of the problem considered here is shown in Figure 1: the aim of the active control system is to isolate the free end of a cantilever beam of unit mass per unit length m and bending stiffness B from the effects of an external disturbance F_d. Bernoulli-Euler bending theory describes the Fourier components of lateral displacements along the beam as solutions of the differential equation

$$m\frac{d^4W(x,\omega)}{dx^4} - \omega^2 B\,W(x,\omega) = Q(x,\omega) \tag{1.1}$$

where $Q(x,\omega)$ is the force distribution acting on the beam. Motion at co-ordinates at which $Q(x,\omega) = 0$ can then be described as the superposition of wave components:

$$W(x,\omega) = W^+(\omega)e^{-ikx} + W_n^+(\omega)e^{-kx} + W^-(\omega)e^{ikx} + W_n^-(\omega)e^{kx} \tag{1.2}$$

The coefficients W^+, W_n^+, W^- and W_n^- represent amplitudes of propagating and evanescent waves in the positive and negative x-directions, and the frequency dependent wavenumber

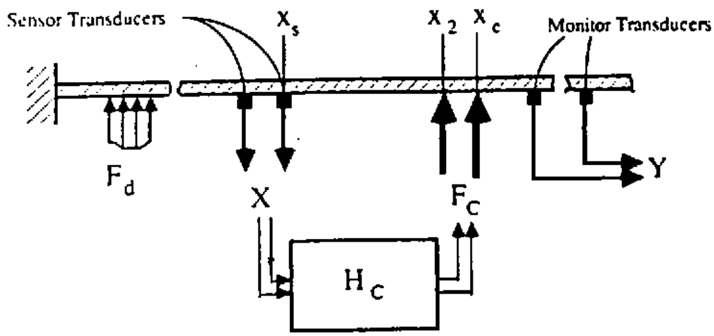

Figure 1: Geometry of Cantilever Isolation problem

k characterises the vibration transmission behaviour of the beam. It is the solution to the dispersion relation for flexural waves $k = (m/B)^{\frac{1}{4}}\sqrt{\omega}$.

The behaviour of the structure may be altered by actively introducing cancelling waves in analogy to the anti-sound method. The possibility of using these concepts in the control of flexural vibrations was recognised by Scheuren [6] in the design of an active system for absorbing propagating flexural wave components. His scheme ignored the effect of the evanescent flexural waves generated at discontinuities in the structure and by the control forces themselves. Active vibration isolation is implemented by detecting waves generated by the disturbance F_d as they pass through sensor coordinate x_s, then driving controlling forces $\mathbf{F}_C$ to cancel both propagating and evanescent waves entering the isolation region in the positive x-direction. This cancellation scheme provides isolation of the region of the beam beyond the cancelling forces from effects of any disturbance excitation to the left of control force location x_c, and is independent of the structure in the region to be isolated, offering great flexibilty in implementation. These principles were introduced in [7] and further investigated by Mace [8]. The systems synthesised in these papers were described in terms of flexural wave models, and the results presented suggested firstly that irrational controller transfer functions were required to match the dispersive wave transmission characteristics and secondly that the practical application of the isolation approach would require identification of these characteristics in finite systems where measurable vibrations are a superposition of four flexural wave components. As the performance of the feedforward wave-cancellation system is limited only by the accuracy of matching controller transfer functions to the beam transmission characteristics, these requirements would limit the effectiveness of isolation to the extent that the proposed scheme would not compete with well-known feedback methods.

The work described below involved the development of the idea of using active flexural wave cancellation to isolate one part of a finite beam from disturbances introduced elsewhere in the beam, to the stage where realistic implementation became possible. The configuration of the isolation system was chosen to remove the requirement for irrational controller transfer functions. Limitations introduced by error and noise in an imperfect control system were quantified and a procedure for the design of the controller using measured frequency response functions was developed. This made implementation possible without the use of detailed (and therefore expensive) theoretical models. The work culminated in the application of active isolation to a cantilever beam subjected to continuous, broadband excitation.

2. IMPLEMENTATION OF ACTIVE ISOLATION IN A FINITE SYSTEM

2.1 Wave-Cancellation Synthesis of Isolation System

We consider the block-diagram representation of the wave behaviour of the system of Figure 1 in the frequency domain (Figure 2). The wave components of Equation (1.2) are grouped into positive- and negative-going vectors $\mathbf{W}_X^+$, $\mathbf{W}_X^-$, $\mathbf{W}_C^+$ and $\mathbf{W}_C^-$ containing propagating and evanescent components defined at the sensor and controller reference coordinates x_s and x_c respectively.

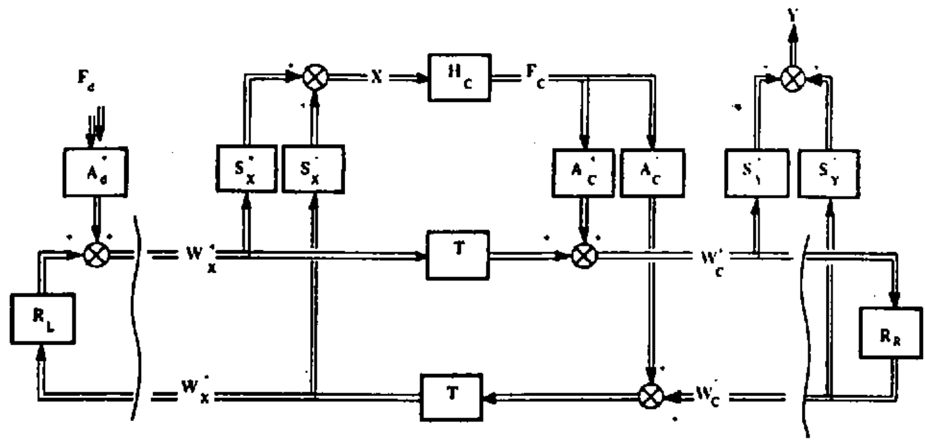

Figure 2: Flexural wave representation of beam vibration response

The disturbance excitation introduces flexural waves into the system according to the semi-infinite beam admittance matrix $\mathbf{A}_d^+$ which includes effects of wave reflections at discontinuities to the left of sensor location x_s. Transmission matrix $\mathbf{T}$ describes the dispersive transmission characteristics of the beam between sensor and controller regions; $\mathbf{A}_C^+$ and $\mathbf{A}_C^-$ are the infinite-beam admittance matrices describing waves introduced by the two control forces in vector $\mathbf{F}_C$. The "wave-superposition" matrices $\mathbf{S}_X^+$, $\mathbf{S}_X^-$, $\mathbf{S}_Y^+$ and $\mathbf{S}_Y^-$ represent the combination of wave components into the physical quantities measured by the controller sensors and further transducers monitoring vibration levels in the region to be isolated $x \geq x_c$. $\mathbf{R}_L$ and $\mathbf{R}_R$ are reflection matrices describing transmission and reflection to the left of the sensor and right of the active forces respectively. Specific entries in each matrix defined are not relevant at this stage, as they depend on the specific structure involved; we do however note that all matrices contain irrational functions of frequency resulting from the dispersive nature of flexural waves and are thus not suitable for direct implementation in signal processing hardware.

The isolation requirement for the active forces may be written as

$$\mathbf{W}_C^+ = \mathbf{A}_C^+ \mathbf{F}_C + \mathbf{T}\mathbf{W}_X^+ = 0 \qquad (2.1)$$

If the active system is arranged so that the disturbance, sensor and active force regions (each simply connected) do not overlap, we can express the wave components $\mathbf{W}_X^+$ in terms of causal functions of both the sensor output vector $\mathbf{X}$ and the control forces $\mathbf{F}_C$. In terms of admittance, transmission and reflection matrices:

$$\mathbf{W}_X^- = \mathbf{T}\mathbf{R}_R\mathbf{T}\mathbf{W}_X^+ + (\mathbf{T}\mathbf{A}_C^- + \mathbf{T}\mathbf{R}_R\mathbf{A}_C^+)\mathbf{F}_C. \qquad (2.2)$$

Equations (2.1) and (2.2) may be combined to yield an expression for the forward-going waves at the sensor position in terms of the sensor signal $\mathbf{X}$ and the control forces $\mathbf{F}_C$:

$$\mathbf{W}_X^+ = \mathbf{S}_X^{+^{-1}}\mathbf{X} - \mathbf{S}_X^{+^{-1}}\mathbf{S}_X^-\mathbf{T}\mathbf{A}_C^-\mathbf{F}_C. \qquad (2.3)$$

The wave-detection phase of the control system is thus implemented by subtracting contributions to $\mathbf{X}$ from the known control forces. Finally substituting this expression into the control requirement (2.1) gives the transfer matrix of a feed-forward isolation system:

$$\mathbf{H}_C = -(\mathbf{I} + \mathbf{A}_C^{+^{-1}}\mathbf{T}\mathbf{S}_X^{+^{-1}}\mathbf{S}_X^-\mathbf{T}\mathbf{A}_C^-)^{-1}\mathbf{A}_C^{+^{-1}}\mathbf{T}\mathbf{S}_X^{+^{-1}}. \qquad (2.4)$$

This result emphasises the fact that the isolation system is independent of the structure beyond the controlling forces: effects of this region of the structure, described by reflection matrix $\mathbf{R}_R$, have been cancelled from the governing equations just as the flexural waves have been cancelled by the active forces.

2.2 Isolation in Terms of Measurable Quantities
Although the above formulation reveals useful properties of the proposed isolation scheme, it is not yet in an easily implementable form. To synthesise the controller in terms of measurable quantities, we use the second set of transducers monitoring physical quantities in the region to be isolated, with outputs $\mathbf{Y}$. Under the original assumption that all vibrations in this region result from waves transmitted through the reference co-ordinate x_c, the monitor vector $\mathbf{Y}$ may be expressed in terms of superposition matrices and wave components at x_c:

$$\mathbf{Y} = \mathbf{S}_Y^+\mathbf{W}_C^+ + \mathbf{S}_Y^-\mathbf{W}_C^- \qquad (2.5)$$

Negative-going waves $\mathbf{W}_C^-$ must result only from reflections of components $\mathbf{W}_C^+$ in the isolation region; including this effect in (2.5) gives

$$\mathbf{Y} = (\mathbf{S}_Y^+ + \mathbf{S}_Y^-\mathbf{R}_R)\mathbf{W}_C^+. \qquad (2.6)$$

Transducer types and positions must be selected to ensure that linearly independent measurements are possible. The requirement that the forward-going wave components be reduced to zero is then identical to the requirement that two or more quantities resulting from the superposition of these components and their reflections be reduced to zero.

Including reflections described by matrices $\mathbf{R}_L$ and $\mathbf{R}_R$ closes the loop travelled by flexural waves in a finite structure. The block diagram of Figure 2 may then be reduced to that of Figure 3 (with noise inputs Z_X, Z_Y and Z_C set to zero), where all transfer functions relate disturbance and controller inputs to the outputs $\mathbf{X}$ and $\mathbf{Y}$ produced by each input. We

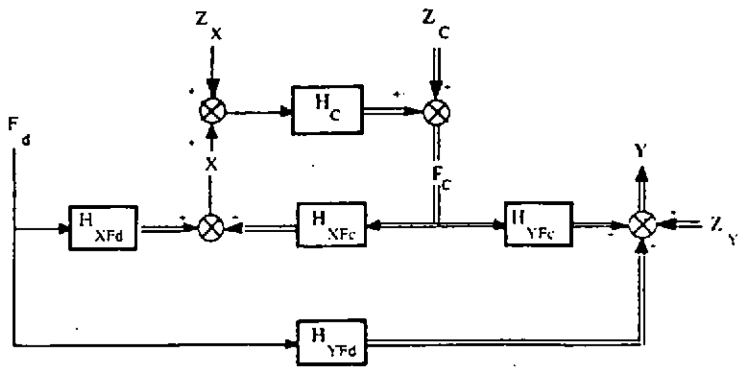

Figure 3: Beam vibration response in terms of measurable quantities

can now express controller inputs $\mathbf{X}$ and monitor outputs $\mathbf{Y}$ in terms of disturbance and controlling forces:

$$\mathbf{X} = \mathbf{H}_{XF_d}\mathbf{F}_d + \mathbf{H}_{XF_c}\mathbf{F}_C \qquad (2.7)$$

$$\mathbf{Y} = \mathbf{H}_{YF_d}\mathbf{F}_d + \mathbf{H}_{YF_c}\mathbf{F}_C \qquad (2.8)$$

Applying the isolation criterion developed in terms of monitor outputs $\mathbf{Y} = 0$, a formulation for controller transfer function matrix $\mathbf{H}_C$ in terms of measurable system input-output response functions emerges:

$$\mathbf{H}_C = -\left(\mathbf{I} - \mathbf{H}_{YF_c}^{-1}\mathbf{H}_{YF_d}\mathbf{H}_{XF_d}^{-1}\mathbf{H}_{XF_c}\right)^{-1}\mathbf{H}_{YF_c}^{-1}\mathbf{H}_{YF_d}\mathbf{H}_{XF_d}^{-1} \qquad (2.9)$$

This expression for the controller transfer function is exactly equivalent to that of (2.4) above — the two formulations are related through the equivalence of the block diagrams of Figures 2 and 3.

It is evident that with formulation (2.9) for the controller transfer functions, an active isolation system based on the principle of flexural wave cancellation may be designed using response functions measuring the superposition of underlying wave components. Any requirement for detailed modelling of the structure on which such an isolation system is to be installed is thus removed. Details of the dispersion relation resulting from different theories of dynamic bending, such as the Bernoulli-Euler or Timoshenko theories, need only be considered when the order of the isolation system, corresponding to the number of modes of wave activity producing significant vibration levels in the beam, is being established.

2.3 A Parametrization for Controller Transfer Functions

A solution of equation (1.1) for finite beam motion as a superposition of natural modes rather than wave components may be applied to each of the physical response functions in (2.9), giving each transfer function as an infinite-order rational polynomial

$$H(\omega) = \frac{K \prod_{k=1}^{\infty}(\omega^2 - z_k^2)}{\prod_{l=1}^{\infty}(\omega^2 - p_l^2)} \tag{2.10}$$

Note that the denominator of each transfer function is the same: the poles p_l describe the natural frequencies of the structure. The zeroes z_k depend on specific sensor locations.

When infinite-order rational polynomials of the form of (2.10) are substituted for each of the transfer functions in the controller design equation (2.9), the denominator polynomial of the original system ($\prod_{l=1}^{\infty}(\omega^2 - p_l^2)$) disappears from the controller denominator $\det|\mathbf{I} - \mathbf{H}_{YF_c}^{-1}\mathbf{H}_{YF_d}\mathbf{H}_{XF_d}^{-1}\mathbf{H}_{XF_c}|$. The resonant behaviour of the original structure without control is thus irrelevant to the design of the isolation system. Because all the factors involved in (2.9) result in real or complex conjugate pole or zero pairs, the infinite-order rational polynomial resulting from their manipulation must similarly result in real or complex conjugate pole and zero pairs. We therefore conclude that controller transfer functions to be implemented in our isolation system do not require the approximation of irrational, dispersive transmission or admittance characteristics suggested by the original wave formulation of the isolation system.

In implementing these transfer functions, the infinite order polynomials must be truncated, leaving only those poles and zeroes within the bandwidth for which isolation is to be achieved. For realistic bandwidths, a low-order model may be used to accurately represent the required transfer functions; the implication is that an inexpensive, low-order signal processor may fulfil the demands for effective active isolation.

When the geometry of the disturbance region ($x < x_c$) of the structure is fixed, a fixed relationship exists between the two forward-going wave components generated by F_d. This relationship may be used in the isolation system by reducing the sensor vector to a single element X, thereby reducing the order of the controller to a single-input two-output form. The dimensions of transfer matrices $\mathbf{H}_{XF_d}$, $\mathbf{H}_{YF_d}$ and $\mathbf{H}_{XF_c}$ in (2.9) are similarly reduced.

3. EXPERIMENTAL RESULTS

3.1 Description of Equipment

Experimental work was carried out on a thin cantilever beam of length 0.56m. The external disturbing force was applied to the beam by means of an elecrodynamic shaker, driven by broadband input signals. Active control forces were produced by two similar shakers, while the control sensor was implemented in the form of a strain-gauge pair, measuring dynamic bending strain of the beam at x_s. A second strain-gauge pair was used alongside a miniature accelerometer to monitor vibration levels in the region to be isolated between the controlling shakers and the free end of the cantilever. The signal processing stage of active control was implemented in the form of a two-channel real-time recursive digital filter running on a general-purpose minicomputer equipped with 12-bit analogue-digital and digital-analogue convertors. The speed of this computer limited the order of the filter to only 30 coefficients (for both channels) with a sampling time of 1000μs, giving a useful bandwidth of 500 Hz. To reduce the order of compensation required, no anti-aliasing filters were used on the input to the ADC as the random disturbance input was similarly limited to a 500 Hz bandwidth, but filters were included between the DACs and the control force shakers.

3.2 Attenuation of the Effects of a Broad-Band Disturbance

When the disturbance shaker was driven by a random noise input with a bandwidth from 0 Hz to 500 Hz, the response at one of the monitors Y_1 (the strain-gauge pair situated close to

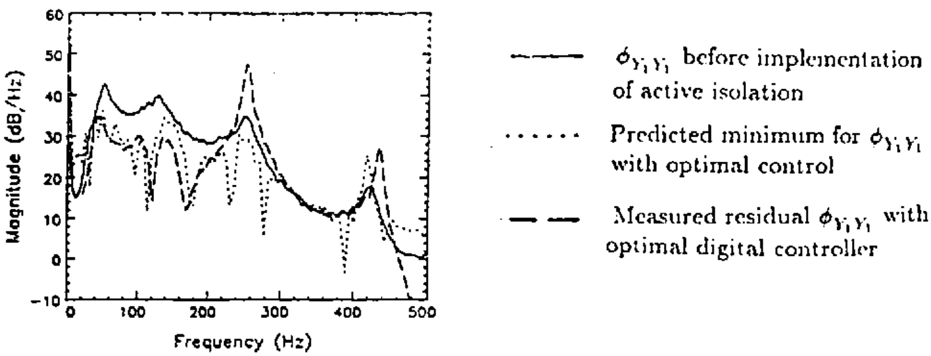

Figure 4: Autospectra of monitor output Y_1 with and without active isolation

controlling force F_{C_1}) was measured using standard spectral analysis techniques. The auto power spectral density for the monitor output with active control switched off, but with all hardware associated with the control system in place, is presented in Figure 4.

Uncorrelated noise in the system, consisting of transducer and instrumentation noise, inaccuracies in controller implementation and any non-linearities present, may be represented by inputs Z_X, Z_Y and Z_C in Figure 3. When active isolation is implemented in the noisy system, the minimum residual vibration level will be that resulting from these noise inputs. At frequencies where noise levels are high, control will be less effective and the optimal controller transfer function must be modified accordingly. This was achieved in the experimental system by using the minicomputer intended for digital filter implementation to generate random signals, of 500 Hz bandwidth, to drive the active force transducers in an open-loop test. The random disturbance excitation was simultaneously applied to the disturbance shaker from an external noise source. Input channels of the minicomputer collected response data from sensor output X and monitor outputs $\mathbf{Y}$ during this test; multi-input-multi-output digital spectral analysis then provided the spectra and correlated parts of the response functions required to derive the minimum residual auto power spectrum of monitor outputs with optimal active isolation implemented, and the optimal controller transfer functions themselves. Details of this procedure may be found in [9]. The residual autospectrum predicted for monitor output Y_1 with active isolation removing the maximum possible contribution from the disturbance force, as given by the noisy sensor input $X + Z_X$, is shown in Figure 4.

Coefficients to be implemented in the digital filter were found by parametrizing the compensation in the form described in Section 2.4 and fitting the frequency response of the parametrisation to the optimal transfer functions derived from the testing procedure described above. These transfer functions corresponded closely to the optimal transfer functions in the frequency range 0–200 Hz, but not for higher frequencies. Active isolation cannot be expected at these higher frequencies. Measurements carried out on the experimental rig with active isolation in operation bore this out: Figure 4 shows that up to 200 Hz, the residual auto spectrum at monitor Y_1 corresponds closely to that predicted from the measured data without control action. Above 200 Hz, phase errors in controller transfer functions caused vibration response levels to be increased by controller action. Because the controller gain was minimised over the upper frequency range, the system did remain stable.

For a random disturbance excitation covering a reduced bandwidth of 0–200 Hz, the reduction of mean square vibration levels in the region to be isolated by the active controller was found to be approximately -6 dB. The attenuation achieved was limited by noise and non-linearity in the apparatus used in experiments.

4. CONCLUSION

The wave cancellation approach to vibration isolation was useful in exploring the possibilities offered by the system as it led to the conclusion that isolation could be achieved without consideration of effects outside the transmission path to be blocked, and that the performance of the isolation system is independent of the structure in the isolated region. In implementation, however, the wave cancellation approach was abandoned as dispersive wave characteristics obscured the relatively simple feedforward compensation controller required. Instead, a formulation in terms of physical vibration responses enabled the optimal controller to be derived from measured data. Experimental tests of the implementation techniques developed confirmed their validity and emphasised that active vibration isolation by wave cancellation is a practical method, limited not by difficulties inherent in the wave-cancellation approach but instead by accuracy limitations of the hardware used in its implementation.

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