METHODS FOR MEASURING POWER INPUT TO A STRUCTURE

R.J. PINNINGTON

Institute of Sound and Vibration Research
University of Southampton

1. INTRODUCTION

Power flow techniques are being used increasingly to tackle vibration control problems. Three methods of measuring the power input to a structure are presented here together with experimental results. The three methods are: power input via a single force, power transmitted through a vibration isolator power absorbed by a finite structure subjected to undefined forces.

2. POWER INPUT TO A STRUCTURE BY SINGLE POINT EXCITATION

A force with a harmonic time dependence $|F|$ sin ωt, acting at a point on a structure of mobility $|\bar{M}| = |M|e^{i\phi}$ causes a velocity of $|V|$ sin $(\omega t+\phi)$ at that point. The time averaged input power P is given by

$$P = \frac{1}{T}\int_0^T |F|\ |V|\ \sin\omega t.\ \sin(\omega t+\phi)\ dt.$$

Integrating this expression leads to the alternative forms

$$P = \frac{1}{2}|F|\ |V|\cos\phi = \frac{1}{2}|F|^2.\ \mathrm{Re}\ \{\bar{M}\} = \frac{1}{2}\mathrm{Re}\ \{\bar{F}\bar{V}*\} = \frac{1}{2}|V|^2.\ \mathrm{Re}\ \{\bar{z}\}. \quad 2.1\text{-}2.3$$

where $\bar{z}$ is the point impedance.

Likewise, if the structure is driven by a force of random time dependence of spectral density G_{FF}, giving rise to an acceleration at that point of spectral density G_{aa}, the power/Hz is

$$P/Hz = (1/\omega).\ G_{FF}.\ \mathrm{Im}\ \{\bar{I}\} = (1/\omega).\ \mathrm{Im}\ \{\bar{G}_{Fa}\} = (1/\omega)\ G_{aa}.\ \mathrm{Im}\ \{\bar{A}\} \quad 2.4\text{-}2.6$$

where G_{Fa} is the cross spectrum between the force and acceleration, $\bar{I}$ is the inertance and $\bar{A}$ the apparent mass. Expressions 2.1-2.6 all represent practical means of measuring power for simple point excitation but only expressions 2.2 and 2.5 can be used for multipoint excitation as the other methods require definition of transfer mobility and impedance terms, which make them impractical.

3. THE DETERMINATION OF POWER TRANSMITTED THROUGH AN ISOLATOR VIA MEASUREMENT OF THE ACCELERATIONS AT THE ENDS OF THE ISOLATOR

A method of measuring the power transmitted through an isolator is presented, which requires a knowledge of the isolator properties and the accelerations at each end of the isolator.

The power transmitted through an isolator can be calculated with reference to Fig. 1, in which the end faces of the isolator are permitted to move with only one degree of freedom.

For harmonic excitation, the force at the base of the isolator, $\bar{F}_2$, can be written as

$$\bar{F}_2 = \bar{A}_{12}\ \bar{a}_1 + \bar{A}_2\ \bar{a}_2 \quad 3.1$$

METHODS FOR MEASURING POWER INPUT TO A STRUCTURE

where $\bar{A}_2$ and $\bar{A}_{12}$ are the isolator direct and transfer 'apparent mass'

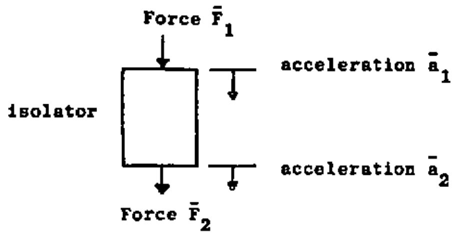

Figure 1

Using equation 2.2 the transmitted power can be written as $(1/2\omega)\,\bar{F}_2\,\bar{a}_2^*$ which on substituting for $\bar{F}_2$ gives

$$P \;=\; (1/2\omega).\;\; \mathrm{Im}\;\{\bar{A}_{12}\,\bar{a}_1\,\bar{a}_2{}^* + \bar{A}_2\,|\bar{a}_2|^2\}. \qquad 3.2$$

In practice the second term in { } is small enough to be neglected. Therefore the time averaged power is approximately

$$P \;\simeq\; (1/2\omega).\;\; \mathrm{Im}\;\{\,\bar{A}_{12}.\,\bar{G}_{a1\;a2}\,\} \qquad 3.4$$

where $\bar{G}_{a1\;a2}$ is the cross spectral density of $\bar{a}_1$ and $\bar{a}_2$. This expression can be used to measure the total vibration power transmitted from a multisupported machine by summing the individual power contribution through each isolator. Only the transfer apparent mass of each isolator is required, which at low frequencies is simply

$$\bar{A}_{12} \;=\; (K/\omega^2) + i\eta\;\; K/\omega^2$$

where K is the dynamic stiffness and η the hysteretic damping coefficient.

4. MEASUREMENT OF POWER ABSORBED BY A FINITE STRUCTURE SUBJECT TO MULTIPOINT EXCITATION

A method of measuring the power absorbed by a finite structure subjected to undefined forces is presented. The method is only applicable when the structure is sufficiently lightly damped for the vibrations to be governed by modal behaviour. Under this condition the power input to the structure can be equated to the sum of the powers absorbed by each vibrational mode. The power absorbed by each vibrational mode is calculated from the resonance peak value of the point or transfer inertance at selected positions on the structure and the acceleration spectra at those points. In order to gain an estimate of the power levels between resonances, cubic spline curves are fitted through the resonance peak inertance values. Experimental work has been carried out on a damped plate in order to study the practicability of the method.

4.1 General Expressions: if a structure is excited by N forces $\bar{F}_1 e^{i\omega t}$ ... $\bar{F}_R e^{i\omega t}$ ... $\bar{F}_N e^{i\omega t}$, the generalised force to the p'th mode f_p of the structure can be written as

$$f_p \;=\; \sum_{R=1}^{N} F_R\;\alpha_{Rp}\,, \qquad 4.1$$

where α_{Rp} is the eigenvector of the p'th mode at position R. The velocity at any point s, $\bar{V}_s$ is given as

$$\bar{V}_s = \sum_{p=1}^{m} \bar{M}_p \; \alpha_{sp} \cdot \bar{f}_p \qquad 4.2$$

where

$$\bar{M}_p = (1/m_p)(\eta\omega\omega_p^2 + i\omega(\omega_p^2 - \omega^2))/((\omega_p^2 - \omega^2)^2 + (\eta\omega_p^2)^2) \qquad 4.3$$

where m_p and ω_p are the generalized mass and angular natural frequency of the p'th mode; η is the loss factor.

From equation 4.3 an important relationship can be written.

$$|M_p|^2 = \text{Re}\;\{M_p\}.\;\hat{M}_p \qquad 4.4$$

where $\hat{M}_p$ is the resonance peak value of M_p when $\omega = \omega_p$.
The power input to the system from N forces can be expressed as

$$P = \frac{1}{2} \sum_{R=1}^{N} \text{Re}\;\{\,F_R^* \, V_R\,\},$$ which on substituting from equations 4.2, 4.1 becomes

$$P = \frac{1}{2} \sum_{p=1}^{m} |f_p|^2 \;\text{Re}\;\{M_p\}. \qquad 4.5$$

Thus it can be seen that the total power input is the sum of the powers to each mode.

4.2 Estimating power input from velocities at two points

Using equation 4.2 the product of the velocities at two arbitrary points r and s can be written as

$$\bar{V}_s \bar{V}_r^* = \sum_{\substack{p=1\\ p=q}}^{m} |M_p|^2 . \,\alpha_{sp} . \alpha_{rp} \, |f_p|^2 + \sum_{\substack{p=1\\ p\neq q}}^{m} \sum_{q=1}^{m} \bar{M}_p \bar{M}_q^* \, \bar{f}_p \bar{f}_q^* \, \alpha_{rp} \alpha_{sq} \qquad 4.6$$

(i) At frequencies near to the p'th mode only the p'th term in the left hand summation contributes significantly to the velocity product, provided the modes are well separated. Making use of equations 4.4 and 4.5 the power to the p'th mode, P_p, near to resonance can be written as

$$P_p = (\tfrac{1}{2})\; |\bar{V}_s \bar{V}_r^*| . \;/\; \hat{M}_{prs} \quad \text{if} \quad r \neq s \qquad 4.7$$

or

$$P_p = (\tfrac{1}{2})\; |V_s|^2 \;/\; \hat{M}_{ps} \quad \text{if} \quad r = s \qquad 4.8$$

where $\hat{M}_{ps}$ and $\hat{M}_{prs}$ are the resonance peak values of the point mobility at s and the modulus of the transfer mobility between points r and s.

(ii) Between two adjacent modes p and q the power cannot be accurately predicted, but from inspection of expression 4.6 it can be shown that an average estimate can be obtained using the arithmetic mean of the two adjacent peaks i.e.

$$P_p + P_q \simeq \frac{1}{2} \quad |\bar{V}_s \bar{V}_r^*| \quad 2 / (\hat{M}_{prs} + \hat{M}_{qrs}) \qquad 4.9$$

The proposed method:

For random excitations, the power/Hz absorbed by a structure at any frequency can be estimated from a two point measurement of acceleration using

$$E(P)/Hz = (1/\omega) \quad |\bar{G}_{rs}| \; / \; \hat{I}_{rs} \qquad 4.10$$

where $\bar{G}_{rs}$ is the cross spectrum of the acceleration signals between two points, and I_{rs} is the envelope of the peaks in the transfer inertance between points r and s. In practice this envelope is a cubic spline curve fitted through the resonance peaks, constrained to zero slope in the region of each resonance.

Likewise using a single point measurement

$$E(P)/Hz = (1/\omega) \quad |G_s| \; / \; \hat{I}_s \qquad 4.11$$

where G_s is the power spectrum of the acceleration at point s and $\hat{I}_s$ is the envelope at the peaks of the imaginary component of inertance. The limitation of this method is that individual resonance peaks must be detected, which is not always possible if two modes are close. However any errors resulting from the non detection of a mode leads to an underestimation of the power. Thus if two separate measurements are made the larger at any frequency can be taken as the more accurate. The transfer measurement method (equation 4.10) is preferable to the point measurement method because: the measurement is easier to make, the modes are more evenly excited and detectable.

References

1. R.J. Pinnington & R.G. White Power flow through machine isolators to resonant and non resonant beams JSV (1981) 75(2).

2. R.J. Pinnington . Using the envelope of resonance peaks to estimate power absorbed by a finite structure. ISVR Technical Report No. 115. (to be published).